New Curriculum

A *Practical* Approach to Mathematics

'O' Level Revision

- *With summary notes covering syllabus objectives*
- *Model ZIMSEC questions and answers*

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Editor in Chief: Munyaradzi Gunduza
Development Editor: Alistair T. Dzamara and Martha Daka
Text and design layout: Nesbert T. Gotekote

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Preface

A Practical Approach to Mathematics Ordinary Level Revision Book aims to deliver the best revision content which complies with the current Form 1-4 Mathematics syllabus. The author and publishers are pleased to present the revision book in which all the topics in the Mathematics Form 1-4 Syllabus were thoroughly revised. We are grateful to the many readers and users of the Complete Practical Approach Mathematics O-Level Revision Book whose valuable interactions will help to ensure that the book remains relevant and on course.

About this book

- The revision book contains 14 main topics which are: **Real numbers, Sets, Financial Mathematics, Measures and Mensuration, Graphs, Variation, Algebra, Geometry, Statistics, Trigonometry, Vectors, Matrices, Transformation and Probability.** Every main topic is accompanied by subtopics which are followed by **exercises**. At the end of each topic, there is a **revision exercise** which covers all aspects of the topic covered.
- After the discussion of all 14 topics, the revision guide gives you a set of 10 complete specimen examination papers modeled from ZIMSEC standards. Each set is loaded with full Paper 1 and Paper 2 numbered from Examination 1–10.
- Throughout the book, there are worked examples to show the important techniques required to tackle questions. The examples and exercises put mathematics in a real-world context, with a truly Zimbabwean focus.
- The revision guide contains detailed answers to all subtopic exercises, end of topic revision exercises and specimen examination papers. The answers given at the back of the book are concise. However, when answering questions, you should show as many steps in your working as possible.

TOPIC 1 **REAL NUMBERS**

Real numbers

- Real numbers are a union of **rational** and **irrational** numbers.
- These real numbers can be both positive and negative.
- There are various types of numbers under the umbrella term "**real numbers**".

Number types

Integers

- All positive or negative whole numbers are called **integers**.
- Examples include –15; –1; 0; 5; 7; …

Rational numbers

- These are numbers that can be expressed as a **fraction,** in the form $\frac{a}{b}$ where $b \neq 0$.
- Examples include –7; 3; $\sqrt{16}$; 0,8; …

Irrational numbers

- These are numbers that **cannot be expressed** as **fractions**.
- Examples are π and all other **surds** that have no exact square roots such as $\sqrt{3}$; $\sqrt{13}$; $\frac{\sqrt{2}}{3}$, …

Prime numbers

- These are numbers that are divisible only by 1 and itself.
- Examples include 2; 3; 5; 7; 11; …
- **Note**: 1 is not a prime number. The numbers 2 or 3 are prime numbers but –2 or –3 are not because they have four factors which are –1; 1; 2 and –2, the same is true for –3.

Odd numbers

- These are numbers that are **not divisible** by 2.
- They include numbers such as 1; 3; 5; 7; …
- A number can also be considered odd if its unit digit is odd. Examples are but not limited to 12<u>1</u>; 24<u>3</u>; 63<u>7</u>; …

Even numbers

- These are numbers that are **divisible** by 2.
- Examples of even numbers include all multiples of 2.
- A number can also be even if the units digit is even, for instance, 12<u>2</u>; 24<u>4</u>.

Relationship between odd and even numbers

Odd and even numbers on the **number line** are arranged as shown below.

Odd	**Even**	**Odd**	**Even**
1	2	3	4
Odd	**Even**	**Odd**	
5	6	7	

Factors and Multiples

Factors

- These are numbers that wholly **divide** into a given number without leaving a remainder.
- For example 1; 2; 5; and 10 are all factors of 10.

Multiples

- A multiple is a **product** of multiplying a number by an integer.
- For example, the number 4 has the multiples 4; 8; 12; 16; …

Prime factorisation

- This is a **method** of expressing a given number as a product of its **prime numbers**.
- It can also be referred to as **integer factorisation**.

Example 1.1

Express 576 as a product of its prime factors in index form.

Solution

When expressing a number as a product of its prime factors we divide the number repeatedly starting with the smallest prime factor until you get 1.

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Therefore, 576 as a product of its prime factors

$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\mathbf{576 = 2^6 \times 3^2}$

Example 1.2

Find the smallest integer that can be multiplied by 540 to make it a perfect square.

Solution

In order to find the smallest integer, express 540 as a product of its prime factors in index form. All the powers should be even for a number to be a perfect square.

2	540
2	270
3	135
3	45
3	15
5	5
	1

Therefore, $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 \times 3^3 \times 5^1$

Note that the index of 3 and 5 are not even. Therefore, we make use of indices, that if we multiply by 3^1 and 5^1, powers will be added and become even as illustrated.

$540 = 2^2 \times 3^3 \times 3^1 \times 5^1 \times 5^1$

$540 = 2^2 \times 3^4 \times 5^2$

The result is now a perfect square because now all the powers are even.

The smallest value that should multiply 540 to make it a perfect square.

Smallest number = 3×5

Smallest number **= 15**

Exercise 1.1

1. Express 252 as a product of its prime factors. [2]
2. List the prime numbers found between 30 and 40. [2]
3. Write down all common factors of 30 and 36. [2]
4. Find the smallest whole number that is needed to multiply $2^3 \times 3^2 \times 7$ to make it a perfect square. [2]

Example 1.5

Find the HCF and LCM of 72, 96 and 108.

Solution

2	72
2	36
2	18
3	9
3	3
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	108
2	54
3	27
3	9
3	3
	1

$72 = 2^3 \times 3^2$

$96 = 2^5 \times 3^1$

$108 = 2^2 \times 3^3$

It can be noted that the lowest indices are 2^2 and 3^1 whilst the highest indices are 2^5 and 3^3.

HCF $= 2^2 \times 3^1$

$= 4 \times 3$

$=$ **12**

LCM $= 2^5 \times 3^3$

$= 32 \times 27$

$=$ **864**

Exercise 1.2

1. Given that $294 = 2^1 \times 3^1 \times 7^2$ and $784 = 2^4 \times 7^2$ respectively. Find the:

(a) largest integer that is a factor of both 294 and 784. [2]

(b) square root of 784. [2]

2. Write down the smallest number which is both a multiple of 12 and 18. [2]

3. $2^1 \times 3^2 \times 11^1$ and 90 is $2^1 \times 3^2 \times 5^1$. Use these results to find the:

(a) smallest integer k, such that $198k$ is a perfect square. [2]

(b) HCF of 198 and 90. [2]

Surds

- **Surds** are numbers or terms, **under the root**, such as $\sqrt{2}$ $\sqrt{15}$, $\sqrt{5}$.
- If we cannot find the exact square root of a number, use a calculator and leave the number to a certain degree of accuracy or just leave it in **surd form**.
- There are some numbers that may need to be simplified even if they have no exact square root value.
- For example $\sqrt{20}$. 20 has no exact square root but can be simplified to 5×4, where 4 has an exact square root value.

$$\sqrt{20} = \sqrt{5 \times 4}$$
$$= \sqrt{4} \times \sqrt{5}$$
$$= 2 \times \sqrt{5}$$
$$= 2\sqrt{5}$$

Example 1.6

Simplify $\sqrt{75}$, leaving your answer in the form $a\sqrt{b}$.

Solution

$$\sqrt{75} = \sqrt{25 \times 3}$$
$$= \sqrt{25 \times 3}$$
$$= \sqrt{25} \times \sqrt{3}$$
$$= 5 \times \sqrt{3}$$
$$\sqrt{75} = \mathbf{5\sqrt{3}}$$

Addition and subtraction of surds

- Note that we can only **add** or **subtract** the surd if the numbers inside the **roots** are the **same**.
- For example, $\sqrt{x} + \sqrt{x} = 2\sqrt{x}$.

Example 1.7

Simplify the following.

(a) $\sqrt{2} + \sqrt{8}$

(b) $\sqrt{20} - \sqrt{5}$

Solution

1. $\sqrt{2}+\sqrt{8} = \sqrt{2}+\sqrt{8}$

$= \sqrt{2}+\sqrt{2}\times\sqrt{4}$

$= \sqrt{2}+\sqrt{2}\times 2$

$= \sqrt{2}+2\sqrt{2}$

$\sqrt{2} + \sqrt{8} = \mathbf{3\sqrt{2}}$

2. $\sqrt{20}-\sqrt{5} = \sqrt{20}-\sqrt{5}$

$= \sqrt{4\times 5}-\sqrt{5}$

$= \sqrt{4}\times\sqrt{5}-\sqrt{5}$

$= 2\times\sqrt{5}-\sqrt{5}$

$= 2\sqrt{5}-\sqrt{5}$

$\sqrt{20} - \sqrt{5} = \mathbf{\sqrt{5}}$

Note: Add or subtract the coefficients of the surds after making the digits inside the roots the same. The solution will be in terms of the surds as $2x + x = 3x$ or $5x – 2x = 3x$.

Exercise 1.3

1. Simplify the following leaving your answer in the form $a\sqrt{b}$.

(a) $\sqrt{(5p+p)^2}$ [2]

(b) $\sqrt{50}$ [2]

2. Evaluate each of the following.

(a) $\sqrt{3}+\sqrt{12}$ [2]

(b) $3\sqrt{2}+5\sqrt{2}$ [2]

3. Find the value of $\sqrt{0,0081}$ [2]

Note: A square root of a decimal number for example, $\sqrt{0,000004}$ is the same as $\frac{\sqrt{4}}{\sqrt{1\,000\,000}}$.

Multiplication of surds

- If the numbers under the roots are the **same**, then we get that **same number** as our solution when we multiply.
- For example $\sqrt{a}\times\sqrt{a}=a$.
- If the numbers inside the roots are **different** then we multiply to get a **single value** under the root.
- For instance, $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ but this only applies if the coefficient of the surd is one.

Example 1.8

Simplify **(a)** $\sqrt{3}\times\sqrt{3}$

(b) $\sqrt{2}\times\sqrt{7}$

(c) $3\sqrt{2}\times 4\sqrt{2}$

Solution

(a) $\sqrt{3}\times\sqrt{3} = 3$ *(from the laws of surds)*

(b) $\sqrt{2}\times\sqrt{7} = \sqrt{7\times 2}$

$= \sqrt{14}$

(c) $3\sqrt{2}\times 4\sqrt{2} = 3\times 4\times\sqrt{2}\times\sqrt{2}$

$= 12\times 2$

$= \mathbf{24}$

Exercise 1.4

Evaluate and leave your answer in surd form where necessary.

1. $3\sqrt{2}\times 5\sqrt{2}$ [2]

2. $7\sqrt{3}\times 3\sqrt{2}$ [2]

3. $\sqrt{a}\times 2\sqrt{a}$ [2]

4. $\sqrt{20}\times\sqrt{5}$ [2]

5. $3\sqrt{8}\times\sqrt{12}$ [2]

Number patterns

- Number patterns are a series of numbers which display a **recurring** and **consistent behavioural** pattern.

