ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/1
PAPER 1

JUNE 2023 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae MF7
Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

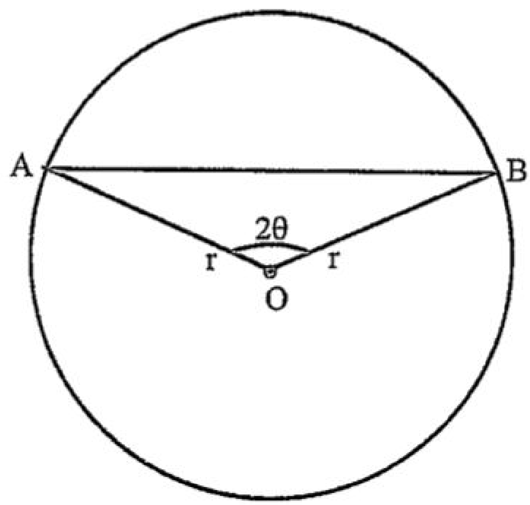
This question paper consists of 5 printed pages and 3 blank pages.

1 A sequence U_n is defined by $U_n = (-1)^{n+1} + 2$.

(a) Find the first 3 terms of the sequence. [3]

(b) State the behaviour of the sequence. [1]

2



A and B are two points on the circumference of a circle centre O and radius r.
The minor arc AB subtends an angle of 2θ radians at O.
If the area of the minor segment is one third of the area of the minor sector AOB, show that $2\theta = 3\sin\theta\cos\theta$. [4]

3 A variable p is inversely proportional to the square of $2q + 1$. Given that $p = \frac{3}{4}$ when $q = 2$. Find the value of q when $p = \frac{1}{3}$ [4]

4 (a) Solve the inequality $(1.05)^{n-4} < 60$. [3]

(b) Hence state the largest integral value of n, for the inequality in **(a)** [1]

5 Solve the following simultaneous equations:

$xy = 1$

$2x + y = 3$ [4]

6 Differentiate with respect to x

(a) $e^{2x}\sin 3x$, [2]

(b) $ln(1 + x^2)$. [2]

7 A curve C has parametric equations

$x = 2t + \frac{1}{2t}$, $y = 2t - \frac{1}{2t}$, where t is a parameter.

(a) Find x^2 and y^2 in terms of t. [4]

(b) Hence by evaluating $x^2 - y^2$ find the cartesian equation of C. [2]

8 **(a)** Express $\frac{1}{(\sqrt{x})^{\frac{4}{3}}}$ in the form x^n. [2]

(b) Solve $\frac{2^{1,5}}{2^7 \times 16} = 2^x$. [3]

9 **(a)** Divide $a^3 - b^3$ by $a - b$. [2]

(b) Given that $x^4 + x^2 + x + 1 \equiv (x^2 + A)(x^2 - 1) + Bx + C$,

determine the numerical values of A, B and C. [4]

10 Functions $h(x)$ and $g(x)$ are defined by:

$h(x) = 3x + 7$ for $x \in \mathbb{R}$

$g(x) = \frac{6}{2x-4}$ $x \neq 2$ for $x \in \mathbb{R}$

(a) Write down in terms of x

(i) $h^{-1}(x)$,

(ii) $g^{-1}(x)$ stating the values of x for which $g^{-1}(x)$ is not defined. [5]

(b) Sketch the graph of $h(x)$ and $h^{-1}(x)$ on the same axes making clear the relationship between the two graphs. [3]

11 Prove by Induction

$$\sum_{r=1}^{n} ap^{r-1} = \frac{a(p^n - 1)}{p - 1}$$

[7]

12 Given that the matrix $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & -5 & 3 \\ 4 & p & 7 \end{pmatrix}$,

find the

(a) value of p for which the determinant of $A = 54$, [3]

(b) inverse of matrix A using the value of p obtained in **(a)** above. [5]

13 **(a)** Use Taylor series to expand $\frac{1}{x}$ as series of ascending powers of $(x - a)$, up to and including the term in $(x - a)^3$. [6]

(b) Use the expansion to evaluate $\frac{1}{1{,}01}$ correct to 3 significant figures. [2]

14 **(a)** Express $4\cos\theta - 3\sin\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 360°$, giving α to the nearest degree. [3]

(b) Solve the equation $4\cos\theta - 3\sin\theta = 3$, for $0° \le \theta \le 360°$. [3]

(c) Find the least value of $\frac{1}{4\cos\theta - 3\sin\theta + 9}$. [2]

15 **(a)** Given that $z_1 = 3 + 4i$, and $z_2 = 1 + i$,

find

(i) $z_1 - z_2$, [2]

(ii) argument of $z_1 - z_2$. [2]

(b) **(i)** Solve the equation $z^2 - 4z + 53 = 0$, expressing the roots in the form $a + bi$, where $a, b \in \mathbb{R}$ [3]

(ii) State the sum and product of the roots of $z^2 - 4z + 53 = 0$. [2]

16 The points A, B and C have position vector $2\boldsymbol{i} - \boldsymbol{k}$, $3\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $5\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$ respectively.

(a) Calculate the angle ABC correct to the nearest $0.1°$ [4]

(b) Given that N is a point on vector $\overrightarrow{AC}$ such that $\overrightarrow{AN} : \overrightarrow{NC} = 1:2$, find the

(i) position vector of N, [3]

(ii) unit vector in the direction of $\overrightarrow{NC}$. [3]

17 The straight line ℓ has equation $2y - x - 5 = 0$.
Another straight line m passes through the point $P(-2, 6)$ and is perpendicular to ℓ.

Find the

(a) equation of m giving the answer in the form $ax + by + c = 0$, [3]

(b) co-ordinates of the point of intersection of ℓ and m. [3]

(c) perpendicular distance from P to the line ℓ. [2]

(d) It is given that the points Q$(7; 6)$ and R $(-7; -1)$ lie on line ℓ.
Find the area of the triangle PQR. [3]

18 **(a)** Sketch on the same axis the graphs of $y = e^{-x}$ and $y = \sin x$ where

$0 \leq x \leq \pi$. [2]

(b) Hence state the number of roots of the equation $e^{-x} - Sin\, x = 0$ [1]

(c) Show that the smallest root of $e^{-x} - Sin\, x = 0$ lies between $x = 0$ and $x = \frac{\pi}{2}$. [3]

(c) Starting with $x_0 = 0{,}8$, use the Newton -Raphson method twice to estimate the smallest root of $e^{-x} - Sin\, x = 0$, giving the answer correct to three decimal places. [4]

DE ~ MANUE SOLUTIONS

SUGGESTED MARKING GUIDE

PURE MATHEMATICS

JUNE 2023 SESSION

SYLLABUS CODE : 6042/1

THE ESSENCE OF MATH IS NOT TO MAKE SIMPLE THINGS COMPLICATED, BUT TO MAKE COMPLICATED THINGS SIMPLE.

DE~MANUE--0717256148 / 0774459409

zivanayiemmanuel@gmail.com

Feel free to contact us

ACKNOWLEDGEMENT: TRINITY CHIGUBHU

Suggested marking guide

1.) $U_n = (-1)^{n+1} + 2$

$U_1 = (-1)^2 + 2$

$= 3$

$U_2 = (-1)^3 + 2$

$= 1$

$U_3 = (-1)^4 + 2$

$= 3$

b) The sequence is periodic

2. Area of the minor segment

Area of the segment $= \frac{r^2}{2}(\theta - \sin\theta)$ Area of the sector $= \frac{1}{2}r^2\theta$

Let A_1 be the area of the minor sector A_2 be the area of the minor segment

$A_1 = \frac{A_2}{3}$

$\frac{r^2}{2}(2\theta - \sin 2\theta) = \frac{r^2\theta}{3}$

$3(2\theta - \sin 2\theta) = 2\theta$

$6\theta - 3\sin 2\theta = 2\theta$

$6\theta - 2\theta = 3\sin 2\theta$

$4\theta = 3(2\sin\theta\cos\theta)$

$\frac{4\theta}{2} = \frac{6\sin\theta\cos\theta}{2}$

$2\theta = 3\sin\theta\cos\theta$ as required

3. $P \propto \frac{1}{(2q+1)^2}$

$P = \frac{k}{(2q+1)^2}$

$P = \frac{3}{4}$ when $q = 2$

$\frac{3}{4} = \frac{k}{(4+1)^2}$

$\frac{3}{4} = \frac{k}{25}$

$K = \frac{75}{4}$

Therefore

$P = \frac{75}{4(2q+1)^2}$

When $p = \frac{1}{3}$,

$\frac{1}{3} = \frac{75}{(2q+1)^2}$

$(2q + 1)^2 = \frac{75\times3}{4}$

$2q + 1 = \sqrt{\frac{225}{4}}$

$2q = \pm\frac{15}{2} - 1$

$2q = \frac{15}{2} - 1$ or $-\frac{15}{2} - 1$

$2q = \frac{13}{2}$ or $-\frac{17}{2}$

$q = \frac{13}{4}$ or $-\frac{17}{4}$

4a) $(1.05)^{n-4} < 60$

$(n-4)\ln(1.05) < \ln 60$

$n - 4 < \frac{\ln 60}{\ln 1.05}$

$n < \frac{\ln 60}{\ln 1.05} + 4$

$n < 87.9$

b) $n = 87$

5) $xy = 1$

$2x + y = 3$

$x = \frac{1}{y}$

$2\frac{1}{y} + y = 3$

$2 + y^2 = 3y$

$y^2 - 3y + 2 = 0$

$y^2 - y - 2y + 2 = 0$

$y(y-1) - 2(y-1) = 0$

$(y-2)(y-1) = 0$

$y = 2 \text{ or } 1$

$For\ y = 1$

$x = \frac{1}{1}$

$x = 1$

$For\ y = 2$

$x = \frac{1}{2}$

$Therefore\ x = 1 \text{ or } \frac{1}{2} \text{ and } y = 1 \text{ or } 2$

6a) $Differentiate\ \boldsymbol{e^{2x}Sin2x}$

$\frac{d}{dx}e^{2x}Sin3x$

Applying Implicit Rule

$e^{2x}\frac{d}{dx}Sin3x + Sin3x\frac{d}{dx}e^{2x}$

$3e^{2x}Cos3x + 2e^{2x}Sin3x$

$e^{2x}[3Cos3x + 2Sin3x]$

b) $Differentiate\ \ln(1 + x^2)$

$\frac{2x}{1 + x^2}$

7a) Find x² and y² in terms of t.

