Section A (40 marks).

Answer **all** questions in this section.

1 (a) Simplify $\frac{3^{2x-3}.81^{x+\frac{1}{2}}}{27^x}$. [3]

(b) Hence or otherwise solve the equation

$$\frac{3^{2x-3}.81^{x+\frac{1}{2}}}{27^x} = \frac{1}{5^{x-1}}$$ [4]

giving the answer in exact form.

2 (a) Show that 1 is a root of the polynomial $Q(x) = 2x^3 + 3x^2 - 4x - 1$. [2]

(b) Hence find the polynomial $P(x)$ such that $Q(x) = (x-1)P(x)$ [3]

(c) (I) Express $P(x)$ in the form $A(x+B)^2 + C$. [3]

(II) Hence state the minimum value of $P(x)$. [1]

3 The curved surface area, A of a cylindrical drum varies jointly as its circular base diameter d, and its height l. Given that the curved surface area of the drum is $2.64m^2$ when its base radius is 0,35m and its height is 1.2m,

(a) find the relationship between A, d and l, [5]

(b) calculate the radius of a cylinder whose surface area is $14,52m^2$ and height is 3m to 2 decimal places. [4]

4 Given that $f(x) = kx^2 + 3x + 3$ and $g(x) = kx + 7$, find the range of values of k for which $f(x) = g(x)$ has two distinct solutions. [5]

5 (a) Given that $2x^3 + ax^2 + x - 3 \equiv (bx^2 + c)(x-3)$, find the constants a, b and c. [4]

(b) Hence express $\frac{1+6x}{2x^3+ax^2+x-3}$ in partial fractions. [6]

Section B (80 marks)

Answer any **five** questions from this section. Each question carries **16** marks.

6 Functions $f(x)$, $t(x)$ and $h(x)$ are defined by

$f(x) = 12x + 20$
$t(x) = e^x$
$h(x) = 2x + 3$.

(a) (i) Find $th(x)$. [1]

(ii) On the same axes, sketch the graphs of $y = f(x)$ and $y = th(x)$. [5]

(b) Calculate by integration the area between the graph of $y = f(x)$, $y = th(x)$ and the lines $x = -1.5$ and $x = 0$. [3]

(c) (i) Estimate the area in **(b)** using the trapezium rule with 4 ordinates. [4]

(ii) Calculate the relative error made in estimating the area in **(b)** using the trapezium rule. [3]

7 **(a)** Prove the Identity $\frac{2\,Cot\,2\theta}{1-Tan^2\theta} \equiv Cot\,\theta$. [4]

(b) Hence or otherwise solve the equation $\frac{2\,Cot\,2\theta}{1-Tan^2\theta} = 2\,Cosec^2\theta - 1$, for θ in the range $0° \le \theta \le 360°$. [12]

8 Matrices A and B^{-1} are given by

$$A = \begin{pmatrix} 1 & 2 & 1 \\ x & 1 & x+1 \\ 2 & x-2 & 0 \end{pmatrix}, x > 0$$

$$B^{-1} = \frac{1}{8}\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

(a) Calculate the value of x given that the determinant of A is equal to 13 [5]

(b) Hence find

(i) A^{-1}. [7]

(ii) $(AB)^{-1}$. [4]

9 The sum of n terms of a sequence, S_n is given by

$S_n = \frac{n}{4}(5 - n)$, for $n = 1; 2; 3 \ldots$

(a) Find the second and third terms of the sequence. [4]

(b) Hence or otherwise, find the general term of the sequence in term of n. [3]

(c) Calculate the number of terms needed for the Sum, S_n, to be equal to -9. [5]

(d) Find the range of value of n for which $S_n > 0$ [4]

10 A quadrilateral ABCD has vertices at points A(-1;1); B(3;7); C(4;2) and D(2; −1), respectively.

(a) Find the equation of line AB. [4]

(b) Show that line AB is perpendicular to line AD. [4]

(c) Find the equation of a circle with center at point D and radius CD in the form $x^2 + y^2 + ax + by = c$ where a, b and c are constants. [5]

(d) Show that the circle in (c) passes through point A. [2]

(e) Explain why line AB is a tangent to the circle in (c). [1]

11 Triangle ABC has vertices with position vectors $i + 2j + 2k$, $2i + 5j - 2k$ and $5i + 4j + k$ respectively.

Find

(a) $\overrightarrow{AC}$. [2]

(b) the cartesian equation of line AC. [4]

(c) (i) the perpendicular distance of point B from line AC, [8]

(ii) the area of triangle ABC. [2]

12 (a) Express $g(x) = \frac{4x+5}{2(x+1)}$ in the form $A + \frac{B}{2(x+1)}$, where A and B are constants. [2]

(b) With the aid of diagrams, in each case, illustrate the transformations which maps the graph of $y = \frac{1}{x}$ on to the graph of $y = g(x)$. [10]

(c) Find the inverse of $g(x)$ and state its domain. [4]