The third *dp* is higher than 5, the second *dp* will be rounded off upwards and the final answer becomes 365,88.

(b) 685,62<u>3</u>

The third *dp* is less than 5, the second *dp* will be rounded off downwards to 685,62.

Significant figures (sf)

- The first **significant figure** is counted from the **left** moving to the **right**.
- The digits are significant starting from a **non-zero** digit but a zero which is between non-zeros is **significant**.

Example 1.11

Round off 53,781 to:

(a) 1*sf.*
(b) 2*sf.*
(c) 3*sf.*
(d) 4*sf.*

Solution

(a) 53,781 to 1*sf* = **50**
(b) 53,781 to 2*sf* = **54**
(c) 53,781 to 3*sf* = **53,8**
(d) 53,781 to 4*sf* = **53,78**

Note: Therefore, to maintain the value of a number with zeros that are found before the comma after rounding off a number such as in example **(a)**, the zeros are considered significant and must be written. However, those that are found after the comma are not considered significant and are thus neglected.

Example 1.12

Round off 138 756 to :

(a) 1*sf.*
(b) 2*sf.*
(c) 3*sf.*
(d) 4*sf.*
(e) 5*sf.*

Solution

(a) 138 756 to 1*sf* = **100 000**
(b) 138 756 to 2*sf* = **140 000**
(c) 138 756 to 3*sf* = **139 000**
(d) 138 756 to 4*sf* = **138 800**
(e) 138 756 to 5*sf* = **138 760**

Exercise 1.6

1. Express 0,9083 to the nearest tenth. [1]

2. **(a)** Round off 784,709 correct to 4*sf.* [1]
(b) Correct 7,369 to 3*sf.* [1]
(c) Express 49,9973 correct to 4*sf.* [2]

3. A cube has an edge of length 1,99*cm*; express the length of the edge correct to 1*sf.* [2]

4. Find the value of 10,03 × 0,17 correct to 3*sf.* [3]

Estimations

When estimating the value of a number, the number should be **rounded off** to the **nearest degree of stated accuracy** and in its simplified form.

Example 1.13

Estimate the value of each number and write the answer correct to 1*sf.*

$$\frac{0,387 \times (7,03)^2}{\sqrt[3]{8,11}}$$

Example 1.18

$-1 + 4 = \mathbf{3}$

Solution

- Take –1 as the initial point for movement.
- The second number is +4, which has a positive sign; the direction of movement is to the right and moving 4 steps.
- Moving 4 steps to the right direction from –1 gives +3.

Example 1.19

$2 - 3 = \mathbf{-1}$

Solution

- Take +2 as initial point for movement.
- The second number is –3, which has a negative sign, the direction of movement is to the left and moving 3 steps.
- Moving 3 steps to the negative direction from –1 gives the difference between the two numbers, that is, –1.

Example 1.20

$-1 - 1 = \mathbf{-2}$

Solution

- Take –1 as our initial point for movement.
- The second number is –1, which has a negative sign therefore the direction of movement is to the left and moving just 1 step.
- Moving 1 step to the left direction from –1 gives us the sum of the two numbers, that is, –2.

Exercise 1.9

1. $5 + 6$ [1]
2. $3 - 6$ [1]
3. $-12 + 5$ [1]
4. $-7 + 3$ [1]
5. $-25 - 13$ [1]
6. $-9 - 6$ [1]
7. $0 - 20$ [1]
8. $-205 - 22$ [1]
9. $-58 + 50$ [1]
10. $72 - 12$ [1]

Multiplication and division of directed numbers

- If numbers have the **same signs**, the solution for multiplication or division gives a **positive answer**.
- If numbers are of **different signs**, the solution for multiplication or division gives a **negative answer**.

Example 1.21

(a) $4 \times 4 = \mathbf{16}$

(b) $-10 \times -5 = \mathbf{50}$

(c) $\frac{100}{5} = \mathbf{20}$

(d) $-30 \div -6 = \mathbf{5}$

Notice that the signs are the same between the numbers, therefore multiply or divide to get the answer with a positive sign.

Example 1.22

(a) $12 \times -3 = \mathbf{-36}$

(b) $-9 \times 8 = \mathbf{-72}$

(c) $28 \div -7 = \mathbf{-4}$

(d) $-18 \div 6 = \mathbf{-3}$

Notice that the signs are different between the numbers, multiply or divide and get the answer with a negative sign.

Exercise 1.10

1. 315×-26 [1]
2. -48×35 [1]
3. 92×-65 [1]
4. -132×-58 [1]
5. -18×-16 [1]
6. $225 \div -5$ [1]
7. $-72 \div -9$ [1]
8. $-117 \div 9$ [1]
9. $625 \div -25$ [1]
10. $-102 \div -17$ [1]

Fractions and percentages

- A **fraction** represents a part of a whole or more generally any number of equal parts.
- A **percentage** is a number or ratio expressed as a fraction of 100. The sign % denotes percentage.

Example 1.23

Express 0,341 as a:

(a) common fraction.
(b) percentage.

Solution

(a) $0{,}341 = \frac{341}{1000}$

(b) $0{,}341 \times 100\% = \frac{341}{1000} \times 100\%$
$= \mathbf{34{,}1\%}$

Example 1.24

Simplify $\frac{2}{7} + \frac{2}{3} - \frac{5}{42}$ giving your answer as a percentage.

Solution

$$\frac{2}{7} + \frac{2}{3} - \frac{5}{42} = \frac{2 \times 6 + 2 \times 14 - 5 \times 1}{42}$$
$$= \frac{12 + 28 - 5}{42}$$
$$= \frac{5}{6}$$

Example 1.25

Find 12% of 128g.

Solution

Notice that, to remove a number from a percentage we simply divide by 100%.

$$\frac{12\%}{100\%} \times 128g = \frac{12}{100} \times 128g$$
$$= \mathbf{15{,}36g}$$

Exercise 1.11

1. Express 0,096 as a:
   (a) common fraction in its simplest form. [2]
   (b) percentage. [1]
2. Evaluate 36 minutes as a percentage of two hours. [2]
3. Convert $\frac{3}{8}$ to a percentage. [1]
4. Find 15% of \$270. [1]
5. **(a)** Evaluate $\frac{2}{5} + \frac{1}{6} - \frac{1}{30}$, giving your answer as fraction in its simplest form. [2]
   (b) Hence, express $\frac{2}{5} + \frac{1}{6} - \frac{1}{30}$ as a percentage. [1]

Exercise 1.14

1. Express the following in standard form.
   (a) 39 000 [1]
   (b) 0,00387 [1]
   (c) 754, 96 [1]
2. Write 160×10^{-4} in ordinary form. [1]
3. Evaluate $3{,}25 \times 10^4 \times 10^{-6}$ and express your answer:
   (a) in ordinary form. [1]
   (b) in standard form. [1]
   (c) as a decimal fraction. [1]
   (d) as a common fraction in its lowest terms. [1]

Addition and subtraction of numbers in standard form

- Two basic methods can be used to add or subtract numbers in standard form. The methods are:
  - converting the given numbers to ordinary form is followed by adding or subtracting and subsequent converting the solution back to standard form.
  - applying the factorisation method.

Example 1.30

Given that $M = 3{,}6 \times 10^2$ and $N = 8 \times 10^{-1}$, find in standard form the value of $M + N$:

Solution

Converting numbers to ordinary form and back to standard form after operating them.

$M = 3{,}6 \times 10^2$
$= 360$ (in ordinary form)
$N = 8 \times 10^{-1}$
$= 0{,}8$ (in ordinary form)

Therefore, $M + N = 360 + 0.8$
$= 360{,}8$

Converting to a standard form produces the following: $\mathbf{360{,}8 = 3{,}608 \times 10^2}$

Example 1.31

Given that $M = 3{,}6 \times 10^2$ and $N = 8 \times 10^{-1}$, find in standard form the value of $M + N$.

Solution

Using factorisation method for evaluation

$$2 \times 10^3 - 8 \times 10 = 10^2\left(\frac{2 \times 10^3}{10^2} - \frac{8 \times 10^2}{10^2}\right)$$
$$= 10^2 (2 \times 10^3 \div 10^2 - 8 \times 10^2 \div 10^2)$$
$$= 10^2 (2 \times 10^{3-2} - 8 \times 10^{2-2})$$
$$= 10^2 (2 \times 10^1 - 8 \times 10^0)$$
$$= 10^2 (20 - 8)$$
$$= 10^2 (12)$$
$$= 100 \times 12$$
$$= 1\ 200$$
$$= \mathbf{1{,}2 \times 10^3}$$

Note: Any method of choice can be used when solving the addition or subtraction of numbers in standard form unless stated otherwise.

Exercise 1.15

1. Evaluate $(4 \times 10^2) + (6 \times 10^3) + (1 \times 10^5)$ and express your answer in standard form. [4]
2. Given that $M = 4 \times 10^5$ and $N = 5 \times 10^2$, express the value of $M + N$ in standard form. [3]
3. Solve $(20 \times 10^2) - (8 \times 10^2)$. [3]

Division and multiplication of numbers in standard form

- In division and multiplication of numbers in standard form, the solution must be expressed in the form $\boldsymbol{A \times 10^n}$, $\boldsymbol{1 \leq A < 10}$ unless stated otherwise.
- When multiplying numbers in standard notation, the new $\boldsymbol{A}$ is found by multiplying $A \times A$ of the two or more given numbers in standard form. For the term 10^n, we add the powers to find the new 10^n.
- In division, the new A is found by dividing $A \div A$ from the two or more given numbers in standard form. For the term 10^n, we subtract the powers to find the new 10^n.

Example 1.32

Simplify and leave your answer in standard form.

(a) $(9{,}6 \times 10^5) \div (3 \times 10^3)$
(b) $(1{,}2 \times 10^9) \times (3 \times 10^5)$

Solution

(a) $(9{,}6 \times 10^5) \div (3 \times 10^3) \quad = \frac{9{,}6}{3} \times 10^{5-3}$
$= \boldsymbol{3{,}2 \times 10^2}$

(b) $(1{,}2 \times 10^9) \times (3 \times 10^5) \quad = (1{,}2 \times 3)\ 10^{9+5}$
$= \boldsymbol{3{,}6 \times 10^{14}}$

Note: After division or multiplication of numbers in standard form, the new $\boldsymbol{A}$ should be maintained in the range $\boldsymbol{1 \leq A < 10}$. If the value of the new $\boldsymbol{A}$ is found to be less than 1 or greater than or equal to 10 which is not in the required range $\boldsymbol{1 \leq A < 10}$, then we are required to move the decimal point in accordance, to control over our $\boldsymbol{A}$ in the range.