$x = 2t + \frac{1}{2t}$

$y = 2t - \frac{1}{2t}$

$x^2 = (2t + \frac{1}{2t})^2$

$x^2 = 4t^2 + 2 + \frac{1}{4t^2}$

$x^2 = 4t^2 + \frac{1}{4t^2} + 2$

$y^2 = (2t - \frac{1}{2t})^2$

$y^2 = 4t^2 - 2 + \frac{1}{4t^2}$

$y^2 = 4t^2 + \frac{1}{4t^2} - 2$

b) $x^2 - y^2 = 4t^2 + \frac{1}{4t^2} + 2 - \left(4t^2 + \frac{1}{4t^2} - 2\right)$

$x^2 - y^2 = 4t^2 - 4t^2 + \frac{1}{4t^2} - \frac{1}{4t^2} + 2 + 2$

$x^2 - y^2 = 4$

8a) $\frac{1}{(\sqrt{x})^{\frac{4}{3}}}$

$(\sqrt{x})^{-\frac{4}{3}}$

$x^{-\frac{1}{2}\times\frac{4}{3}}$

$x^{-\frac{2}{3}}$

b) $\frac{2^{1.5}}{2^7 \times 16} = 2^x$

$\frac{2^{1.5}}{2^7 \times 2^4} = 2^x$

$\frac{2^{1.5}}{2^{11}} = 2^x$

$2^{1.5} = 2^x \times 2^{11}$

$2^{1.5} = 2^{x+11}$

$1.5 = x + 11$

$x = 1.5 - 11$

$x = -9\frac{1}{2}$

$$a^2 + ab + b^2$$

9a) $a - b \,\overline{)\, a^3 - b^3}$

$$\underline{-(a^3 - a^2b)}$$

$$a^2b - b^3$$

$$\underline{-(a^2b - ab^2)}$$

$$ab^2 - b^3)$$

$$\underline{-(ab^2 - b^3)}$$

$$0$$

$Therefore\ (a^3 - b^3) \div a - b = \quad a^2 + ab + b^2$

b) $x^4 + x^2 + x + 1 \equiv (x^2 + A)(x^2 - 1) + Bx + C$

$R.H.S = x^4 + x^2 + Ax^2 - A + Bx + C$

$x^4 + (A - 1)x^2 + C - A$

$Comparing\ Coeffs$

$1 = A - 1$ -------------(i)

$1 + 1 = A$

$A = 2$

$B = 1$

$C - A = 1$

$C = 1 + A$

$C = 1 + 2 \quad since\ A = 2$

$C = 3$

$Therefore\ A = 2\ ; B = 1; C = 3$

10) $h(x) = 3x + 7; x \in R$

$g(x) = \dfrac{6}{2x - 4}; x \neq 2, x \in R$

$ai)\ h(x) = 3x + 7$

Replace $h(x)$ with y

$y = 3x + 7$

Make x subject of formular

$y - 7 = 3x$

$x = \dfrac{y - 7}{3}$

Replace x by $h^{-1}(x)$ and y with x

$h^{-1}(x) = \dfrac{x - 7}{3}; x \in R$

ii) $g(x) = \frac{6}{2x-4}$

Replace $g(x)$ with y

$y = \dfrac{6}{2x - 4}$

$y(2x - 4) = 6$

$2xy - 4y = 6$

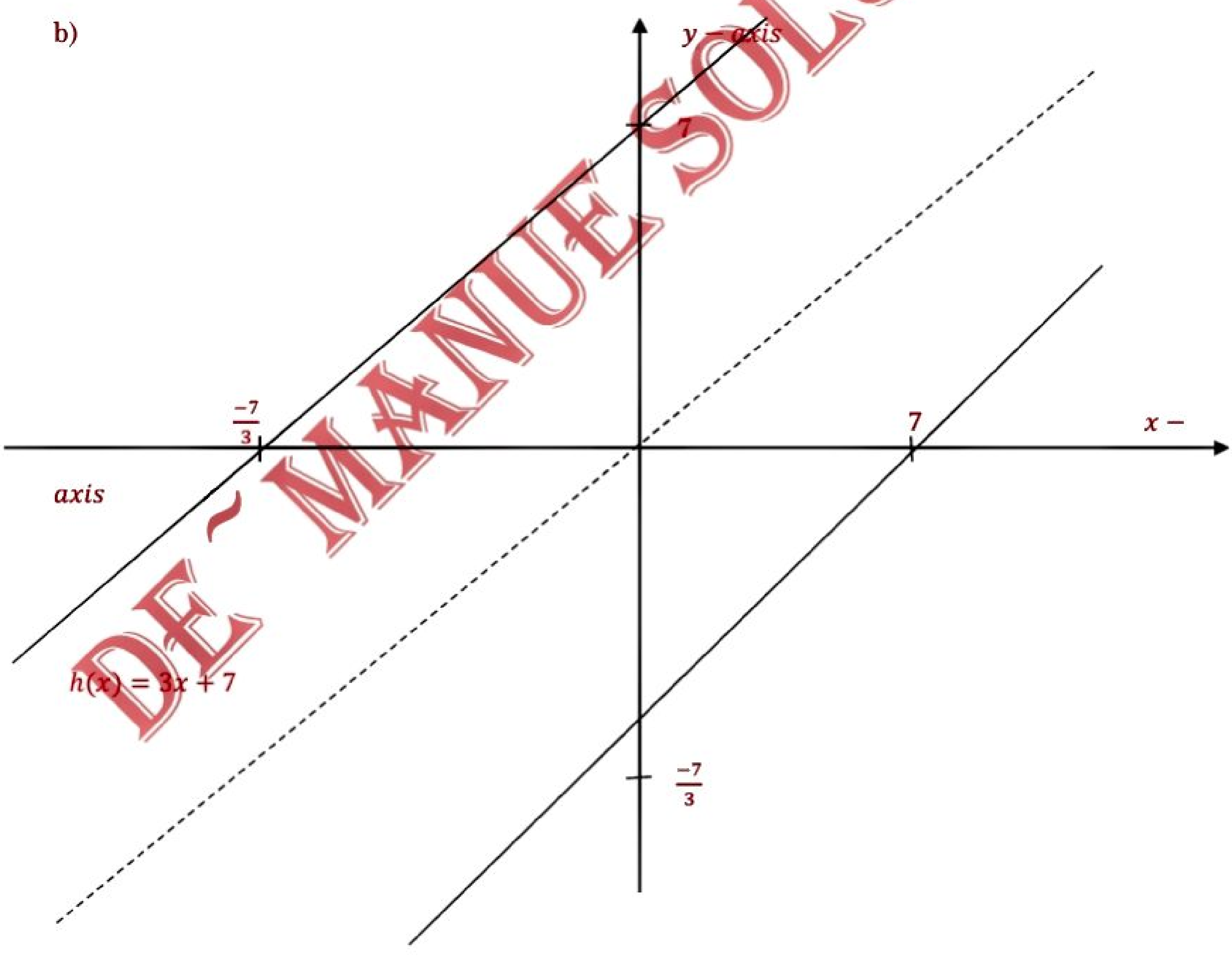
$2xy = 6 + 4y$

$x = \frac{6 + 4y}{2y}$

Replace x *by* $g^{-1}(x)$ *and* y *with* x

$g^{-1}(x) = \frac{6 + 4x}{2x}$

b)



$y = x$ $\qquad\qquad$ $h^{-1}(x) = \frac{x-7}{3}$

11) Prove by induction that $\sum_{r=1}^{n}(ap^{r-1}) = \frac{a(p^n - 1)}{p - 1}$

Let Pn denote the expression $\sum_{r=1}^{n}(ap^{r-1}) = \frac{a(p^n - 1)}{p - 1}$

Proof for $n = 1$

$$\sum_{r=1}^{1}(ap^{1-1}) = \frac{a(p^1 - 1)}{p - 1}$$

$$ap^0 = a$$

$$a = a$$

Since L.H.S = R.H.S ; Pn is true for $n = 1$

Assume Pn to be true for $n = k$

$$\sum_{r=1}^{k}(ap^{r-1}) = \frac{a(p^k - 1)}{p - 1}$$

Proof for $n = k + 1$

$$\sum_{r=1}^{k+1}(ap^{r-1}) = \frac{a(p^{k+1} - 1)}{p - 1}$$

Breaking The sum

$$\sum_{r=1}^{k+1}(\mathrm{ap}^{\mathrm{r}-1}) = \sum_{r=1}^{k}(\mathrm{ap}^{\mathrm{r}-1}) + \mathrm{ap}^{(\mathrm{k}+1)-1}$$

$$\sum_{r=1}^{k+1}(\mathrm{ap}^{\mathrm{r}-1}) = \sum_{r=1}^{k}(\mathrm{ap}^{\mathrm{r}-1}) + \mathrm{ap}^{\mathrm{k}}$$

$$= \frac{\mathrm{a}(\mathrm{p}^{\mathrm{k}} - 1)}{\mathrm{p} - 1} + \mathrm{ap}^{\mathrm{k}}$$

$$= \frac{\mathrm{a}(\mathrm{p}^{\mathrm{k}} - 1) + \mathrm{ap}^{\mathrm{k}}(\mathrm{p} - 1)}{\mathrm{p} - 1}$$

$$= \frac{\mathrm{ap}^{\mathrm{k}} - \mathrm{a} + \mathrm{ap}^{\mathrm{k}+1} - \mathrm{ap}^{\mathrm{k}}}{\mathrm{p} - 1}$$

$$= \frac{\mathrm{ap}^{\mathrm{k}+1} - \mathrm{a}}{\mathrm{p} - 1}$$

$$= \frac{\mathrm{a}(\mathrm{p}^{\mathrm{k}+1} - 1)}{\mathrm{p} - 1}$$

$Pn\ is\ true\ for\ n = k + 1$

$Since\ Pn\ is\ true\ for\ n = 1, n = k\ and\ n = k + 1; Therefore\ by\ mathematical\ induction, Pn\ is$

