
ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/1

PAPER 1

JUNE 2023 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

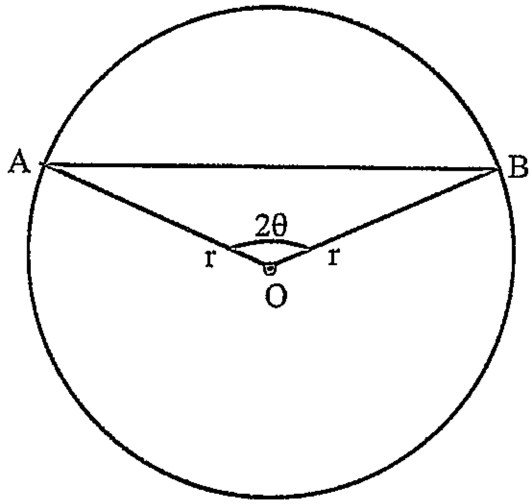
This question paper consists of 5 printed pages and 3 blank pages.

1 A sequence U_n is defined by $U_n = (-1)^{n+1} + 2$.

(a) Find the first 3 terms of the sequence. [3]

(b) State the behaviour of the sequence. [1]

2



A and B are two points on the circumference of a circle centre O and radius r.
The minor arc AB subtends an angle of 2θ radians at O.
If the area of the minor segment is one third of the area of the minor sector AOB, show that $2\theta = 3\sin\theta\cos\theta$. [4]

3 A variable p is inversely proportional to the square of $2q + 1$. Given that $p = \frac{3}{4}$ when $q = 2$. Find the value of q when $p = \frac{1}{3}$ [4]

4 (a) Solve the inequality $(1.05)^{n-4} < 60$. [3]

(b) Hence state the largest integral value of n, for the inequality in **(a)** [1]

5 Solve the following simultaneous equations:

$xy = 1$

$2x+y = 3$ [4]

6 Differentiate with respect to x

(a) $e^{2x}\sin 3x$, [2]

(b) $ln(1 + x^2)$. [2]

7 A curve C has parametric equations

$x = 2t + \frac{1}{2t}$, $y = 2t - \frac{1}{2t}$, where t is a parameter.

(a) Find x^2 and y^2 in terms of t. [4]

(b) Hence by evaluating $x^2 - y^2$ find the cartesian equation of C. [2]

8 **(a)** Express $\frac{1}{(\sqrt{x})^{\frac{4}{3}}}$ in the form x^n. [2]

(b) Solve $\frac{2^{1.5}}{2^7 \times 16} = 2^x$. [3]

9 **(a)** Divide $a^3 - b^3$ by $a - b$. [2]

(b) Given that $x^4 + x^2 + x + 1 \equiv (x^2 + A)(x^2 - 1) + Bx + C$,

determine the numerical values of A, B and C. [4]

10 Functions $h(x)$ and $g(x)$ are defined by:

$h(x) = 3x + 7$ for $x \in \mathbb{R}$

$g(x) = \frac{6}{2x-4}$ $x \neq 2$ for $x \in \mathbb{R}$

(a) Write down in terms of x

(i) $h^{-1}(x)$,

(ii) $g^{-1}(x)$ stating the values of x for which $g^{-1}(x)$ is not defined. [5]

(b) Sketch the graph of $h(x)$ and $h^{-1}(x)$ on the same axes making clear the relationship between the two graphs. [3]

11 Prove by Induction

$$\sum_{r=1}^{n} ap^{r-1} = \frac{a(p^n - 1)}{p - 1}$$

[7]

12 Given that the matrix $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & -5 & 3 \\ 4 & p & 7 \end{pmatrix}$,

find the

(a) value of p for which the determinant of $A = 54$, [3]

(b) inverse of matrix A using the value of p obtained in (a) above. [5]

13 (a) Use Taylor series to expand $\frac{1}{x}$ as series of ascending powers of $(x - a)$, up to and including the term in $(x - a)^3$. [6]

(b) Use the expansion to evaluate $\frac{1}{1{,}01}$ correct to 3 significant figures. [2]

14 (a) Express $4\cos\theta - 3\sin\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 360°$, giving α to the nearest degree. [3]

(b) Solve the equation $4\cos\theta - 3\sin\theta = 3$, for $0° \leq \theta \leq 360°$. [3]

(c) Find the least value of $\frac{1}{4\cos\theta - 3\sin\theta + 9}$. [2]

15 (a) Given that $z_1 = 3 + 4i$, and $z_2 = 1 + i$,

find

(i) $z_1 - z_2$, [2]

(ii) argument of $z_1 - z_2$. [2]

(b) (i) Solve the equation $z^2 - 4z + 53 = 0$, expressing the roots in the form $a + bi$, where $a, b \in \mathbb{R}$ [3]

(ii) State the sum and product of the roots of $z^2 - 4z + 53 = 0$. [2]

16 The points A, B and C have position vector $2\boldsymbol{i} - \boldsymbol{k}$, $3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $5\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ respectively.

(a) Calculate the angle ABC correct to the nearest 0.1° [4]

(b) Given that N is a point on vector $\overrightarrow{AC}$ such that $\overrightarrow{AN} : \overrightarrow{NC} = 1:2$, find the

(i) position vector of N, [3]

(ii) unit vector in the direction of $\overrightarrow{NC}$. [3]

17 The straight line ℓ has equation $2y - x - 5 = 0$.
Another straight line m passes through the point $P(-2, 6)$ and is perpendicular to ℓ.

Find the

(a) equation of m giving the answer in the form $ax + by + c = 0$, [3]

(b) co-ordinates of the point of intersection of ℓ and m. [3]

(c) perpendicular distance from P to the line ℓ. [2]

(d) It is given that the points Q(7; 6) and R $(-7; -1)$ lie on line ℓ.
Find the area of the triangle PQR. [3]

18 **(a)** Sketch on the same axis the graphs of $y = e^{-x}$ and $y = \sin x$ where

$0 \le x \le \pi$. [2]

(b) Hence state the number of roots of the equation $e^{-x} - Sin\, x = 0$ [1]

(c) Show that the smallest root of $e^{-x} - Sin\, x = 0$ lies between $x = 0$ and $x = \frac{\pi}{2}$. [3]

(c) Starting with $x_0 = 0{,}8$, use the Newton -Raphson method twice to estimate the smallest root of $e^{-x} - Sin\, x = 0$, giving the answer correct to three decimal places. [4]