Exercise 1.16

1. Given that $m = 4 \times 10^6$ and $n = 2{,}4 \times 10^{-3}$, calculate the following giving your answer in standard form.

(a) mn [1]
(b) $\frac{m}{n}$ [1]

2. Given that $m = 3{,}6 \times 10^2$ and $n = 8 \times 10^{-2}$, find in standard form the value of mn. [1]

3. Simplify $(1{,}2 \times 10^{-4})^2$ and give your answer in standard form. [1]

Hint: $= (1{,}2 \times 10^{-4})^2 = (1{,}2 \times 10^{-4}) \times (1{,}2 \times 10^{-4})$.

4. Given that $m = 3 \times 10^2$ and $n = 5 \times 10^{-4}$, express in standard form the value of the following.

(a) mn
(b) $\frac{n}{m}$ [3]

5. It is given that $m = 2{,}1 \times 10^7$ and $n = 3 \times 10^4$, expressing your answer in standard form. Find the following:

(a) $m \div n$. [1]
(b) $n^2 + m$. [2]

Number bases

- All the digits of a number can never be greater or equal to the base of that number.
- If we have a number in bases 5, then all the digits on that number begin at 4 going downwards.

Converting any number from any base to base 10

- Two methods can be used which are:
  - expansion method.
  - repeated multiplication.

(b) To find the corresponding area, initially find the ratio of the areas by squaring the given scale factor. This results in $(1cm)^2$ to $(50cm)^2$. However, leave your answer square metres (m^2). Therefore, convert the scale to square metres.

$(1cm)^2$ to $(0{,}5m)^2 = 1cm^2$ to $0{,}25m^2$

Note: Do not neglect to also square the numbers which are the coefficients of units.

$1cm^2 = 0{,}25m^2$

$63cm^2 = ?$ more

$\frac{63\text{cm}^2}{1\text{cm}^2} \times 0{,}25m^2$

$= \mathbf{15{,}75m^2}$

Exercise 1.22

1. Express the scale of $2cm$ to $5m$ in form $1{:}n$, where n is a whole number. [2]

2. All lengths on a map are $\frac{1}{500}$ of their actual lengths. Calculate the:

(a) actual length of the line is represented on the map by a line of $7{,}3cm$. [4]

(b) the area on the map which represents an actual area of $525m^2$. [4]

Revision exercise

1. (a) Write down the value of:

(i) $(0{,}06)^2$. [1]

(ii) $\sqrt{0{,}0144}$. [1]

(b) Find the LCM of 18 and 24. [2]

2. (a) Simplify

	hours	minutes	seconds
	10	25	42
+	8	41	30

[2]

(b) Express 4,65 minutes in minutes and seconds. [1]

3. (a) State the square of 4. [1]

(b) Evaluate $125 \times \sqrt{144}$. [2]

4. (a) Convert the fraction $\frac{3}{8}$ to a percentage. [1]

(b) Convert 9% to decimal. [1]

5. (a) Find the value of n such that 0,0075 can be expressed in standard form as $7{,}5 \times 10^n$. [1]

(b) Write down the value of $4{,}32 \times 10^4$ in ordinary form. [1]

6. Express 2046,489 to:

(a) the nearest ten. [1]

(b) 2 decimal places. [1]

(c) 2 significant figures. [1]

7. Given that $M = 3{,}6 \times 10^2$ and $N = 8 \times 10^{-1}$, find in standard form the value of :

(a) MN. [2]

(b) $M + N$. [2]

8. (a) Convert 408 to a number in base 6. [1]

(b) Write down $2 \times 3^4 + 1 \times 3^2 + 2 \times 3^1$ as a number in base 3. [2]

(c) Given that $42_x + 53_x = 125_x$, find x. [2]

9. (a) Write down the greatest possible digit of a number in base 8. [1]

(b) Convert 53_6 to a number in base 2. [2]

(c) Evaluate: **(i)** $432_5 + 414_5$ [2]

(ii) $4_5 - 2_3 + 1_2$ [1]

10. Three farmers share 120 hectares of land in the ratio 3:4:5. Calculate the area of the smallest share. [2]

11. The dimensions of a rectangle measured to the nearest centimetre are $42cm$ by $81cm$.

(a) State the least possible width of the rectangle. [1]

(b) Calculate the least possible perimeter of the rectangle. [2]

12. Write down the next two terms in the sequence 12; 8; 4; 0; ☐ and ☐. [2]

TOPIC 2 SETS

Definition of sets

- Sets are a way of categorising or grouping numbers or quantities of **elements**.
- A set is made up of elements that can be presented in **curly brackets** or inside a **venn diagram**.

Set notation

- A set is denoted by a **capital letter** and **elements** by **small letters**.
- All other sets are produced from the **universal set**, which is also called the mother set and denoted by the symbol ξ.
- If an **element** is part of a set, it can be written in set notation belongs to set of vowels can be written as : $a \in V$.
- However, if an element is not part of a set, it can be written in set notation. *b* **does not belong** to a set of vowels V, therefore, $b \notin V$.
- Take the universal set to be a set of all alphabet letters, then a set of vowels, then we can say B is a **subset** of the universal set, which is written in set notation as $B \subset \xi$.
- Set A is being a set of even numbers, it is not contained in the universal set. It is **not a subset** of the universal set. This can be written in set notation as $A \not\subset \xi$.
- If we decide to produce set A, it will not have elements since there are no even numbers in set notation can be represented as $A = \emptyset$ or $A = \{\ \}$ which implies that it is an **empty set.**
- The number of elements in a set can be found by counting the number of elements in a set. It is written in set notation as **n(A)** to imply the numbers of elements in set A. If A = {a; e; i; o; u}, then $n(A) = 5$.

Operation of sets

Union of sets

- This refers to a set that takes all the elements from the given sets without repeating the common elements.
- Union of sets is denoted by the symbol $\cup$.

Example 2.1

Given that: $\xi = \{6; 9; 12 \cdots 30\}$

A = {set of factors of 60}

B = {set of multiples of 6}

Find $A \cup B$.

Solution

- Initially, write down the set A and B separately.
- Remember the elements of set A and set B are taken from the universal set.
- Regarding the universal set, the first element is 6 and ending with 30.

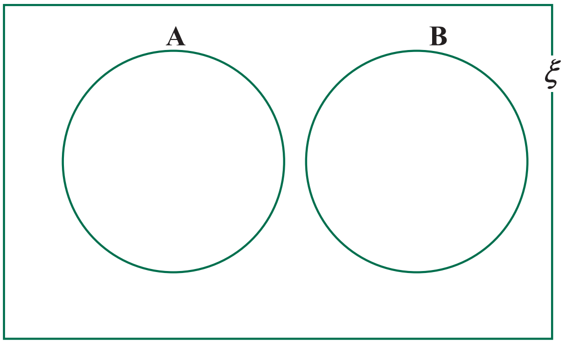


Fig. 2.3 Disjointed sets

- The union of the sets A and B is the sum of the elements in set A and B. It is represented as $n(A) + n(B)$

Independent events

- If sets A and B have common elements, then they are have an intersection.

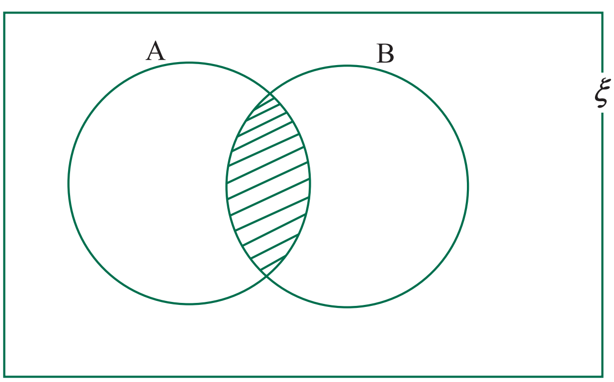


Fig. 2.4 Independent events

- The number of elements in $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- Hence, find the union of set A and set B less the intersection of the two sets, represented by the shaded part on Fig. 2.3.

Example 2.7

Two sets A and B are such that:

$A \cap B \neq \emptyset$.

$A \cup B = \xi$.

$n(A) = 15$.

$n(B) = 25$.

$n(A \cup B) = 30$.

Find $n(A \cap B)$.

Solution

$A \cap B \neq \emptyset$ means the events are independent and there exists an intersection between A and B.

Thus, to find the union of the two sets A and B will bring $n(A \cap B)$ into the equation.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$30 = 15 + 25 - n(A \cap B)$

$30 = 40 - n(A \cap B)$

$\therefore n(A \cap B) = 40 - 30$

$\mathbf{n(A \cap B) = 10}$

Exercise 2.5

1. A and B are independent events. Write down in their simplest form:

(a) $A \cap A'$. [2]

(b) $A \cup A'$. [2]

(c) $(A \cap B) \cup (A \cap B')$. [2]

2. It is given that: $\xi = \{x: 2 \leq x \leq 20, x$ is an integer$\}$. If $P = \{x: x$ is a prime number$\}$ and $Q = \{x: 4 \leq x \leq 17\}$,

(a) list the elements of P. [2]

(b) find $n(Q' \cap P)$. [2]

Revision exercise

1. It is given that:

$\xi = \{x: 31 \leq x \leq 37$ and x is an integer$\}$

$P = \{x: x$ is a multiple of 3$\}$

$Q = \{x: x$ is a factor of 99$\}$

$R = \{x: x$ is a prime number$)$

(a) List all the elements of R. [1]

(b) Write down $n(P \cup R)'$. [1]

(c) List all the elements of $(P \cup Q \cup R)'$.[2]

2. Fig. 2.4 shows three sets A, B and C.

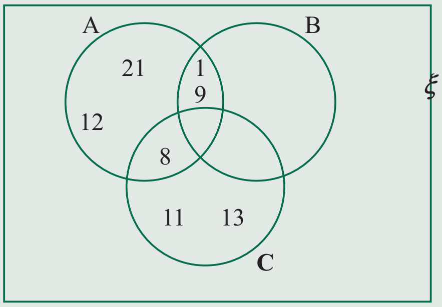


Fig. 2.5

(a) List down all the elements of:

(i) $A \cap B$. [1]

(ii) $(A \cup B)' \cap C$. [1]

(b) Find $n(A \cup C)$. [1]

3. ξ, is a class of 46 learners. B is the set of learners who study Biology whilst C is a set of learners who study Chemistry 23 learners study Biology, 32 learners study Chemistry, x learners study both Biology and Chemistry and 5 learners study neither Biology nor Chemistry.