$true\ for\ all\ positive\ integral\ values\ of\ n!$

12) $\mathrm{A} = \begin{pmatrix} 1 & 2 & 5 \\ 2 & -5 & 3 \\ 4 & p & 7 \end{pmatrix}$

a) $1\begin{bmatrix} -5 & 3 \\ p & 7 \end{bmatrix} - 2\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} + 5\begin{bmatrix} 2 & 5 \\ 4 & p \end{bmatrix} = 54$

$-35 - 3p - 4 + 10p + 100 = 54$

$7p = -7$

$p = \frac{-7}{7}$

$p = -1$

b)*The inverse of matrix A*

$$A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & -5 & 3 \\ 4 & p & 7 \end{pmatrix}$$

Transposing A

$$A^T = \begin{pmatrix} 1 & 2 & 4 \\ 2 & -5 & -1 \\ 5 & 3 & 7 \end{pmatrix}$$

Add the first 2 columns to the right of matrix A^T

Add the resulting top rows below the matrix

Delete the resulting first row and first column

Find the determinant of 2×2 *block entries*

$$\begin{pmatrix} 1 & 2 & 4 & 1 & 2 \\ 2 & -5 & -1 & 2 & -5 \\ 5 & 3 & 7 & 5 & 3 \\ 1 & 2 & 4 & 1 & 2 \\ 2 & -5 & -1 & 2 & -5 \end{pmatrix}$$

$$Adjoint = \begin{bmatrix} \begin{vmatrix} -5 & -1 \\ 3 & 7 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 7 & 5 \end{vmatrix} & \begin{vmatrix} 2 & -5 \\ 5 & 3 \end{vmatrix} \\ \begin{vmatrix} 3 & 7 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 7 & 5 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ -1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -5 \end{vmatrix} \end{bmatrix}$$

$$Adjoint = \begin{pmatrix} -32 & -19 & 31 \\ -2 & -13 & 7 \\ 18 & 9 & -9 \end{pmatrix}$$

$$A^{-1} = \frac{1}{54}\begin{pmatrix} -32 & -19 & 31 \\ -2 & -13 & 7 \\ 18 & 9 & -9 \end{pmatrix}$$

13a) $f(x) = f(a) + \frac{f^{I}(a)(x-a)}{1!} + \frac{f^{II}(a)(x-a)^2}{2!} + \frac{f^{III}(a)(x-a)^3}{3!}$

x	a
$f(x) = \frac{1}{x}$	$f(a) = \frac{1}{a}$
$f^{I}(x) = -\frac{1}{x^2}$	$f^{I}(a) = -\frac{1}{a^2}$
$f^{II}(x) = \frac{2}{x^3}$	$f^{II}(a) = \frac{2}{a^3}$
$f^{III}(x) = -\frac{6}{x^4}$	$f^{III}(a) = -\frac{6}{a^4}$

$f(x) = \frac{1}{a} - \frac{(x-a)}{1!\times a^2} + \frac{2(x-a)^2}{2!\times a^3} - \frac{6(x-a)^3}{3!\times a^4}$

$f(x) = \frac{1}{a} - \frac{(x-a)}{a^2} + \frac{2(x-a)^2}{2a^3} - \frac{6(x-a)^3}{6a^4}$

$f(x) = \frac{1}{a} - \frac{(x-a)}{a^2} + \frac{(x-a)^2}{a^3} - \frac{(x-a)^3}{a^4}$

b) $\frac{1}{1.01} = \frac{1}{1.01} - \frac{0.01}{1.01^2} + \frac{0.01^2}{1.01^3} - \frac{0.01^3}{1.01^4}$

$= 0.9803921$

0.980

14a) $4Cos\theta - 3Sin\theta ------RSin(\theta - \alpha)$; $R > 0,\ 0° \leq \alpha \leq 360°$

For the R.H.S of expression

$RSin(\theta - \alpha)$

$RSin\theta Cos\alpha - RSin\alpha Cos\theta$

Comparing Coeffs

$4 = -RSin\alpha ------(i)$

$-3 = RCos\alpha ------(ii)$

$^{(i)}/_{(ii)}$

$\frac{-4}{3} = \frac{-RSin \propto}{Rcos \propto}$

$tan \propto = \frac{4}{3}$

$\propto = \tan^{-1}(\frac{4}{3})$

$\propto = 53.13010°$

$\propto = 53°$

$R = \sqrt{(4)^2 + (-3)^2}$

$R = 5$

$4Cos\theta - 3Sin\theta \quad = \quad 5Sin(\theta - 53°)$

b) $4Cos\theta - 3Sin\theta \quad = \quad 3$

$5Sin(\theta - 53°) = 3$

$Sin(\theta - 53°) = \frac{3}{5}$

$(\theta - 53°) = \sin^{-1}(\frac{3}{5})$

$(\theta - 53°) = 36.78$

$\theta = (-1)^n \times PV + 180n$

$\theta = (-1)^n \times 36.87 + 180n + 53°$

$For\ n = \ -1\ ;\ \theta = out\ of range$

$For\ n = 0;\ \theta = 90°$

$For\ n = 1;\ \theta = 196.26°$

$For\ n = 2\ ;\ \theta = out\ of\ range$

$Therefore\ \theta = 90°\ ; 196°$

c) $Min\ or\ Max = \frac{A}{k \pm R}$

$Min\ or\ Max = \frac{1}{9 \pm 5}$

$Min\ \frac{1}{9+5}$ & $Max = \frac{1}{9-5}$

$Therefore\ Min = \frac{1}{14}$

15a) $Z_1 = 3 + 4i$ $\qquad$ $Z_2 = 1 + i$

ai) $(Z_1 - Z_2) = (3 - 1) + (4i - i)$

$= 2 + 3i$

ii)) $Arg_{(Z_1 - Z_2)} = \tan^{-1}(\frac{y}{x})$

$Arg_{(Z_1 - Z_2)} = \tan^{-1}(\frac{3}{2})$

$= 0.98279 rads$

$= 0.98\, rads$

$bi)\ z^2 - 4z + 53 = 0$

$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{4 \pm \sqrt{4^2 - 4(1)(53)}}{2(1)}$

$$= \frac{4 \pm \sqrt{16 - 212}}{2}$$

$$= \frac{4 \pm \sqrt{-196}}{2}$$

$$= \frac{4 \pm \sqrt{196i^2}}{2}$$

$$= \frac{4 \pm 14i}{2}$$

$z = 2 + 7i \text{ or } 2 - 7i$

ii) Product & sum

$$Sum = -\frac{b}{a}$$

$$sum = -\left(-\frac{4}{1}\right)$$

$= 4$

$$Product = \frac{(-1)^n \times k}{a}$$

$$= \frac{(-1)^2 \times 53}{1}$$

$= 53$

16) $OA = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$; $OB = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$; $OC = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$

$a)\ Let\ angle\ ABC\ be\ \theta$

$\theta --- AB\ \&\ BC$

$BC = OC - OB$

$AB = OB - OA$

$$BC = \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 5 \end{pmatrix}$$

$$AB = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$$

$$\cos\theta = \frac{\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}\begin{pmatrix} 2 \\ -5 \\ 5 \end{pmatrix}}{\sqrt{(-1)^2 + (2)^2} \times \sqrt{(2^2 + (-5)^2 + (5)^2)}}$$

$$\cos\theta = \frac{8}{\sqrt{270}}$$

$$\cos\theta = \frac{8}{\sqrt{270}}$$

$$\theta = \cos^{-1}\left(\frac{8}{\sqrt{270}}\right)$$

$\theta = 60.8652°$

$b = -1$

$3c + 3 = 5$

$c = \frac{2}{3}$

$ON = \begin{pmatrix} 3 \\ -1 \\ \frac{2}{3} \end{pmatrix}$

$NC = OC - ON$

$= \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ \frac{2}{3} \end{pmatrix}$

$NC = \begin{pmatrix} 2 \\ -2 \\ \frac{10}{3} \end{pmatrix}$

$Unit\ Vector(NC) = \frac{2i - 2j + \frac{10}{3}k}{\sqrt{(2)^2 + (-2)^2 + (\frac{10}{3})^2}}$

