For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS

6042/1

PAPER 1

3 hours

JUNE 2024 SESSION

Additional materials:
Answer paper
Graph paper
List of Formulae MF7
Scientific calculator [Non-Programmable]

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the answer paper/answer booklet.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

Answer **all** questions.

This question paper consists of 4 printed pages.

[Turn over

1 A sequence is defined by $U_{n+1} = 3U_n + 1$

(i) Write down the first four terms of the sequence, given that $U_1 = 0$. [2]

(ii) Hence describe the behaviour of the sequence. [1]

(iii) Express U_3 in terms of U_1. [2]

2 The variables x and y are connected by the equation $y = ae^{bx}$, where a and b are constants. When the values of lny are plotted against values of x, a straight line is obtained passing through the points (0; $ln2$) and (3;$ln5$).

Calculate the values of a and b in exact form. [6]

3 Express $\frac{2x-3}{x^2(x^2-4)}$ in partial fractions. [7]

4 Solve the equation $\frac{e^{2x}+e^{-2x}}{5} = 1$, giving the answer in exact form. [7]

5 Find the set of values of x for which $(x-3)^2 > 2x + 1$, giving answers in exact form. [7]

6 Chord XY subtends an angle of 2θ radians at the centre of a circle radius r and centre O. Given that the area of the minor segment is $\frac{1}{6}$ of the area of the major segment, show that $Sin2\theta = \frac{2}{7}(7\theta - \pi)$. [7]

7 (a) Expand $(1-2x)^{\frac{1}{2}}$ up to and including the term in x^2, simplifying coefficients. [3]

(b) State the values of x for which the expansion is valid. [1]

(c) By putting $x = \frac{1}{9}$ in the expansion in part (a), show that $\sqrt{7} \simeq \frac{143}{54}$. [4]

8 The equation of line l is $y + 3x - 9 = 0$.

Find the

(a) equation of the line m, that is perpendicular to line l and passes through the point A(1:6) in the form $ax + by + c = 0$. [4]

(b) equation of the line parallel to l passing through the point B(5;6). [2]

(c) perpendicular distance of the point B(5;6) from the line m. [3]

9 **(a)** Express $x^2 + 6x + 5$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [2]

(b) Hence or otherwise state the turning point of the graph of $y = x^2 + 6x + 5$. [1]

(c) Find the set of values taken by $y = \frac{1}{x^2+6x+5}$ for real values of x. [7]

10 Given that $f(x) = x^4 + x^3 + 23x^2 + 25x - 50$,

(a) show that $-5i$ is a root of $f(x) = 0$, [3]

(b) hence solve the equation $f(x) = 0$. [7]

11 The functions f, g and h are defined as:

$f: x \to x^2 + 2x \qquad x \in \mathrm{R}, x \geq -1$

$g: x \to 4x + 1 \qquad x \in \mathrm{R}, -2 \leq x \leq 3$

$h: x \to \frac{1}{x-3} \qquad x \in \mathrm{R} \quad x \neq 3$

(a) Find

(i) $gh(x)$, stating clearly its domain, [3]

(ii) $f^{-1}(x)$. [4]

(b) **(i)** Sketch the graph of $y = |g(x)|$, showing clearly the intercepts and end points. [2]

(ii) Hence state the range of $|g(x)|$. [1]

12 **(i)** Prove the identity

$$\frac{\text{Cos}\,\theta}{1-\text{Cos}\,\theta} - \frac{\text{Cos}\,\theta}{1+\text{Cos}\,\theta} \equiv 2Cot^2\theta.$$ [4]

(ii) Hence or otherwise solve the equation

$\frac{\text{Cos}\,\theta}{1-\text{Cos}\,\theta} - \frac{\text{Cos}\,\theta}{1+\text{Cos}\,\theta} = 1\ for\ 0° \leq \theta \leq 360°$ giving answer to the nearest degree. [7]

[Turn over

13 **(a)** **(i)** By sketching two appropriate graphs, show that $x^3 + x - 4 = 0$ has one real root. [4]

(ii) Show that there is a root between 1 and 1.5. [3]

(b) By using the formular $x = \sqrt{\frac{4}{x_n} - 1}$ and taking x_1 as 1.37, approximate the root of $x^3 + x - 4 = 0$ correct to 2 decimal places. [4]

14 The equation of a curve is $y^2 - 3xy + 2x^2 = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3y-4x}{2y-3x}$. [4]

(b) Find the equation of the tangent to the curve at $x = 1$ and $y > 0$. [8]