(a) Complete the venn diagram below. [2]

(b) Find x. [2]

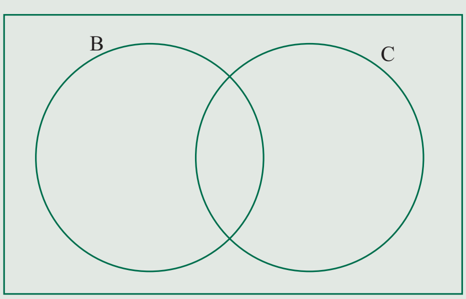


Fig. 2.6

4. $\xi = \{1; 2; 3; 4; 5; 6; 7; 8; 9\}$

P = {prime numbers}

S = {perfect square numbers}

M = {multiples of 3}

(a) List the elements of P. [2]

(b) Write down $(P \cap S \cap M)$. [1]

(c) Complete the venn diagram. [3]

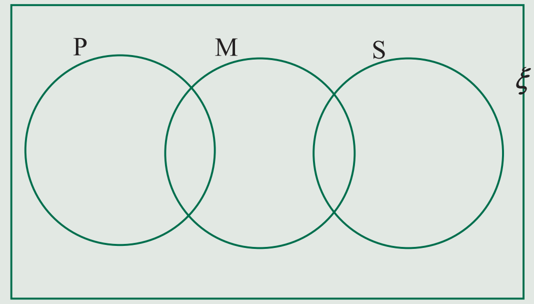


Fig. 2.7

5. There are 25 children in a class, 9 of these are in the debate club whilst 13 are in the history club. ξ = {children in a class}, D = {children in debating club}, H = {children in history class} and $n(D \cap H) = 5$.

(a) Find:

(i) the number of children in the debate club only. [1]

(ii) $n(D \cup H)$. [1]

(b) Write down the number of children who are in neither the debate club nor the history club. [1]

6. It is given that: $n(\xi) = 14$

$n(P) = 7$

$n(P \cap Q) = 2$

$(P \cup Q) = 13$

(a) Show the given information on a venn diagram. [2]

(b) Find the following:

(i) $n(Q)$. [1]

(ii) $n(Q \cap P')$. [2]

TOPIC 3 FINANCIAL MATHEMATICS

Consumer arithmetic

- **Consumer arithmetic** is a branch in mathematics that uses basic math skills in real life situations.
- These real life situations comprise of shopping, calculation of taxes and estimating monthly budget in a household.

Discount, profit and loss

- **Discount** refers to a **deduction** of a certain amount from the marked price of a good or article.
- If an article or a good is sold for more than its cost price then the difference is the **profit**.
  profit = selling price – cost price
- If an article or a good is sold for less than it cost then it is sold at a **loss**.
  loss = cost price – selling price

Example 3.1

A trader bought a tonne of goods worth $2 500.

(a) Calculate the cost price per kilogramme.
(b) If the goods were later sold at $2,10 per kilogramme. Calculate the percentage loss.

Solution

(a) Since they are 1 000*kg* in a tonne.

$$\text{Cost per kilogramme} = \frac{\$2\,500,00}{1000}$$
$$= \mathbf{\$2,50}$$

(b) Loss = Selling price – Cost price
= $2,10 – $2,5
= **–$0,40**

There was a loss of $0,40.

$$\text{Percentage loss} = \frac{\text{loss}}{\text{profit}} \times 100\%$$
$$= \frac{\$0,40}{\$2,50} \times 100\%$$
$$= \mathbf{16\%}$$

Example 3.2

A shop owner made a loss of 5% by selling an article for $11,40. Calculate the cost price of the article.

Solution

Assume that the cost price be 100%. If the shop owner made a loss of 5%, this means the article was sold at 95%.

$11,40 = 95%
? = 100%

$$\therefore \text{cost of the article} = \frac{100}{95} \times \$11,40$$
$$= \$12$$

Example 3.3

During a sale, a shop reduced all its prices by 20%. Calculate the original price of an article that was sold during the sale for $440 000.

Solution

We must understand that initially, the original selling price was **100%.** Therefore, the current selling price after a **20%** reduction becomes **80%**.

Example 3.14

The table below shows Mr Ndhlela's bank statement for October 02 to November 08.

Statement period: 2021-10-02 to 2021-11-08

Date	Description	Ref.	Withdrawals	Deposits	Balance
2021-10-02	Previous balance				0,55
2021-10-08	Payroll deposit–Hotel			694,81	695,36
2021-10-08	Web bill payment – Master card	9685	2 00,00		495,36
2021-10-12	ATM withdrawal–INTERAC	3990	21,25		474,11
2021-10-12	Fees–Interac		1,50		472,61
2021-10-21	Interac Purchase–Electronics	1975	2,99		469,62
2021-10-22	Web bill payment–AMEX	3314	300,00		169,62
2021-10-22	ATM withdrawal–POSB	0064	100,00		69,62
2021-10-25	Interac Purchase–Supermarket	1559	29,08		40,54
2021-10-29	Interac refund–Electronics	1975		2,99	43,53
2021-10-30	Telephone bill payment–VISA	2475	6,77		36,76
2021-10-31	Payroll deposit–HOTEL			694,81	731,57
2021-11-02	Web funds transfer–From Savings	2620		50,00	781,57
2021-11-02	Pre-Auth Payment–INSURANCE		33,55		748,02
2021-11-04	Cheque No. – 409		100,00		648,02
2021-11-06	Mortgage Payment		710,49		-62.47
2021-11- 07	Fees Overdraft		5,00		-67,47
2021-11-08	Fees – Monthly		5,00		-72,47
	Totals		1 515,63	1 442,61	

Exercise 3.8

1. The bank statement below is for Mrs C Mabhiza for 01/10/21 to 01/11/21.

Date	Description	Withdrawals	Deposits	Balance
2021-10-01	Previous balance			**27 584,38**
2021-10-05	Internet bill	**X**		27 508,39
2021-10-06	Electric bill	253,68		27 254,71
2021-10-10	Check No. 4598 (*payment from Nesley Mashongera*)		456,84	**Y**
2021-10-12	Deposit from Credit Card Processor		5 891,26	33 602,81
2021-10-16	Payroll run	3 894,75		29 708,06

2021-10-18	Debit transaction (*main office wholesale*)	243,46		29 464,60
2021-10-18	Rent bill	750,00		28 714,60
2021-10-21	Check No.234 *(payment from Mingo & Mlambo group)*		268,84	28 983,44
2021-10-23	Payroll run	3 743,23		25 240,21
2021-10-25	Deposit		3 656,45	28 896,66
2021-11-01	Debit transaction *(ABC Business supplies)*	1 548,96		27 347,70
	Ending balance			**Z**

Find the value of the following:

(a) X
(b) Y
(c) Z [10]

Revision exercise

1. During a sale, the price of the radio was reduced by 28% to $486.

(a) Calculate the original price of the radio. [3]

(b) When $4 400 amounts to $5 786 in 18 months. Find the rate of simple interest per annum. [4]

(c) The table below shows part of Mr Banda's telephone bill for January.

January monthly rental Cost of metered calls from 010561 to 012777 (---units at---cents per unit)	$ ____________ $886,40
Sub total Sales tax at $17\frac{1}{2}$%	$910,40 $____________
Total amount due	$____________

Calculate the:

(i) number of units used. [1]
(ii) charge per unit in cents. [1]
(iii) sales tax. [1]

2. A shop sells a packet of biscuits for $23,46 making a profit of 15% on the cost price. Calculate the cost price. [3]

3. A car loses 55% of its value after four years. If it costs $8 500 when it was new. Calculate the value of the car after four years. [3]

4. Find the time in which $72 will earn $189 simple interest at $3\frac{1}{2}$% per annum.[3]

5. An advert in a Hair and Beauty shop reads: A hair dressing session was originally marked at $25 each.

TOPIC 4 MEASURES AND MENSURATION

Measures

- This refers to the finding of the size, amount or degree of any discussion of concern by use of an instrument marked as standard units.
- The knowledge of **measures** in mathematics is crucial in making assessments in real life.

Units of measure

Time

- **Time** is a continuous existence of events in a sequential way that occurs is an irreversible succession from the past.
- Time can be measured in any particular time interval used as a standard way of measuring.
- Examples of units of time are second, minutes, hours, days and weeks.

Mass

- **Mass** can be referred to as matter with no definite shape.
- It can be measured in grammes (g), kilogrammes (kg) and tonne (t).
- 1 tonne = 1 000kg.

Length

- This is the measurement of an object from end to end.
- Units used for **length** include m, cm, and mm.

Temperature

- **Temperature** is a physical quantity that measures the degree of hotness or coldness.
- In metric, it is measured in **degrees celsius** ($°C$).
- In the customary unit, it is measured in **degrees Fahrenheit** ($°F$).

Capacity

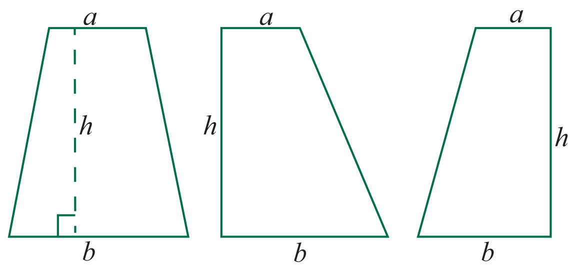
- It is the maximum amount that can be contained.
- It is measured in **litres** (l) and **millilitre** (ml).

Area

- This is the extent of measurement of a surface.
- Area uses every unit of length as it has a corresponding **unit of area**.
- Therefore, areas can be measured in **square metres** (m^2) and **square centimetres** (cm^2).
- 1 hectare = 10 000m^2 = 0,1km^2

Volume

- Volume is the amount of space that a substance is occupying.
- It can be referred to as the measure of the 3 dimensional space occupied by matter.
- Volume is measured in **cubic units**.
- The SI unit of volume is the **cubic metres** (m^3).
- 1m^3 = 1 000 litres



- This shape is very important in velocity time graphs.

Example 4.6

Find the area of a square with a side of 3*cm*.

Solution

Area of a square $= s^2$

$= (3cm)^2$

$= \mathbf{9cm^2}$

Example 4.7

Find the area of a rectangle with a length of 7*cm* and a width of 2*cm*.

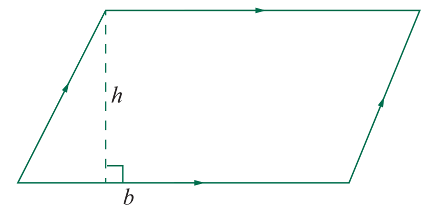
Solution

Area of a rectangle = length × width

= L × W

$= 7cm \times 2cm$

$= \mathbf{14cm^2}$



Example 4.8

Find the area of the parallelogram on Fig. 4.10.

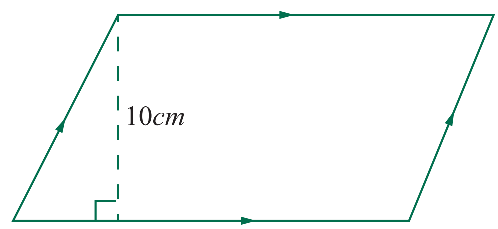


Fig. 4.10

Solution

Area $= 18cm \times 10cm$

$= \mathbf{180cm^2}$

Example 4.9

Find the area of the trapezium on Fig. 4.11.