$= \frac{2i - 2j + \frac{10}{3}k}{(\frac{2\sqrt{43}}{3})}$

$= \frac{3(2i - 2j + \frac{10}{3}k)}{2\sqrt{43}}$

$= \frac{3i - 3j + +5k)}{\sqrt{43}}$

17) $2y - x - 5 = 0$

a)$Using\ differentiation\ to\ find\ the\ gradient$

$2\frac{dy}{dx} - 1 = 0$

$\frac{dy}{dx} = \frac{1}{2}$

$T \times N = -1$

$\frac{1}{2}N = -1$

$N = -2$

$(y - y_1) = N(x - x_1)$

$(y - 6) = -2(x + 2)$

$y - 6 = -2x + 4$

$2x + y - 2 = 0$

b) $y = -2x + 2 -------(i)$

$2y - x - 5 = 0 -------(ii)$

$2(-2x + 2) - x - 5 = 0$

$-4x+4-x-5=0$

$-5x=1$

$x=-\frac{1}{5}$

$y=-2\left(-\frac{1}{5}\right)+2$

$y=\frac{2}{5}+2$

$y=\frac{12}{5}$

$\left(-\frac{1}{5}\ ;\ \frac{12}{5}\right)$

c) *Let d be the distance*

$d=\sqrt{(-2+\frac{1}{5})^2+(6-\frac{12}{5})^2}$

$=\sqrt{(-\frac{9}{5})^2+(\frac{18}{5})^2}$

$=\frac{9\sqrt{5}}{5}\,units$

d) $P(-2;6)$ & $Q(7;6)$ & $R(-7;-1)$

$PQ = \sqrt{(7+2)^2 + 0} = 9$

$QR = \sqrt{(7+7)^2 + (6+1)^2} = 7\sqrt{5}$

$PR = \sqrt{(-2+7)^2 + (6+1)^2} = \sqrt{74}$

Let θ be angle PQR

$$Cos\theta = \frac{(7\sqrt{5})^2 + 9^2 - (\sqrt{74})^2}{2(7\sqrt{5})(9)}$$

$$Cos\theta = \frac{2}{\sqrt{5}}$$

$$Cos^2\theta = \left(\frac{2}{\sqrt{5}}\right)^2$$

$$Cos^2\theta = \frac{4}{5}$$

$$Sin^2\theta = 1 - \frac{4}{5}$$

$$Sin^2\theta = \frac{1}{5}$$

$$Sin\theta = \sqrt{\frac{1}{5}}$$

$$Sin\theta = \frac{1}{\sqrt{5}}$$

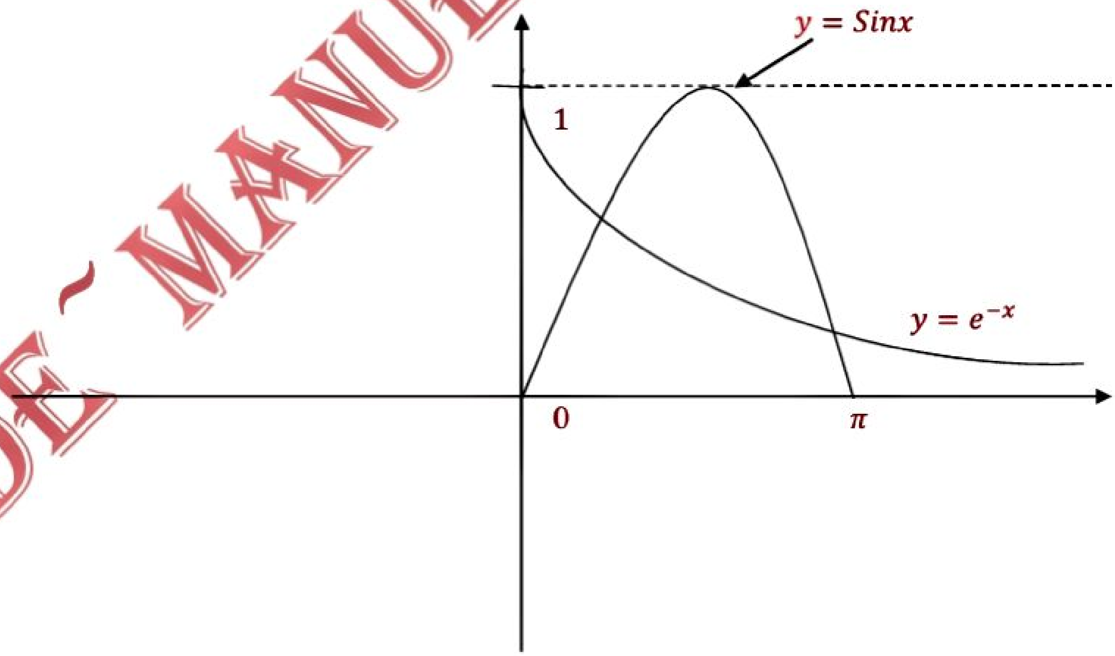
$$Area = \frac{ab}{2} Sin\theta$$

$= \frac{9(7\sqrt{5})}{2} \times \frac{1}{\sqrt{5}}$

$= \frac{9}{2} \times 7$

$= 31.5 units^2$

18a)

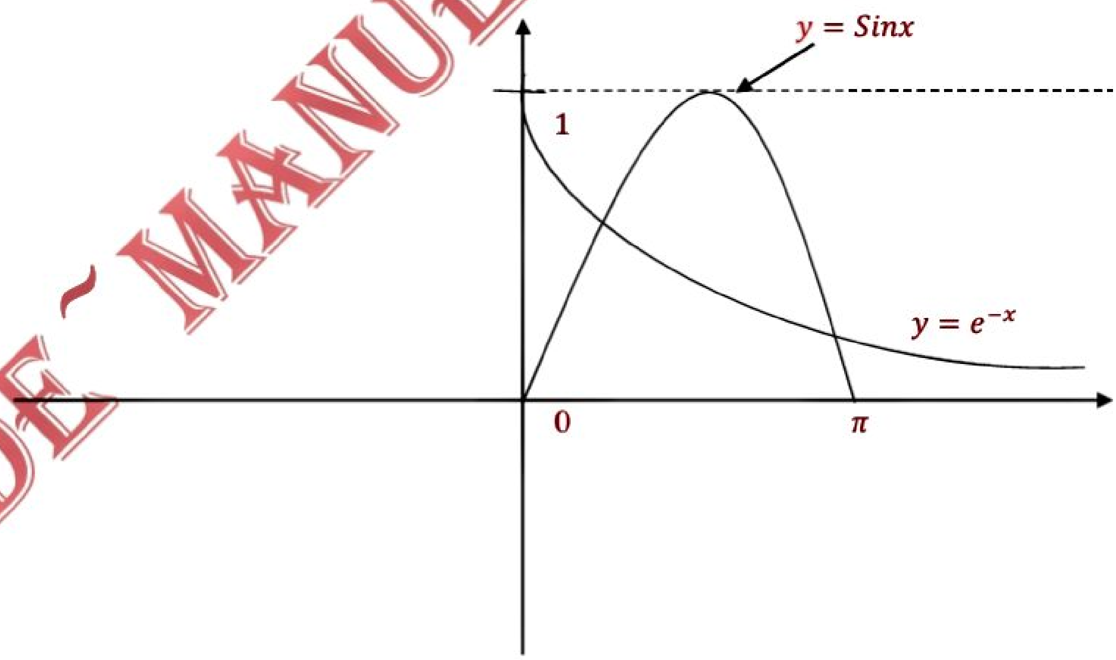


$= \frac{9(7\sqrt{5})}{2} \times \frac{1}{\sqrt{5}}$

$= \frac{9}{2} \times 7$

$= 31.5units^2$

18a)



b) $x_0 = 0.8$

$f(x) = e^{-x} - Sinx$

$f^I(x) = -e^{-x} - Cosx$

$$x_{n+1} = x_n - \left(\frac{f(x_n)}{f^I(x_n)}\right)$$

$$x_1 = 0.8 - \left(\frac{e^{-0.8} - Sin0.8}{-e^{-0.8} - Cos0.8}\right)$$

$x_1 = 0.57$

$$x_2 = 0.57 - \left(\frac{e^{-0.57} - Sin0.57}{-e^{-0.57} - Cos0.57}\right)$$

$x_2 = 0.589$

$Therefore\ the\ root\ is\ 0.589$

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

PURE MATHEMATICS — 6042/2
PAPER 2

JUNE 2023 SESSION — **3 hours**

Additional materials:
Answer paper
Graph paper
List of Formulae MF7
Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and any **five** questions from Section B

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

Section A (40 marks).

Answer **all** questions in this section.

1 A curve has parametric equations:

$x = 2 + 5\cos\theta$
$y = 3 + 5\sin\theta$

(a) Find the cartesian equation of the curve. [2]

(b) Describe fully in geometrical terms the type of equation the curve represents. [3]

2 The expression $ax^3 + x^2 - bx - 6$, is divisible by $x - 2$ and $2x^2 + 5x + 3$.

(a) Find the values of a and b. [2]

(b) When the values of a and b have the values found in **(a)** above, solve the equation $ax^3 + x^2 - bx - 6 = 0$. [2]

(c) Solve the inequality $ax^3 + x^2 - bx - 6 < 0$, using the values of a and b in **(a)** above. [2]

3 Circles with centres A and B have equations $x^2 + \left(y - \frac{5}{2}\right)^2 = \frac{37}{4}$ and $\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{13}{2}$, respectively.

(a) State the coordinates of A and coordinates of B. [2]

(b) Find the equation of the line AB in the form $ay + bx + c = 0$ [3]

(c) Hence or otherwise find the equation of the perpendicular bisector of the line segment AB in the form $y = mx + c$. [4]

4 A hollow circular cylinder open at one end has a height $h\ cm$ and radius $r\ cm$. The cylinder has a total external surface area of $1\ 200 cm^2$.

(a) Show that $h = \frac{600}{r} - \frac{r}{2}$ [2]

(b) Express the volume of the cylinder, $v\ cm^3$, in terms of r. [2]

(c) Find the

(i) value of r for which v has a stationary point, [2]

(ii) stationary value. [2]

(d) Determine the nature of the stationary point. [2]

5 **(a)** Express in partial fractions

$$\frac{x^3 + 4x^2 + 3}{x^2 + 2x - 3}.$$

[6]

(b) Hence or otherwise evaluate the exact value of

$$\int_2^3 \frac{x^3 + 4x^2 + 3}{x^2 + 2x - 3}\,dx$$

[4]

Section B (80 marks)

Answer any **five** questions from this section. Each question carries **16** marks.

6 **(a)** Find the equation of the image of the line $y = 2x + 1$, under the transformation with matrix [5]

$$\begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}.$$

(b) It is given that matrix $A = \begin{pmatrix} -4 & -3 & 5 \\ -5 & -4 & 7 \\ 1 & 1 & -1 \end{pmatrix}$.

(i) Find $|A|$.

(ii) Find A^{-1}.

(iii) Hence, or otherwise solve the simultaneous equations:

$$-4x - 3y + 5z = 3$$
$$-5x - 4y + 7z = 4$$
$$x + y - z = 0$$

[11]

7 **(a)** It is given that $f(x) = x^3 + x^2 - x + 15$.
Given that $1 + 2i$ is a root of $f(x) = 0$,

(i) state the other complex root of $f(x) = 0$, [1]

(ii) hence or otherwise solve the equation $f(x) = 0$. [5]

(b) (i) Using DeMoivre's Theorem or otherwise prove the identity
$\cos 5\theta \equiv \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$. [5]

(iii) Hence or otherwise, express $\cos 5\theta$ as powers of $\cos\theta$. [2]

(c) On an argand diagram sketch and shade the region whose points represent complex number z which satisfy the inequality $|z - 3i| < 2$. [3]

8 **(a)** Find the equation of the plane in cartesian form containing the vectors $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ given that point A has position vector $\begin{pmatrix} 1 \\ 0 \\ 6 \end{pmatrix}$. [4]

(b) Two lines r_1 and r_2 have vector equations.

$$r_1 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} \text{ and } r_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}, \text{ respectively.}$$

(i) Calculate the angle between the lines r_1 and r_2, correct to the nearest $0.1°$ [4]

(ii) Show that the lines r_1 and r_2 intersect. [4]

(iii) The point P has position vector $\boldsymbol{i} + 5\boldsymbol{j} + 6\boldsymbol{k}$.
Find the perpendicular distance from P to the line r_1. [4]

9 **(a)** **(i)** Using integration by parts, evaluate exactly $\int_2^4 lnx dx$. [5]

(ii) Use the trapezium rule with 5 ordinates to find an approximate value of $\int_2^4 lnx dx$. [4]

(b) **(i)** Use the graphical argument to show that the equation $e^{2x} = 5 - 4x$ has only one real root. [3]

(ii) Taking 0.5 to be the first approximation to the root, use the Newton-Raphson method 2 times to obtain a better approximation correct to 3 decimal places. [4]

10 **(a)** By differentiating from first principles, find the gradient function of the expression $x^2 + 2x$. [4]

(b) Given that $y^2 \cos x - y \sin 2x = 4$, find $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{4}; 2\right)$. [6]

(c) Find the coordinates of the stationary points of the curve $y = x^2 e^{-2x}$. [6]

11 (a) (i) State any **four properties** of a group. [4]

(ii) Hence show that $(\mathbb{R}, \times)$ is a group. [7]

(b) (i) Construct group tables for (Z_{2+}), addition module 2. [3]

(ii) Verify that (Z_{2+}) is commutative [2]

12 (a) Given that $I = \int \frac{x^2}{\sqrt{4-x^2}} dx$, using the substitution $x = 2\sin\theta$, show that

$I = \int 4\sin^2\theta d\theta$. [4]

(b) Use Taylor's series to expand $\cos x$ as a series of ascending powers of $(x - a)$ up to and including the term in $(x - a)^2$ [5]

(c) Hence or otherwise

(i) state the expansion of $\cos x$ as ascending powers of $x - \frac{\pi}{6}$, up to and including the term in $\left(x - \frac{\pi}{6}\right)^2$ [3]

(ii) find an approximate value for $\cos 31°$, given that $1° = 0{,}017$ radians, giving the answer to 5 decimal places. [4]

DE ~ MANUE SOLUTIONS

SUGGESTED MARKING GUIDE

PURE MATHEMATICS

JUNE 2023 SESSION

SYLLABUS CODE : 6042/2

THE ESSENCE OF MATH IS NOT TO MAKE SIMPLE THINGS COMPLICATED. BUT TO MAKE COMPLICATED THINGS SIMPLE.

DE~MANUE--0717256148 / 0774459409

zivanayiemmanuel@gmail.com

Feel free to contact us

ACKNOWLEDGEMENT: TRINITY CHIGUBHU

Suggested marking guide

SECTION A

1) $x = 2 + 5Cos\theta$

$y = 3 + 5Sin\theta$

a) $x - 2 = 5Cos\theta$

$$\frac{x-2}{5} = Cos\theta$$

$$\left(\frac{x-2}{5}\right)^2 = Cos^2\theta$$

$y - 3 = 5Sin\theta$

$$\frac{y-3}{5} = Sin\theta$$

$$\left(\frac{y-3}{5}\right)^2 = Sin^2\theta$$

$Cos^2\theta + Sin^2\theta = 1$

$$\left(\frac{x-2}{5}\right)^2 + \left(\frac{y-3}{5}\right)^2 = 1$$

$$\left(\frac{x^2-4x+4}{25}\right) + \left(\frac{y^2-6y+9}{25}\right) = 1$$

$$\left(\frac{x^2-4x+4+y^2-6y+9}{25}\right) = 1$$

$x^2 + y^2 - 4x - 6y + 13 = 25$

$x^2 + y^2 - 4x - 6y - 12 = 0$

b) $x^2 + y^2 - 4x - 6y - 12 = 0$

$(x-2)^2-4+(y-3)^2-9-12=0$

$(x-2)^2+(y-3)^2=25$

$(x-a)^2+(y-b)^2=r^2$

$\therefore$ *equation is a circle centre*$(2,3)$ *and radius* $=5$

2) ax^3+x^2-bx-6

$factors=(x-2)\ \&\ 2x^2+5x+3$

a) *For quadratic factor*

$2x^2+5x+3$

$2x^2+2x+3x+3$

$(2x+3)(x+1)$

$f(-1)\rightarrow\ -a+1+b-6=0$

$f(2)\rightarrow\ 8a+4-2b-6=0$

$b=a+5----(i)$

$8a-2b=2----(ii)$

$8a-2(a+5)=2$

$8a-2a-10=2$

$6a=12$

$\frac{6a}{6}=\frac{12}{6}$

$a=2$

$b=2+5$

$b=7$

Therefore $a=2\ \&\ b=7$

b)$2x^3 + x^2 - 7x - 6 = 0$

$(x-2)(2x^2+5x+3)$

$(x-2)(x+1)(2x+3)$

$Therefore\ x = -\frac{3}{2}\ \&-1\ \&\ 2$

c) $2x^3 + x^2 - 7x - 6 < 0$

$Critical\ Values : x = -\frac{3}{2}\ \&-1\ \&\ 2$

	$x < -\frac{3}{2}$	$-\frac{3}{2} < x < -1$	$-1 < x < 2$	$x > 2$
$(x-2)$			$-$	
$(x+1)$			$+$	
$(2x+3)$			$+$	
$f(x)$	$-$	$+$	$-$	$+$

$Solution\ Set : x < -\frac{3}{2}\ U -1 < x < 2$

3)$x^2 + \left(y - \frac{5}{2}\right)^2 = \frac{37}{4}$

$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{13}{2}$

$a)\ A\left(0; \frac{5}{2}\right)\ \&\ B\left(\frac{1}{2}; -\frac{1}{2}\right)$

b) *Let the gradient be* $= m$

$$m = \frac{\frac{5}{2} + \frac{1}{2}}{0 - \frac{1}{2}}$$

$$m = 3 \times -2$$

$$m = -6$$

$$(y - y_1) = m(x - x_1)$$

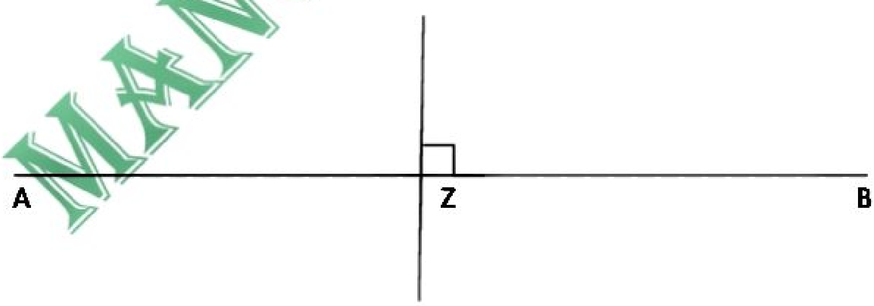
$$\left(y - \frac{5}{2}\right) = -6(x)$$

$$y - \frac{5}{2} = -6x$$

$$2y - 5 = -12x$$

$$2y + 12x - 5 = 0$$

c)



For Gradient Of Bisector(N)

$$T \times N = -1$$

$$-6N = -1$$

$$N = \frac{1}{6}$$

$$Z = \left[\frac{\left(0+\frac{1}{2}\right)}{2};\frac{\left(\frac{5}{2}-\frac{1}{2}\right)}{2}\right]$$

$$Z = \left(\frac{1}{4};1\right)$$

$$(y - y_1) = m(x - x_1)$$

$$(y-1) = \frac{1}{6}\left(x - \frac{1}{4}\right)$$

$$y - 1 = \frac{x}{6} - \frac{1}{24}$$

$$y = \frac{x}{6} - \frac{1}{24} + 1$$

$$y = \frac{1}{6}x + \frac{23}{24}$$

4)

Let A be the total surface area

$$A = 2\pi rh + \pi r^2$$

$$1200 = 2\pi rh + \pi r^2$$

$$2\pi rh = 1200 - \pi r^2$$

$h = \frac{1200}{2\pi r} - \frac{\pi r^2}{2\pi r}$

$h = \frac{600}{\pi r} - \frac{r}{2}$

$b)\ V = \pi r^2 h$

$V = \pi r^2 \left(\frac{600}{\pi r} - \frac{r}{2}\right)$

$V = \frac{600\pi r^2}{\pi r} - \frac{\pi r^3}{2}$

$V = 600r - \frac{\pi r^3}{2}$

$ci)\ V = 600r - \frac{\pi r^3}{2}$

$\frac{dV}{dr} = 600 - \frac{3}{2}\pi r^2$

$0 = 600 - \frac{3}{2}\pi r^2$

$600 = \frac{3}{2}\pi r^2$

$r^2 = \frac{400}{\pi}$

$r = \pm\sqrt{\frac{400}{\pi}}$

$r = \frac{20}{\sqrt{\pi}}\, cm$

$ii) V = 600r - \frac{\pi r^3}{2}$

$$V = 600\left(\frac{20}{\sqrt{\pi}}\right) - \frac{\pi}{2}\left(\frac{20}{\sqrt{\pi}}\right)^3$$

$$V = 4513.516668\ cm^3$$

$$V = 4500cm^3$$

$$d)\ \frac{dV}{dr} = 600 - \frac{3}{2}\pi r^2$$

$$\frac{d^2V}{dr^2} = -3\pi r$$

$$\frac{d^2V}{dr^2} = -3\pi\left(\frac{20}{\sqrt{\pi}}\right)$$

$$\frac{d^2V}{dr^2} = -106.34$$

$$\frac{d^2V}{dr^2} < 0$$

Therefore the nature is Maximum

$$5a)\quad \frac{x^3 + 4x^2 + 3}{x^2 + 2x - 3}$$

Using Long Division

$$\begin{array}{r|l} & x + 2 \\ \hline x^2 + 2x - 3 & x^3 + 4x^2 + 3 \\ & \underline{(x^3 + 2x^2 - 3x)} \\ & 2x^2 + 3x + 3 \\ & \underline{(2x^2 + 4x - 6)} \\ & -x + 9 \end{array}$$