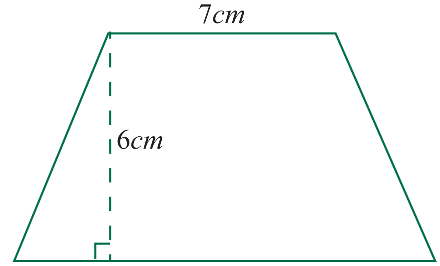


Fig. 4.11

Solution

$a = 7cm$, $b = 9cm$ and $h = 6cm$

Area $= \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (7 + 9) \times 6cm^2$

$= (16) \times 3cm^2$

$= \mathbf{48cm^2}$

- In addition to that, with circular middle part can be spread to produce a rectangle.
- The **separated parts** of the cylinder are shown on Fig. 4.31.

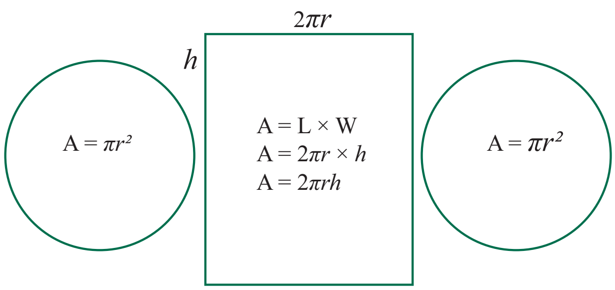


Fig. 4.34

- Note that, the rectangle length is equal to the circumference because it follows the shape of the base.
- Hence, **surface area** $= \pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh$.

Example 4.22

Calculate the surface area of a cylinder that has a base radius of 3cm and 11cm in height.

Solution

Surface area $= 2\pi r^2 + 2\pi rh$

$= 2 \times \frac{22}{7} \times 3^2 + 2 \times \frac{22}{7} \times \frac{3}{1} \times \frac{11}{1}\, cm^2$

$= \mathbf{264cm^2}$

Exercise 4.8

A rectangular concrete slab is $4m$ long, $1\frac{1}{2}m$ wide and $20cm$ thick.

1. Calculate the total surface area of the slab, giving your answer in cm^2. [5]
2. The whole slab is to be painted at a cost of $15.00 per m^2, calculate the cost of painting the slab. [5]

Volume of prisms

- Volume of a cylinder = area of a circle × height.
- Volume of a trapezoidal prism = area of a trapezium × height.

Example 4.23

Find the volume of the rectangular prism in Fig. 4.35.

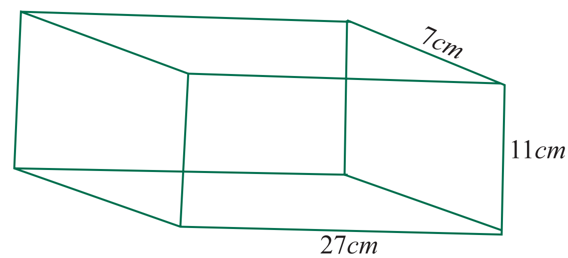


Fig. 4.35

Solution

Volume = base area × perpendicular height

= (length × width) × height

= $(11cm \times 7cm) \times 27cm$

= $\mathbf{2\ 079cm^3}$

Volume of a pyramid

- Volume of a pyramid $= \frac{1}{3} \times$ base area × height.
- The base area may be in the form of a triangle, a square, a rectangle or a pentagon.

Example 4.24

The pyramid measures $11cm$ by $7cm$ by $6cm$, calculate it's volume.

TOPIC 5 GRAPHS

Travel graphs

Distance-time graph

- A **distance-time graph** is a graph to show the motion of an object as well as its direction with the distance being plotted against time.
- **Distance** is represented on the vertical axis.
- **Time** is on the horizontal axis.

Example 5.1

Fig. 5.1 is a distance-time graph. Describe the motion illustrated on the graph.

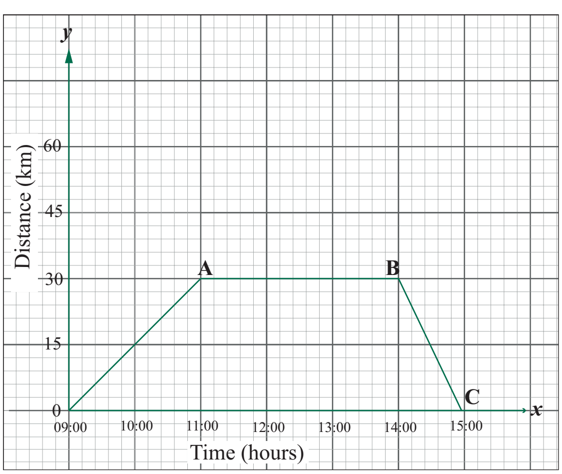


Fig. 5.1

(a) **O – A**.

(b) **A – B**.

(c) **B – C**.

Solution

(a) **O – A**, shows that the object moves for 2 hours covering a distance of 30*km* with a velocity of 15*km/hr*.

(b) **A – B**, shows that the object was stationary for 3 hours.

(c) **B – C**, shows that after being stationary the object returns back to original position covering the same but in less time of 1 hour. Therefore, the speed of the object is 30*km/hr*.

Exercise 5.1

Fig. 5.2 shows the motion of a motorbike from shops to home.

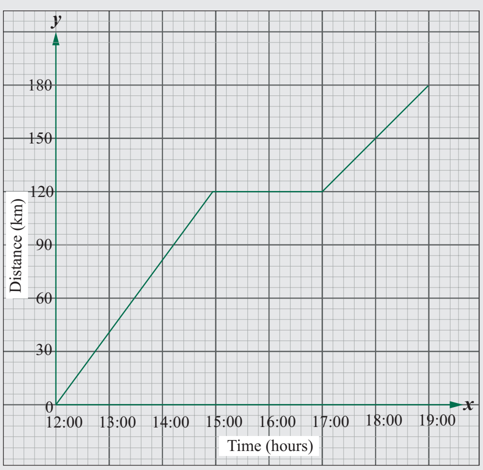


Fig. 5.2

1. Describe the motion of the motorbike between 15:00 to 17:00 [2]
2. Calculate the speed:
   (a) at which the motorbike was moving in the first 2 hours. [3]
   (b) of the motorbike from 17:00 to 19:00. [3]
3. Find the average speed to which the motorbike was moving in the whole journey from home to the shops. [2]

Example 5.8

Plot: $f(x) = 2x + 5$, make use of $-2 < x \leq 4$.

Solution

- The set of values $-2 < x \leq 4$ is the domain, the set of input values produces the corresponding set of y-values.
- According to the inequality, -2 is not included in the values set but 4 is part of the values set, substitute the values $x = -1; 0; 1; 2; 3; 4$.
- Find the values of y by substituting the x values with the given range from the inequality.
- For example, $y = 2(-1) + 5 = 3$. The same process is repeated using all the domain values (x–values) to produce a frequency distribution table.
- The table below shows the frequency distribution table for the given function:

x	-1	0	1	2	3	4
y	3	5	7	9	11	13

Plot the points on the cartesian plane and produce the graph on Fig. 5.10.

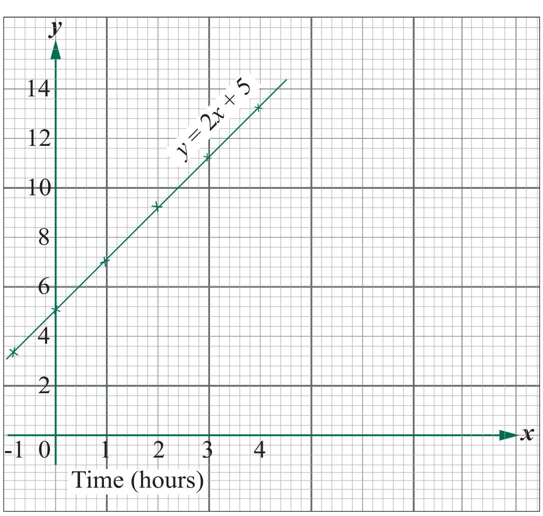


Fig. 5.10

Quadratic graphs

A quadratic graph is drawn using a function in the form $f(x) = ax^2 + bx + c$.

Example 5.9

Using the function $y = x^2 - 2x + 3$,

(a) construct a frequency distribution table, for the domain $-1 \leq x < 6$.

(b) represent the function on a graph.

Solution

(a) Frequency distribution table.

x	-1	0	1	2	3	4	5	6
y	5,5	3	1,5	1	1,5	3	5,5	9

(b)

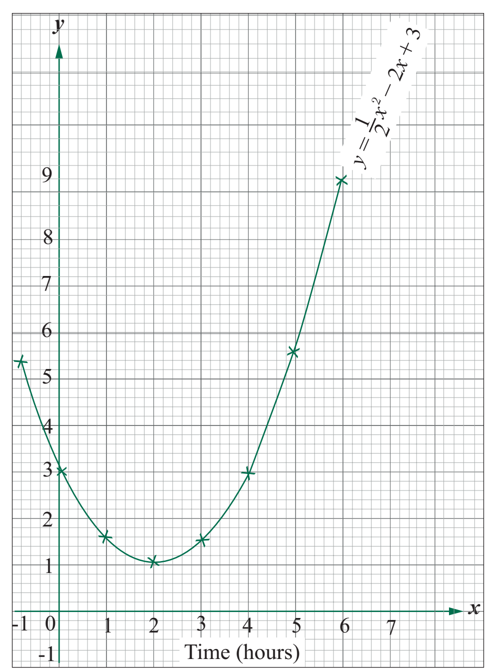


Fig. 5.11

Example 5.10

A particle moves along a straight line at a time t seconds and velocity, v m/s. The function of motion is given by $v = 5 + 7t - 2t^2$.

TOPIC 7 ALGEBRA

Algebra defined

Algebra is a branch of mathematics the part of mathematics in which **letters** and other general **symbols** are used to **represent numbers** and quantities in **formulae** and **equations**.

Algebraic manipulation

Remember we can only **add or subtract like terms**.

Example 7.1

Simplify.

(a) $2x + 3x + y$

(b) $a + 3b - 4a - 7b$

Solution

(a) $2x + 3x + y = \mathbf{5x + y}$

(b) $a + 3b - 4a - 7b = \mathbf{-3b - 4b}$

Note: Group all like terms first then add or subtract them.

Exercise 7.1

Simplify the following.

1. $2x + 5x - 7$ [2]

2. $-21a + 2b - a$ [2]

3. $3x + 2y - 7x$ [2]

4. $-5y - 2b + c - 21c$ [2]

5. $18w - 5x + 13x - 15x$ [2]

Substitution of values

This involves the substitution of **letters** in an algebraic expression with respective **numerical values** and simplifying.