$x + 2 + \frac{9 - x}{x^2 + 2x - 3}$

$For\ x^2 + 2x - 3$

$(x^2 - x + 3x - 3)$

$x(x - 1) + 3(x - 1)$

$(x + 3)(x - 1)$

$x + 2 + \frac{9 - x}{(x + 3)(x - 1)}$

$For\ \frac{9 - x}{(x + 3)(x - 1)} = \frac{A}{x + 3} + \frac{B}{x - 1}$

$9 - x = A(x - 1) + B(x + 3)$

$For\ x = 1$

$9 - 1 = 4B$

$8 = 4B$

$B = 2$

$For\ x = -3$

$9 + 3 = -4A$

$12 = -4A$

$A = -3$

$x + 2 + \frac{2}{x - 1} - \frac{3}{x + 3}$

$b)\ \int_2^3 \left(x + 2 + \frac{2}{x - 1} - \frac{3}{x + 3}\right) dx$

$\int_2^3 x\, dx + \int_2^3 2\, dx + \int_2^3 \frac{2}{x - 1} dx - \int_2^3 \frac{3}{x + 3} dx$

$$\left[\frac{x^2}{2}+2x+2\ln(x-1)-3\ln(x+3)\right]_2^3$$

$$\left(\frac{3^2}{2}+2(3)+2\ln(3-1)-3\ln(3+3)\right)-\left(\frac{2^2}{2}+2(2)+2\ln(2-1)-3\ln(2+3)\right)$$

$$\left(\frac{9}{2}+6+2ln2-3ln6\right)-(2+4+0-3ln5)$$

$$\frac{9}{2}+ln4-ln216+ln125$$

$$\frac{9}{2}+\ln\frac{500}{216}$$

$$\frac{9}{2}+ln\frac{125}{54}$$

SECTION B

6a) $y=2x+1$

$$\begin{pmatrix}x^I\\y^I\end{pmatrix}=\begin{pmatrix}a & b\\c & d\end{pmatrix}\begin{pmatrix}x\\ \cdot \\ mx+c\end{pmatrix}$$

$$\begin{pmatrix}x^I\\y^I\end{pmatrix}=\begin{pmatrix}0 & 3\\1 & -2\end{pmatrix}\begin{pmatrix}x\\2x+1\end{pmatrix}$$

$$\begin{pmatrix}x^I\\y^I\end{pmatrix}=\begin{pmatrix}6x+3\\x-4x-2\end{pmatrix}$$

$$\begin{pmatrix}x^I\\y^I\end{pmatrix}=\begin{pmatrix}6x+3\\-3x-2\end{pmatrix}$$

$x^I=6x+3----(i)$

$y^I = -3x - 2 - - - - (ii)$

$$\frac{x^I - 3}{6} = x$$

$$y^I = -3\left(\frac{x^I - 3}{6}\right) - 2$$

$$6y^I = -3x^I + 9 - 12$$

$$6y^I = -3x^I - 3$$

Therefore $6y = -3x - 3$ *is the image line*

Accept any other method

$$b)\ A = \begin{pmatrix} -4 & -3 & 5 \\ -5 & -4 & 7 \\ 1 & 1 & -1 \end{pmatrix}$$

i) Applying method of co – factors

$+ \quad - \quad +$

$$|A| = -4\begin{vmatrix} -4 & 7 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} -5 & 7 \\ 1 & -1 \end{vmatrix} + 5\begin{vmatrix} -5 & -4 \\ 1 & 1 \end{vmatrix}$$

$$|A| = -4(-3) + 3(-2) + 5(-1)$$
$$|A| = 1$$

Accept any other method

$$ii)\ A = \begin{pmatrix} -4 & -3 & 5 \\ -5 & -4 & 7 \\ 1 & 1 & -1 \end{pmatrix}$$

Add the first 2 columns to the right of matrix A^T

Add the resulting top rows below the matrix

Delete the resulting first row and first column

Find the determinant of 2×2 *block entries*

$$A^T = \begin{matrix} -4 & -5 & 1 & -4 & -5 \\ -3 & -4 & 1 & -3 & -4 \\ 5 & 7 & -1 & 5 & 7 \\ -4 & -5 & 1 & -4 & -5 \\ -3 & -4 & 1 & -3 & -4 \end{matrix}$$

$$Adjoint = \begin{bmatrix} \begin{vmatrix} -4 & 1 \\ 7 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -3 \\ -1 & 5 \end{vmatrix} & \begin{vmatrix} -3 & -4 \\ 5 & 7 \end{vmatrix} \\ \begin{vmatrix} 7 & -1 \\ -5 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 5 \\ 1 & -4 \end{vmatrix} & \begin{vmatrix} 5 & 7 \\ -4 & -5 \end{vmatrix} \\ \begin{vmatrix} -5 & 1 \\ -4 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -4 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} -4 & -5 \\ -3 & -4 \end{vmatrix} \end{bmatrix}$$

$$Adjoint = \begin{pmatrix} -3 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times Adjoint$$

$$A^{-1} = \begin{pmatrix} -3 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$$

Accept any other method

$iii) - 4x - 3y + 5z = 3$

$-5x - 4y + 7z = 4$

$x + y - z = 0$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \times A = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \times \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 & 2 & -1 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$Therefore\ x = -1\ \&\ y = 2\ \&\ z = 1$

$7a) f(x) = x^3 + x^2 - x + 15$

$1 + 2i\ is\ a\ root\ of\ f(x) = 0$

$i)\ The\ other\ root\ is\ the\ conjugate\ of\ 1 + 2i$

$1 - 2i$

$ii)\ Sum\ of\ all\ the\ roots = -\frac{b}{a}$

$Sum = -\left(\frac{1}{1}\right)$

$Sum = -1$

$Let\ the\ third\ root\ be\ \propto$

$1 + 2i + 1 - 2i + \propto = -1$

$2 + \propto = -1$

$\propto = -1 - 2$

$\propto = -3$

$Use\ Formula$

$(x - a)(x - b)(x - \propto) = 0$

$Where\ a; b\ \&\ \propto\ are\ complex\ roots$

$[x - (1 + 2i)][x - (1 - 2i)][x - (-3)] = 0$

Either

$[x-(1+2i)]=0 \quad or \quad [x-(1-2i)]=0 \quad or \quad [x-(-3)]=0$

$Therefore\ x = 1+2i \quad \& \quad 1-2i \quad \& \quad -3$

$b)\ Cos5\theta = Cos^5\theta - 10Cos^3\theta Sin^2\theta + 5Cos\theta Sin^4\theta$

Proving on the LHS

$Cos5\theta$ *is obtained from expansion of* $(Cos5\theta + iSin5\theta)$

$(Cos5\theta + iSin5\theta) \equiv (Cos\theta + iSin\theta)^5$

$(Cos\theta + iSin\theta)^5$

Let x be $Cos\theta$ *and y be* $Sin\theta$

$(x+iy)^5$

Using the pascal's triangle to expand $(x+iy)^5$, *you get*

$x^5 + 5x^4iy + 10x^3i^2y^2 + 10x^2i^3y^3 + 5xi^4y^4 + i^5y^5$

Substitute (-1) *for* i^2

$x^5 + 5x^4iy - 10x^3y^2 - 10x^2iy^3 + 5xy^4 - iy^5$

For $Cos5\theta$, *collect all real numbers*

$Cos5\theta = x^5 - 10x^3y^2 + 5xy^4$

Replacing x with $cos\theta$ *and y with* $Sin\theta$

$Cos5\theta = Cos^5\theta - 10Cos^3\theta Sin^2\theta + 5Cos\theta Sin^4\theta$ *As required*

$ii)\ Cos5\theta = Cos^5\theta - 10Cos^3\theta Sin^2\theta + 5Cos\theta Sin^4\theta$

RHS

$Cos^5\theta - 10Cos^3\theta Sin^2\theta + 5Cos\theta(Sin^2\theta)^2$

$Sin^2\theta = 1 - Cos^2\theta$

$Cos^5\theta - 10Cos^3\theta(1 - Cos^2\theta) + 5Cos\theta(1 - Cos^2\theta)^2$

$Cos^5\theta + 10Cos^5\theta + 5Cos\theta(1 - 2Cos^2\theta + Cos^4\theta)$

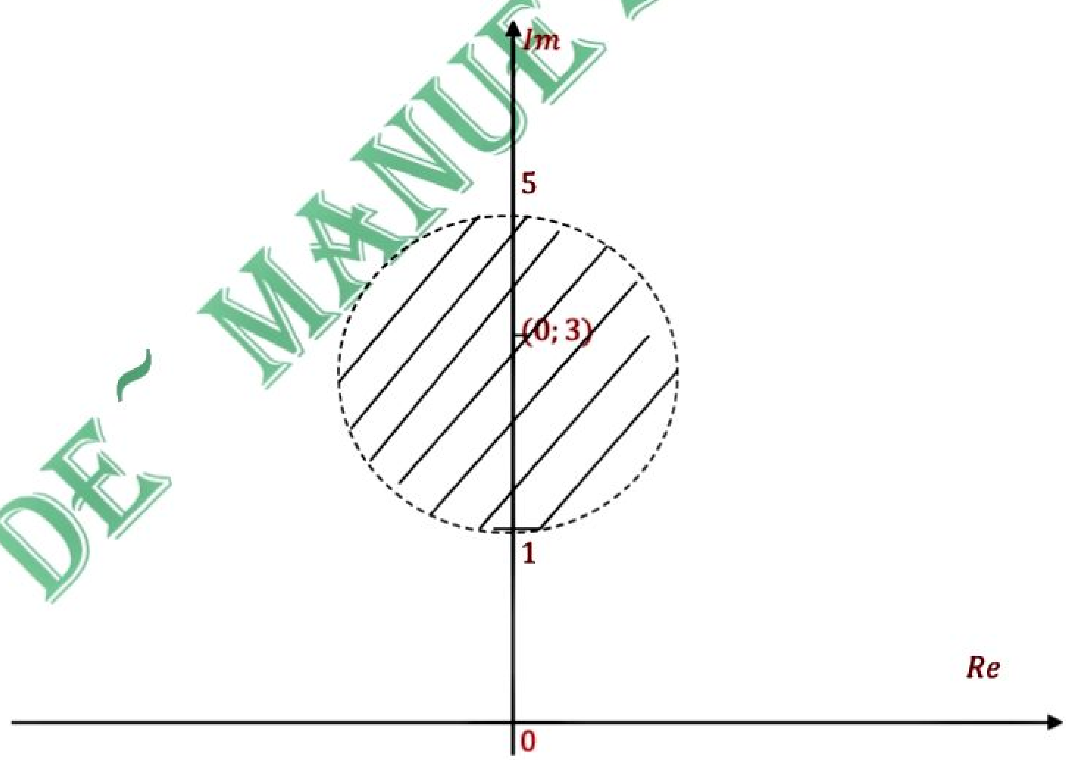
$11 - 10Cos^3\theta + 5Cos\theta - 10Cos^3\theta + 5Cos^5\theta$