Example 7.2

$m = 0{,}5$, $n = 0$ and $r = 3$. Evaluate :

(a) $(mr)^2$.

(b) $(2{,}25)^m$.

(c) $\sqrt[r]{-64}$.

Solution

(a) $(mr)^2 = (0{,}5 \times 3)^2$
$= (1{,}5)^2$
$= \mathbf{2{,}25}$

(b) $(2{,}25)^m = \left(\frac{225}{100}\right)^{0{,}5}$
$= \sqrt{\frac{225}{100}}$
$= \frac{\sqrt{225}}{\sqrt{100}}$
$= \frac{15}{10}$
$= \mathbf{1\tfrac{1}{2}}$ **or 1,5**

$\sqrt[r]{-64} = \sqrt[3]{-64}$
$= \mathbf{-4}$

Note: Express the number as a product of its prime factors in index form to find the root of a number. Multiply the power by $\frac{1}{2}$ to get the square root of the number. Multiply the power or index by $\frac{1}{3}$ to get the cube root.

Factorisation

- The steps followed in basic factorisation of two terms are as follows.
  - Find the highest common term between the given terms.
  - Factor out the highest common factor, introduce a bracket and divide each term by the highest common factor.

Example 7.6

Factorise $3x^2 - 12x$ completely.

Solution

The HCF between $3x^2$ and $-12x$ *is* $3x$.

$$3x^2 - 12x = 3x\left(\frac{3x^2}{3x} - \frac{12x}{3x}\right)$$

$$= \mathbf{3x(x - 4)}$$

Example 7.7

Factorise completely $16a^2 - 6a$.

Solution

The HCF is $2a$.

$16a^2 - 6a = \mathbf{2a(8a - 3)}$

Note that, each element is divided by the HCF.

Factorisation of four terms

- When given four terms, factorise in pairs.
- Factorising four terms implies that the result will have two brackets.
- The two brackets to which we factor out the HCF must be the same.
- If the brackets are not the same after factorising, then there is a need to rearrange the terms.

Example 7.8

Factorise completely $4m - 28 + 5mn - 35n$.

Solution

Find the HCF of $4m$ and -28, which is 4.

Find the common factor to which gets into a pair of $5mn$ and $-35n$.

The resulting bracket must be the same as initially found of $4m$ and -28.

$4m - 28 + 5mn - 35n$

$$= 4\left(\frac{4m}{4} - \frac{28}{4}\right) + 5n\left(\frac{5mn}{5n} - \frac{35n}{5n}\right)$$

$= 4(m - 7) + 5n(m - 7)$

$= \mathbf{(m - 7)(4 + 5n)}$

Note: It can be seen that the other bracket will be created from the common factors.

Example 7.9

Factorise completely $15m + 18 - 10mn - 12m$.

Solution

$15m + 18 - 10mn - 12n$

$$3\left(\frac{15m}{3} + \frac{18}{3}\right) - 2n\left(\frac{-10mn}{-2n} - \frac{12n}{-2n}\right)$$

$3(5m + 6) - 2n(5m - 6)$

$\mathbf{(3 - 2n)(5m + 6)}$

Note: Sometimes use a negative HCF to make the brackets the same as well as the signs.

Example 7.10

Factorise completely $12 - 2ty + 8t - 3y$.

Solution

Note that, if the first two terms are 12 and $8t$ and the last terms are $-3y$ and $-2ty$, we end

Exercise 7.22

Express as a single logarithm.

1. $\log_{10} 27 \div \log_{10}3$ [2]
2. $2 - 2\log 50$ [2]
3. $\log_{10}16 + \log_{10}2$ [2]
4. $2\log_{10}5 + \log_{10}36 - \log_{10}9$ [2]
5. $\log_4 32 + \log_4 2$ [2]

Solving logarithms

Example 7.69

Given that log 3 = 0,477 and log5 = 0,699, find log 45.

Solution

- Base 10 is being used and all the other evaluated logarithms are of base 10.
- log 45 is evaluated by substituting log 3 and log5.

$$\begin{aligned}\log 45 &= \log(3^2 \times 5)\\ &= \log 3^2 + \log 5\\ &= 2\log 3 + \log 5\\ &= 2(0{,}477) + 0{,}699\\ &= 1{,}653\end{aligned}$$

Example 7.70

Given that $\log_{10}3 = 0{,}4771$ and $\log_{10}5 = 0{,}6991$. Find:

(a) $\log_{10} 1\frac{2}{3}$

(b) $\log_{10}30$

Solution

(a) $$\begin{aligned}\log_{10} 1\tfrac{2}{3} &= \log_{10}\left(\frac{5}{3}\right)\\ &= \log_{10} 5 - \log_{10} 3\\ &= 0{,}6991 - 0{,}4771\\ &= 0{,}222\end{aligned}$$

(b) $$\begin{aligned}\log_{10}30 &= \log_{10}(3 \times 10)\\ &= \log_{10}3 + \log_{10}10\\ &= 0{,}4771 + 1\\ &= \mathbf{1{,}4771}\end{aligned}$$

Exercise 7.23

1. Given that $\log_{10} 5 = 0{,}699$, evaluate:
   - **(a)** $\log_{10}125$. [1]
   - **(b)** $\log_{10}50$. [1]
   - **(c)** $\log_{10}0{,}5$. [1]
2. Given that $\log_5 2 = 0{,}431$ and $\log_5 3 = 0{,}683$. Find the value of :
   - **(a)** $\log_5 1\frac{1}{2}$. [1]
   - **(b)** $\log_5\sqrt{3}$. [1]
3. Given that $\log m = -6$ and $n = 5$. Evaluate the following:
   - **(a)** $\log mn$ [1]
   - **(b)** $\log m^{\frac{1}{2}}$ [1]
   - **(c)** $\log\left(\frac{1}{n}\right)$ [2]
4. Given that $\log_{10}2 = 0{,}301$ and $\log_{10}7 = 0{,}845$. Evaluate:
   - **(a)** $\log_{10}3{,}5$. [1]
   - **(b)** $\log_{10}40$. [1]
5. Given that $\log_7 2 = 0{,}3562$ and $\log_7 3 = 0{,}5646$. Calculate the value of:
   - **(a)** $\log_7 6$. [1]
   - **(b)** $\log_7 1{,}5$. [1]
   - **(c)** $\log_7 8$. [1]

Laws of logarithms in solving equations

Example 7.71

Show that $2\log_5(3x + 2) - \log_5 2 = 1$, reduces to $3x^2 + 4x - 2 = 0$.

Solution

$2\log_5 (3x + 2) - \log_5 2 = 1$

$2\log_5(3x + 2) - \log_5 2 = 1$

$\log_5(3x + 2)^2 - \log_5 2 = 1$

$\log_5\left[\frac{(3x+2)^2}{2}\right] = 1$

$\left[\frac{(3x+2)^2}{2}\right] = 5^1$

$(3x + 2)^2 = 5 \times 2$

$9x^2 + 12x + 4 = 10$

$9x^2 + 12x - 6 = 0$

$\therefore \mathbf{3x^2 + 4x - 2 = 0}$ **shown**

Exercise 7.24

1. Express $\log_{10}x + 2\log_{10} y = 1$ as an equation in index form. [3]
2. If $\log_{13} (x^2 + 25) = 2$, find the two possible values of x. [3]
3. Given that $\log_{10}(x + 2) + \log_{10} (x + 4) = 1$, show that $x^2 + 6x - 2 = 0$. [4]

Revision exercise

1. Evaluate.
   (a) $\sqrt[3]{0,027}$ [1]
   (b) $\left(1\frac{7}{9}\right)^{\frac{1}{2}}$ [1]
   (c) $3^0 \times 3^{-2}$ [1]
2. (a) Factorise completely.
   (i) $x^3 - x$ [2]
   (ii) $x^2 + 2x = -1$ [1]
   (b) Hence, find the HCF of $x^3 - x$ and $x^2 + 2x + 1$. [2]
3. (a) Solve the inequality $2(x - 3) < 7$. [1]
   (b) Write down the largest perfect square that satisfies the inequality $2(x - 3) < 7$. [1]
4. (a) Evaluate (i) $\log_3 45 - \log_3 5$ [2]
   (ii) $\frac{\log 0,2}{\log 5}$ [2]
   (b) Express as a logarithm of a single number $3\log 2 + \frac{1}{2}\log 81$. [2]
   (c) If $\log 6 = 0,7781$ and $\log 5 = 0,699$, calculate $\log 1\ 200\ 000$. [2]
5. (a) Solve the equations.
   (i) $0,3x + 1,7 = 1,8 - 0,4x$ [2]
   (ii) $3x = (-64)^{\frac{1}{3}}$ [2]
   (b) Factorise completely $6m^2 n^2 - mn - 15$. [2]
   (c) Express $\frac{x-4}{16-x^2} \div \frac{2}{x+4}$ as a single fraction in its lowest terms. [3]
6. (a) Write down the numerical value of $x^2 - 9y^2$ when $x = 4$ and $y = \frac{1}{3}$. [1]
   (b) Solve the simultaneous equations.
   $x + 3y = 3$
   $x - 3y = 5$
7. The formula for converting a temperature in degrees centigrade (°C) to a temperature in degrees Fahrenheit (F°) is $F = 32 + \frac{9c}{5}$.
   (a) Find F when $C = 30°$. [1]
   (b) Make C the subject of the formula. [2]
8. The cost of making a telephone call on Tenneco is 25 cents per minute. Kuda has p cents and can make a call. Xolani has q cents which is insufficient to make a call. Write down these inequalities in terms of p and/or q, other than $p > 0$ and $q > 0$, that satisfy the given conditions. [3]

Example 8.14

Construct angle 30°.

Solution

We bisect the 60° angle to construct an angle of 30°.

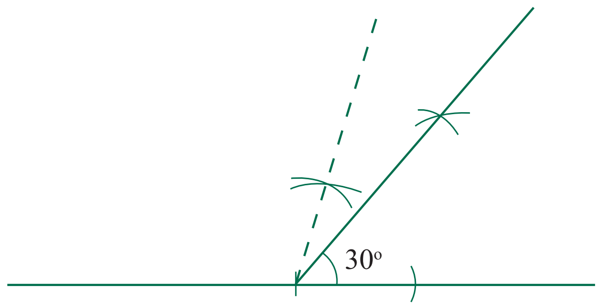


Fig. 8.47

Example 8.15

Construct angle 15°.

Solution

We bisect a 30° angle to construct a 15° angle.

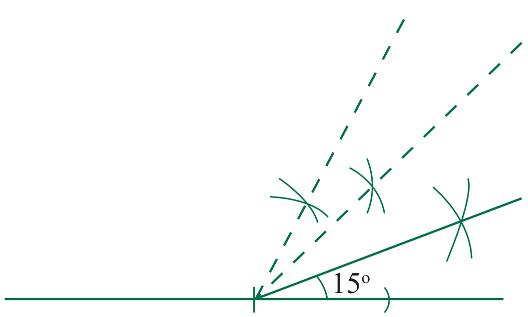


Fig. 8.48

Example 8.16

Construct an angle 105°.