$16Cos^5\theta - 20Cos^3\theta + 5Cos\theta$

c) $|z - 3i| < 2$

$Centre = (0; 3i)$

$radius = 2$



8) $AB = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$ $AC = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ $A = \begin{pmatrix} 1 \\ 0 \\ 6 \end{pmatrix}$

a) Finding the Normal (n) of the plane

$$n = \begin{vmatrix} i & j & k \\ 3 & -4 & 0 \\ 2 & -1 & 4 \end{vmatrix}$$

$+i \quad -j \quad +k$

$$n = i\begin{vmatrix} -4 & 0 \\ -1 & 4 \end{vmatrix} - j\begin{vmatrix} 3 & 0 \\ 2 & 4 \end{vmatrix} + k\begin{vmatrix} 3 & -4 \\ 2 & -1 \end{vmatrix}$$

$n = -16i - 12j + 5k$

$r.n = d$

$d = A.n$

$$d = \begin{pmatrix} 1 \\ 0 \\ 6 \end{pmatrix}\begin{pmatrix} -16 \\ -12 \\ 5 \end{pmatrix}$$

$d = -16 + 30$

$d = 14$

Therefore equation of the plane

$$r.\begin{pmatrix} -16 \\ -12 \\ 5 \end{pmatrix} = 14$$

b) $r_1 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \gamma\begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$ $r_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu\begin{pmatrix} -2 \\ -1 \\ -4 \end{pmatrix}$

i) Let θ be tha angle between r_1 & r_2

$$Cos\theta = \frac{d1.d2}{|d1| \times |d2|}$$

$$Cos\theta = \frac{\begin{pmatrix}1\\-2\\-3\end{pmatrix}\begin{pmatrix}-2\\-1\\-4\end{pmatrix}}{\sqrt{(-2)^2+(-1)^2+(-4)^2}\times\sqrt{1^2+(-2)^2+(-3)^2}}$$

$$Cos\theta = \frac{-2+2+12}{\sqrt{14}\times\sqrt{21}}$$

$$Cos\theta = \frac{12}{\sqrt{294}}$$

$$\theta = Cos^{-1}\left(\frac{12}{\sqrt{294}}\right)$$

$$\theta = 45.58469°$$

$$\theta = 45.6°$$

$ii)\ r_1 = \begin{pmatrix}4+\gamma\\-2\gamma\\1-3\gamma\end{pmatrix}$ $\qquad r_2 = \begin{pmatrix}1-2\mu\\1-\mu\\-4\mu\end{pmatrix}$

At the points where the two lines intersect, $r_1 = r_2$

$$\begin{pmatrix}4+\gamma\\-2\gamma\\1-3\gamma\end{pmatrix} = \begin{pmatrix}1-2\mu\\1-\mu\\-4\mu\end{pmatrix}$$

$$4+\gamma = 1-2\mu$$

$$\gamma = -2\mu - 3 - - - -(i)$$

$$-2\gamma = 1-\mu - - - -(ii)$$

$$1-3\gamma = -4\mu - - - -(iii)$$

Substituting (i) *in* (ii)

$$-2(-2\mu-3) = 1-\mu$$

$$4\mu + 6 = 1-\mu$$

$$5\mu = -5$$

$\mu = -1$

$\gamma = -2(-1) - 3$

$\gamma = -1$

If the lines intersect; γ &μ shall satisfy all the 3 equations.

Substituting γ & μ in (iii)

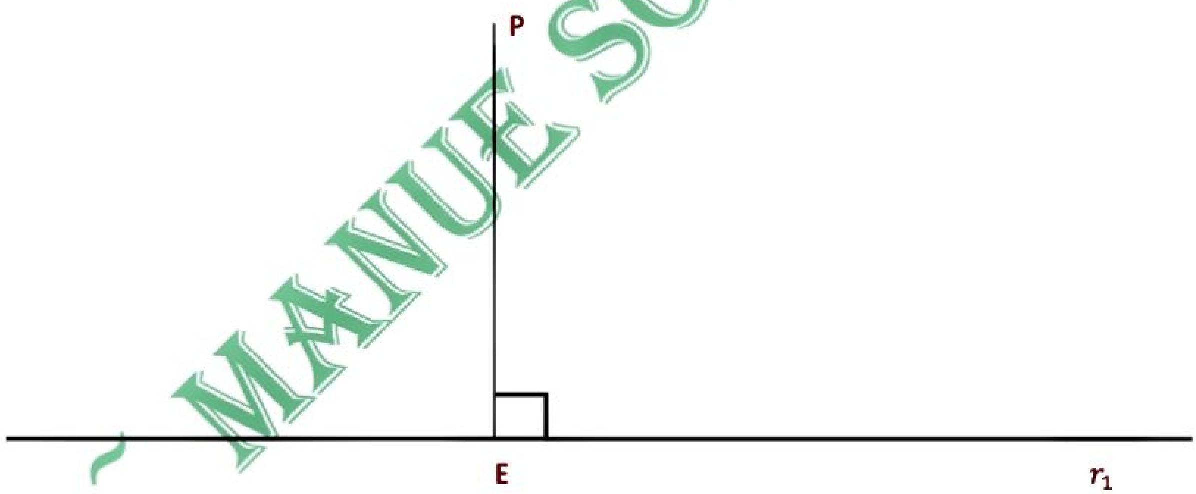
$1 - 3\gamma = -4\mu$

$1 - 3(-1) = -4(-1)$

$4 = 4$

Since the LHS and the RHS of the equation are equal, therefore the two lines intersect

iii)



$$OE = \begin{pmatrix} 4+\gamma \\ -2\gamma \\ 1-3\gamma \end{pmatrix}$$

$PE = OE - OP$

*NB * PE is a direction vector*

$$= \begin{pmatrix} 4+\gamma \\ -2\gamma \\ 1-3\gamma \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ 6 \end{pmatrix}$$

$$PE = \begin{pmatrix} 3+\gamma \\ -5-2\gamma \\ -5-3\gamma \end{pmatrix}$$

Direction vector PE is perpendicular to the direction(d)of the line r_1

$$d.PE = 0$$

$$\begin{pmatrix} 3+\gamma \\ -5-2\gamma \\ -5-3\gamma \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix} = 0$$

$$3+\gamma+10+4\gamma+15+9\gamma = 0$$

$$14\gamma = -28$$

$$\gamma = -2$$

Therefore $\quad OE = \begin{pmatrix} 4-2 \\ -2(-2) \\ 1-3(-2) \end{pmatrix}$

$$OE = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$$

Let d be the distance from P to E

Using coordinates of OE & P

$$d = \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}$$

$$d = \sqrt{(2-1)^2+(4-5)^2+(7-6)^2}$$

$$d = \sqrt{3}\ units$$

9a) $\int_2^4 lnx\,dx$

$$\int_2^4 1 \times lnx\,dx$$

$$let\ u = lnx\ \&\ \frac{dv}{dx} = 1$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$dv = 1.dx$$

$$\int dv = \int 1dx$$

$$v = x$$

$$\int f(x) = uv - \int v\frac{du}{dx}$$

$$\int_2^4 lnx\,dx = xlnx - \int_2^4 \frac{1}{x} \times xdx$$

$$xlnx - \int_2^4 1dx$$

$$[xlnx - x]_2{}^4$$

$$(4ln4 - 4) - (2ln2 - 2)$$

$$ln256 - 4 - ln4 + 2$$

$$\ln\left(\frac{256}{4}\right) - 2$$

$$ln64 - 2$$

$$ii) \int_2^4 lnxdx$$

$$y = lnx$$

$$no\ of\ trapezia = 5 - 1$$

$trapezia = 4$

$$h = \frac{upper - lower}{num\ of trapezia}$$

$$h = \frac{4-2}{4}$$

$$h = \frac{1}{2}$$

$x_0 = 2$

$x_1 = 2.5$

$x_2 = 3$

$x_3 = 3.5$

$x_4 = 4$

$y_0 = ln2$

$y_1 = ln2.5$

$y_2 = ln3$

$y_3 = ln3.5$

$y_4 = ln4$

$$A = \frac{h}{2}\left(y_0 + y_{last} + 2\left(\sum other\ terms\right)\right)$$

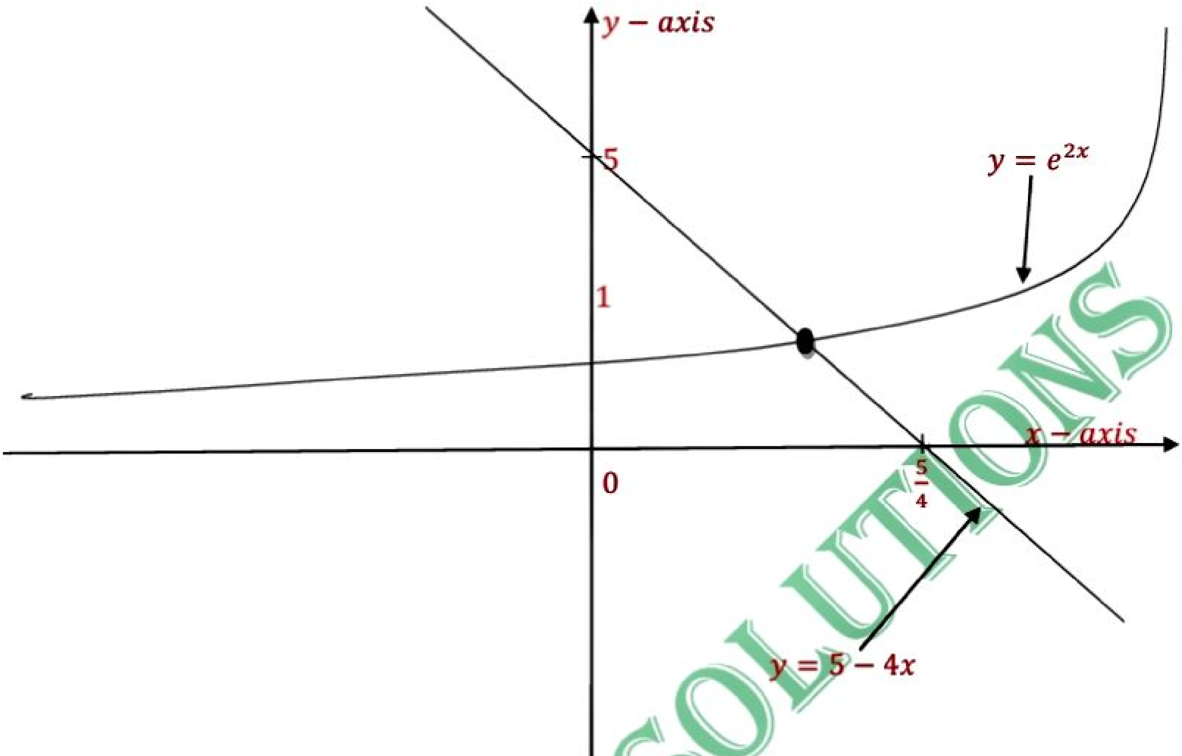
$$A = \frac{\frac{1}{2}}{2}(ln2 + ln4 + 2(ln2.5 + ln3 + ln3.5))$$