Solution

90° + 15° or 60° + 45° gives 105°.

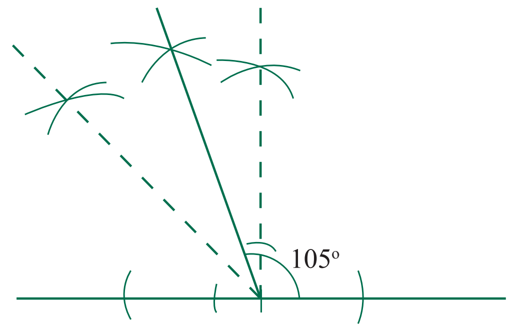


Fig. 8.49

Exercise 8.11

Construct the following angles using a compass.

1. 135°
2. 150°
3. 75°
4. 67,5°

Loci

- A **locus** is a set of points or paths which adheres to a certain condition or rule of movement.
- It is very useful in analytical geometry.

Loci theorems

- **Theorem 1** − the locus of points equidistant from one fixed point is a circle.

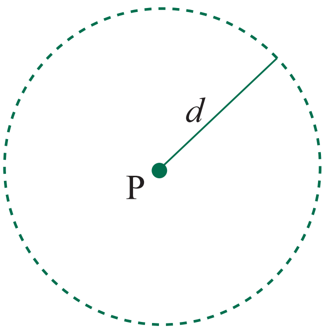


Fig. 8.50

Example 10.8

Find the area of triangle **PQR** in Fig. 10.22.

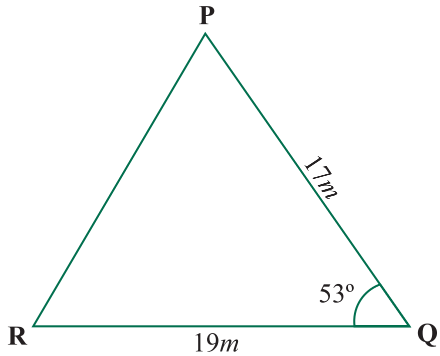


Fig. 10.22

Solution

Area $= \frac{1}{2} \times 19 \times 17 \times \sin 53°$

$= \mathbf{129m^2}$ **(3sf)**

Example 10.9

Find the area of the triangle in Fig. 10.23.

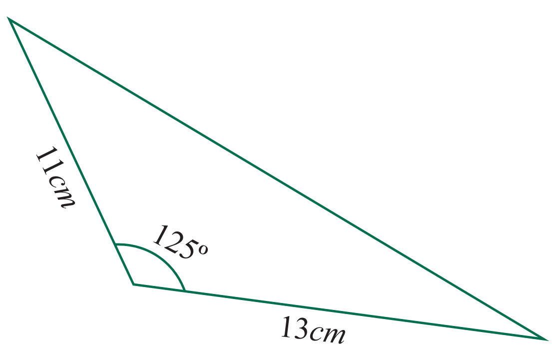


Fig. 10.23

Solution

Area $= \frac{1}{2} \times 11 \times 13 \times \sin 125°$

Area $= \mathbf{58{,}6cm^2}$ **(3s.f)**

Example 10.10

Write down three formulae for the area of the triangle in Fig. 10.24. Use each formula to determine:

(a) Area $= \frac{1}{2}\, ab \sin C$

(b) Area $= \frac{1}{2}\, ac \sin B$

(c) Area $= \frac{1}{2}\, bc \sin A$

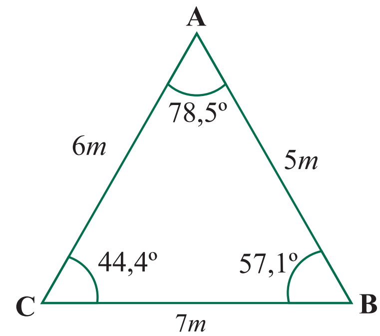


Fig. 10.24

Solution

(a) Area $= \frac{1}{2} \times 76 \times 6 \times \sin 44{,}4°$

$= \mathbf{14{,}7m^2}$ **(3 sf)**

(b) Area $= \frac{1}{2} \times 7 \times 5 \times \sin 57{,}1°$

$= \mathbf{14{,}7m^2}$ **(3 sf)**

(c) Area $= \frac{1}{2} \times 6 \times 5 \times \sin 78{,}5°$

$= \mathbf{14{,}7m^2}$ **(3 sf)**

Exercise 10.4

1. Fig. 10.25 shows triangle **XYZ** with **XY** = 6*cm*, **XZ** = 10*cm* and $Y\hat{X}Z = 30°$.
[sin 30° = 0,5; cos 30° = 0,87;
tan 30° = 0,58]

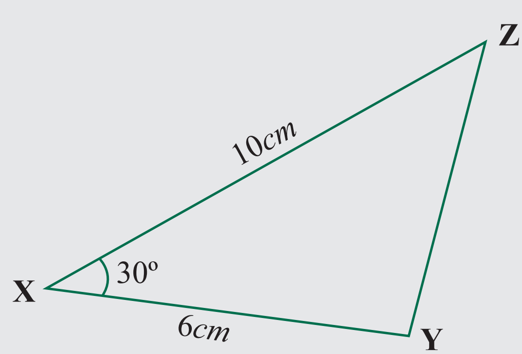


Fig. 10.25

(a) Find the area of the triangle **XYZ**. [2]
(b) Calculate the length of **YZ**, leaving your answer in surd form. [3]

2. **ABC** is a triangle with **AB** = 9*cm*, **BC** = 4*cm* and **ABC** = 120°. Given that : tan 60° = 1,73; sin 60° = 0,87; cos 60° = 0,5, find the area of a triangle **ABC**. [2]

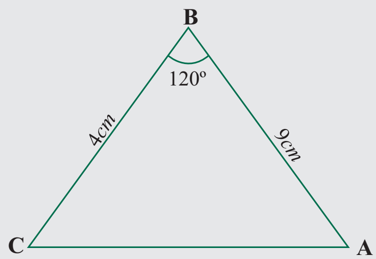


Fig. 10.26

Revision Exercise

1. In Fig. 10.27, **AB** = **BC** = *xcm*, **AC** = $\sqrt{128}$ *cm* and **ABC** = 90°.

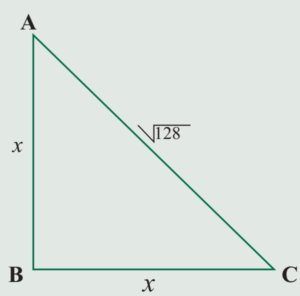


Fig. 10.27

(a) Form an equation in *x*. [1]
(b) Find the value of *x*. [3]

2. Triangle **ACD** is right angled at **C**. **AD** = 6*cm*, **DBC** = 45°, **DAC** = 30° and **ABC** in a straight line.

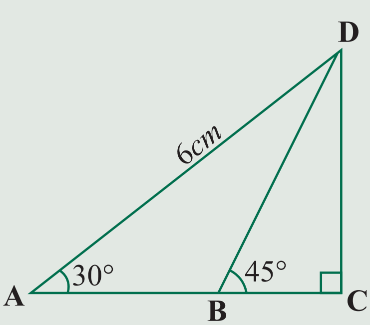


Fig. 10.28

Calculate:
(a) CD. [1]
(b) AB, giving the answer correct to 1 decimal place. [2]
[sin 30º = 0,50; cos 30º = 0,87; tan 30º = 0,58].
[sin 45º = 0,71; cos 45º = 0,71; tan 45º = 1,00].

3. In Fig. 10.29, **AB** = 12*cm*, **AC** = 9*cm* and **BC** = 7*cm*.

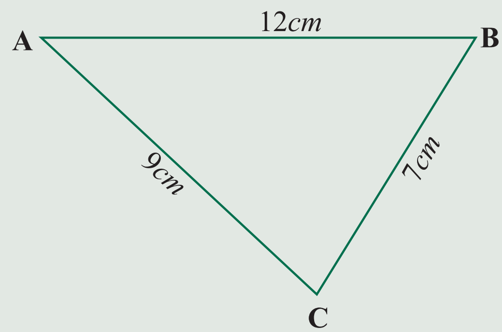


Fig. 10.29

Using as much information given as is necessary,
(a) express the ratio AC:AB is the simplest form. [1]
(b) find the area of triangle **ABC**. [3]
[sin A = 0,58; cos A = 0,81; tan A = 0,71]

4. In Fig. 10.30, A**BC** is a triangle in which **AB** = *x*; AC = 2*x*; **BC** = 140*cm* and BÂC = 120°.

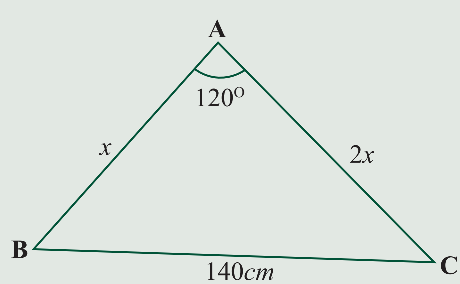


Fig. 10.30

Using as much information given below as is necessary, calculate the:
(a) value of *x*, leaving your answer in surd form. [4]

TOPIC 11 VECTORS

Vector definition and notation

- A **vector** is a quantity that has both size and direction.
- Vectors can be written in **capital letters** as : $\overrightarrow{AB}$, **AB** or $\overline{AB}$.
- **Small letters** are also used to represent vectors as : $\overline{a}$ or **a.**
- In addition to that, vectors can be given in **cartesian form or column vector** form as $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$.
- It is important to consider the **direction of movement** when dealing with vectors.
- If a point is moved along to the **right direction** of the *x-axis*, the *x value* is **positive**, but if moved in the **left direction** of the *x-axis*, the *x value* becomes **negative**.
- In the *y-axis* a point is **positive** if moved **upwards** and is **negative** if moved **downwards**.

Example 11.1

Express the following in column vector form as shown in Fig. 11.1.

(a) $\overrightarrow{PQ}$

(b) $\overrightarrow{QR}$

(c) $\overrightarrow{PR}$

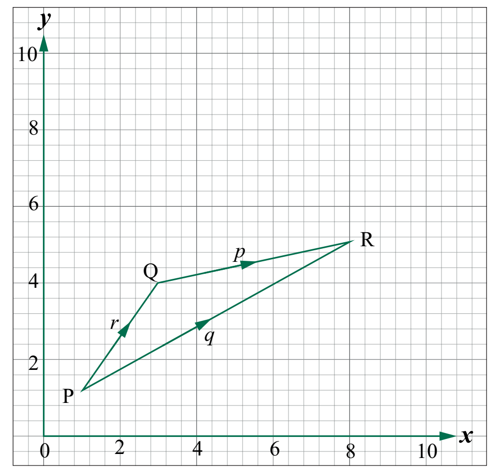


Fig. 11.1

Solution

Fig. 11.1 shows the shape QPR.