$$A = \frac{1}{4}(ln2 + ln4 + 2(ln2.5 + ln3 + ln3.5))$$

$A = 2.15369338$

$A = 2.2 units^2$

$bi)\ e^{2x} = 5 - 4x$



→ *Since the two graphs intersect only once; therefore there is one real root*

ii) $e^{2x} = 5 - 4x$

$f(x) = e^{2x} + 4x - 5$

$f^I(x) = 2e^{2x} + 4$

$x_{n+1} = x_n - \left(\frac{f(x_n)}{f^I(x_n)}\right)$

$x_0 = 0.5$

$x_1 = 0.5 - \left(\frac{e^{2(0.5)} + 4(0.5) - 5}{2e^{2(0.5)} + 4}\right)$

$x_1 = 0.52985$

$x_2 = 0.52985 - \left(\frac{e^{2(0.52985)} + 4(0.52985) - 5}{2e^{2(0.52985)} + 4}\right)$

$x_2 = 0.52934$

$x_2 = 0.53$

$10a)\ y = x^2 + 2x$

$$\frac{dy}{dx} = \lim_{x=0}\left[\frac{f(x+\delta x) - f(x)}{\delta x}\right]$$

$f(x) = x^2 + 2x$

$f(x+\delta x) = (x+\delta x)^2 + 2(x+\delta x)$

$f(x+\delta x) = x^2 + 2x\delta x + \delta x^2 + 2x + 2\delta x$

$$\frac{dy}{dx} = \lim_{x=0}\left[\frac{(x^2 + 2x\delta x + \delta x^2 + 2x + 2\delta x) - x^2 + 2x}{\delta x}\right]$$

$$\frac{dy}{dx} = \lim_{x=0}\left[\frac{2x\delta x + 2\delta x + \delta x^2}{\delta x}\right]$$

$$\frac{dy}{dx} = \lim_{x=0}\left[\frac{\delta x(2x + 2 + \delta x)}{\delta x}\right]$$

$$\frac{dy}{dx} = \lim_{x=0}[2x + 2 + \delta x]$$

$$\frac{dy}{dx} = 2x + 2 + 0$$

$$\frac{dy}{dx} = 2x + 2$$

$b)\ y^2 Cosx - ySin2x = 4$

$$2yCosx\frac{dy}{dx} - y^2 Sinx - \frac{dy}{dx}Sin2x - 2yCos2x = 0$$

$$2yCosx\frac{dy}{dx} - \frac{dy}{dx}Sin2x = y^2 Sinx + 2yCos2x$$

$$\frac{dy}{dx}(2yCosx - Sin2x) = y^2 Sinx + 2yCos2x$$

$$\frac{dy}{dx} = \frac{y^2 Sinx + 2yCos2x}{2yCosx - Sin2x}$$

$$\frac{dy}{dx}=\frac{(2)^2 Sin\frac{\pi}{4}+2(2)Cos\frac{\pi}{2}}{2(2)Cos\frac{\pi}{4}-Sin\frac{\pi}{2}}$$

$$\frac{dy}{dx}=\frac{2\sqrt{2}-0}{2\sqrt{2}-1}$$

$$\frac{dy}{dx}=\frac{8+2\sqrt{2}}{7}$$

$c)y=x^2e^{-2x}$

$$\frac{dy}{dx}=2xe^{-2x}-2x^2e^{-2x}$$

$0=2xe^{-2x}-2x^2e^{-2x}$

$0=2x(e^{-2x}-xe^{-2x})$

$2x=0 \; or \; e^{-2x}-xe^{-2x}=0$

$x=0$

$$\frac{xe^{-2x}}{e^{-2x}}=\frac{e^{-2x}}{e^{-2x}}$$

$x=1$

$For \; x=0$

$y=(0)^2e^{-2(0)}$

$y=0$

$For \; x=1$

$y=(1)^2e^{-2(1)}$

$$y=\frac{1}{e^2}$$

Let A & B be te turning points

$A(0;0)$ & $B\left(1;\frac{1}{e^2}\right)$

$11a)$ *The 4 propertiesof a group are*

Closure

Associativity

Identity

Inverse

b)bi) Inorder to show that $(\boldsymbol{R}, \boldsymbol{x})$ *is a group, we need to verify that it satisfies all 4 group axioms*
1. ***Closure***: *For any* $\boldsymbol{a}, \boldsymbol{b}$ *in* $\boldsymbol{R}$, *the product of* $\boldsymbol{a} * \boldsymbol{b}$ *is also in* $\boldsymbol{R}$
2. *Associativity*: *For any* $\boldsymbol{a}, \boldsymbol{b}$ *and* $\boldsymbol{c}$ *in* $\boldsymbol{R}$, *the product* $(\boldsymbol{a} * \boldsymbol{b}) * \boldsymbol{c}$ *is equal to* $\boldsymbol{a} * (\boldsymbol{b} * \boldsymbol{c})$
3. *Identity*: *There exists an element* $\boldsymbol{e}$ *in* $\boldsymbol{R}$ *such that for any* $\boldsymbol{a}$ *in R, the product* $\boldsymbol{a} * \boldsymbol{e}$ *is equal to* $\boldsymbol{a}$ *and* $\boldsymbol{e} * \boldsymbol{a}$ *is equal to* $\boldsymbol{a}$
4. *Inverse*: *For any* $\boldsymbol{a}$ *in* $\boldsymbol{R}$, *there exists an element* $\boldsymbol{a}^{-1}$ *in* $\boldsymbol{R}$ *such that the product* $\boldsymbol{a} * \boldsymbol{a}^{-1}$ *is equal to the identity element* $\boldsymbol{e}$, *and* $\boldsymbol{a}^{-1} * \boldsymbol{a}$ *is also equal to e. So* $\boldsymbol{a}^{-1} = \frac{1}{\boldsymbol{a}}$

bi) $z_{2+} = \{0; 1\}$

+	0	1
0	0	1
1	1	0

$ii)\ (a * b) = (b * a)$

$Set = \{0; 1\}$

$a = 0; b = 1$

$(a * b) = (b * a)$

$(0 * 1) = (1 * 0)$

$1 = 1$

Therefore z_{2+} *is commutative*

$$12a)\ I = \int \frac{x^2}{\sqrt{4 - x^2}} dx$$

$x = 2Sin\theta$

$$\frac{dx}{d\theta} = 2Cos\theta$$

$$dx = 2Cos\theta d\theta$$

$$I = \int \frac{(2Sin\theta)^2 \times 2Cos\theta d\theta}{\sqrt{4 - (2Sin\theta)^2}}$$

$$I = \int \frac{4Sin^2\theta \times 2Cos\theta d\theta}{\sqrt{4 - 4Sin^2\theta}}$$

$$I = \int \frac{4Sin^2\theta \times 2Cos\theta d\theta}{\sqrt{4(1 - Sin^2\theta)}}$$

$$1 - Sin^2\theta = Cos^2\theta$$

$$I = \int \frac{4Sin^2\theta \times 2Cos\theta d\theta}{2\sqrt{Cos^2\theta}}$$

$$I = \int \frac{4Sin^2\theta \times Cos\theta d\theta}{Cos\theta}$$

$I = \int 4Sin^2\theta \, d\theta$ *As Required*

$b)$ $f(x)$	$f(a)$
$f(x) = Cosx$	$f(a) = Cosa$
$f^I(x) = -Sinx$	$f^I(a) = -Sina$
$f^{II}(x) = -Cosx$	$f^{II}(a) = -Cosa$

$$f(x) = f(a) + \frac{f^I(a)(x-a)}{1!} + \frac{f^{II}(a)(x-a)^2}{2!}$$

$$f(x) = Cosa + \frac{-Sina(x-a)}{1!} + \frac{-Cosa(x-a)^2}{2!}$$

$$f(x) = Cosa - (x-a)Sina - \frac{(x-a)^2 Cosa}{2}$$

$ci) f(x) = Cosa - (x-a)Sina - \frac{(x-a)^2 Cosa}{2}$

$a = \frac{\pi}{6}$

$f(x) = Cos\frac{\pi}{6} - \left(x - \frac{\pi}{6}\right) Sin\frac{\pi}{6} - \frac{(x-\frac{\pi}{6})^2 Cos\frac{\pi}{6}}{2}$

$f(x) = \frac{\sqrt{3}}{2} - \left(x - \frac{\pi}{6}\right) \times \frac{1}{2} - \frac{(x-\frac{\pi}{6})^2(\frac{\sqrt{3}}{2})}{2}$

$f(x) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}(x - \frac{\pi}{6})^2$

$ii)\ f(x) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(x - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}(x - \frac{\pi}{6})^2$

$x = 31°$

$= 0.527$

$f(0.527) = Cos(0.527)$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}\left(0.527 - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}(0.527 - \frac{\pi}{6})^2$

$= 0.864319782$

$= 0.86$