(a) $\overrightarrow{PQ} = r$

$\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

(b) $\overrightarrow{QR} = p$

$= \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

(c) $\overrightarrow{PQ} = q$

$= \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

Exercise 11.1

The vectors on Fig. 11.2-5 are drawn using a scale of 1*cm* to 1unit. Find vectors ***b*** and ***c*** in column vector form.

TOPIC 13 TRANSFORMATION

Transformation basics

- A **transformation matrix** is a matrix that is pre-multiplied by the **object matrix** to give an **image matrix**.
- A transformation matrix may be representing a certain **type of transformation** which can be a **translation**, **reflection**, **enlargement**, **stretch** or **shear**.
- An **object** is referred to as the **original shape** before any transformations occur.
- An **image** is an outcome when a transformation occurs.
- When drawing shapes and diagrams in transformations ensure that :
  - **(a)** the *x*-axis and *y*-axis are clearly labelled.
  - **(b)** a suitable scale is used along each axis when the scale is not given. However, in most cases, the scale is given.
  - **(c)** the plotted points and shapes are labelled and the vertices are in the correct order.
  - **(d)** faint construction lines are used and avoid making the cartesian plane dirty.
- There are two kinds of transformations namely **isometric** and **non-isometric**.

Matrix of a shape from given vertices

Example 13.1

Write down the matrix of triangle **ABC** which has vertices **A** (–4;2), **B** (-3; –1) and **C** (–1,0).

Solution

The columns are formed by the vertices of the shape with each point being represented vertically starting with the values of the *x*-axis at the top and that of the *y*-axis at the bottom.

$$\Delta ABC = \begin{pmatrix} \overset{A}{-4} & \overset{B}{-3} & \overset{C}{-1} \\ 2 & -1 & 0 \end{pmatrix}$$

Isometric transformations

- If a transformation occurs to an object, the resulting image is identical to the object in shape, though the positions and directions they face may differ.
- Therefore, they can be called **congruent transformations**.
- Examples of isometric transformations are **translation**, **reflection** and **rotation**.

Translation

- Note that, one can count boxes of the graph to find the vertices of images.
- This formula can be used in all translation problems: **Object Matrix + Translation Vector = Image matrix**

Example 13.2

A point **M** has coordinates (3; -5), find the:

(a) coordinates of ***Q***, where the translation vector of point **M** is $\begin{pmatrix} -2 \\ 8 \end{pmatrix}$.

(b) translation vector that maps **M** onto point **Q** (1;3).

(c) Enlargement of Δ**DEF** onto $\Delta \mathbf{D_2E_2F_2}$ centre (0;4) and scale factor 2.

Solution

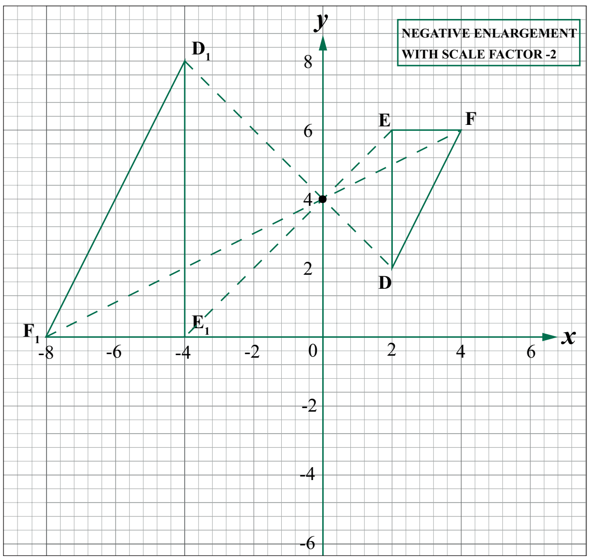


Fig. 13.7

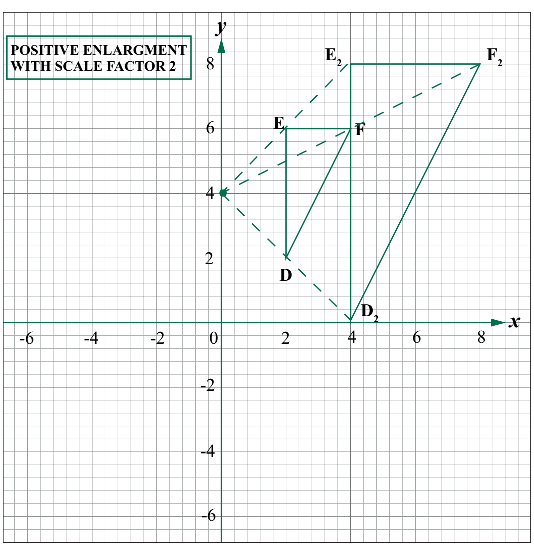


Fig. 13.8

Fig. 13.7 and Fig. 13.8 show the differences between negative and positive enlargement.

Positive enlargement on Fig. 13.8

- The lines of the image must be twice longer than those from the shape because the scale factor is 2.
- Lines from the image must be twice longer than those from the shape since the scale factor is twice longer from the centre.
- The shapes must be on the same side or quadrant as shown on Fig. 13.8.
- The images are longer than the objects only because in both cases the scale factor used was a whole number. If the scale factor was a fraction less than 1 then the images will be smaller than the objects.
- Therefore, it can be concluded that when it's positive enlargement the images and objects face the same direction. If it is negative enlargement, they face the opposite directions.

Stretch

- A **stretch** is a transformation of a plane in which all the points, except those on the invariant/fixed line, move perpendicular to that fixed-line, in such a way that the distance that each point moves from the invariant line is proportional to the points original distance from the same invariant line.
- This **constant proportionality** is called the **stretch or scale factor** which can either be a positive or negative real number.
- In stretch transformations, always consider the :

  (a) **invariant line or axis**.

  (b) **stretch scale factor**.

  (c) **stretch matrix**.

Example 13.10

Δ**A** has vertices (–5;2), (–2;2) and (–2;4) and Δ**B** is a stretch with a scale factor -2 and *y*-axis

Solution

The dimensions of the shapes are neither identical and they overlap. It is either stretch or shear.

Transformation matrix × object matrix = image matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 2 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 4 \\ 2 & 2 & 1 \end{pmatrix}$$

$a + 2b = 2$ __________(1), from the first row by the first column.

$3a + 2b = 6$_________(2), from the first row by the second column.

Solving simultaneously gives $a = 2$ and $b = 0$.

Note that only two equations are required to solve simultaneously.

$c + 2d = 2$___________(1), from the second row by the first column.

$3c + 2d = 2$__________(2), from the second row by the second column.

Solving simultaneously gives $c = 0$ and $d = 1$.

Hence, substituting for a, b, c and d in the matrix gives the transformation matrix as $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$. **It is a stretch with a scale factor of 2 and a *y*-axis invariant**.

Exercise 13.3

1. Study the diagram below.

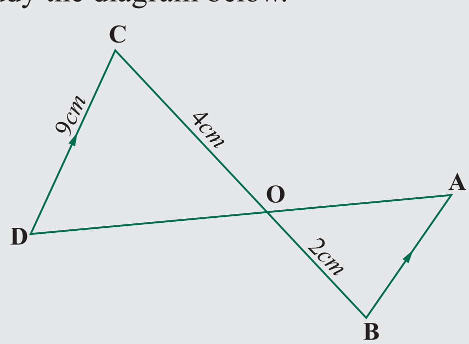


Fig. 13.19

Fig. 13.19 shows two straight lines **AD** and **BC** that meet at **O** such that **OB** = 2*cm*, **OC** = 4*cm* and **CD** = 9*cm*, **AB** is parallel to **CD**.

(a) Name a triangle that is similar to triangle **ABO**. [1]

(b) Describe the single transformation that maps triangle **OCD** onto triangle **OBA.** [3]

2. Use Fig. 13.20 to answer the following questions.

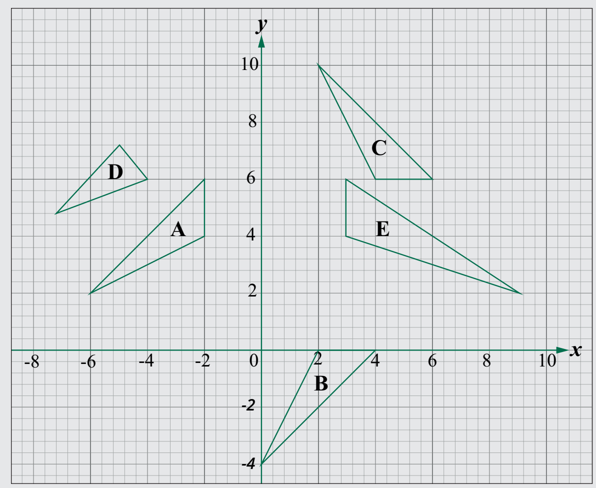


Fig. 13.20

(a) Δ**B** is a reflection of Δ**A**.

(i) Write down the equation of the mirror line. [2]

(ii) Given that (k ; 8) is one of the invariant points under this reflection, find the value of k. [1]

(b) Describe fully the single transformation which maps Δ**A** onto Δ**C**.

(c) Δ**D** is the image of Δ**A** under enlargement, centre origin. Write down the scale factor of enlargement. [1]

(d) Describe fully the single transformation which maps Δ**A** onto Δ**E**. [3]

SPECIMEN EXAMINATION 1

PAPER 1

TIME: 2 hours 30 minutes

Answer **all** questions.

1. Express :

(a) $3,75 \times 10^{-3}$ as a decimal fraction. [2]

(b) 3,75 as a fraction to its lowest terms. [2]

(c) 0,124 in the form $\mathbf{A} \times \mathbf{10}^n$, where **A** is a real number and ***n*** is an integer.[3]

2. State the number of lines of symmetry of the following letters.

(a) **(i)** **N**

(ii) **M**

(iii) **H**

(iv) **O** [6]

(b) The sum of interior angles of a regular polygon is 540°. State the order of rotational symmetry of the regular polygon. [2]

3. **(a)** Convert $3m^2$ in cm^2. [3]

(b) Given that $\mathbf{P} = 1,2 \times 10^4$ and $\mathbf{Q} = 0,6 \times 10^{-3}$. Find $\mathbf{P} \div \mathbf{Q}$, giving your answer in standard form.

4. **F** varies directly as the square of **T**.

(a) Express **F** in terms of **T** and a constant *m*. [2]

(b) Given that **F** = 3 when **T** = 2. Find the numerical value of *m*. [3]

(c) Find the value of **T** when **F** = 126,75 where **T** > 0. [4]

5. Fig. 1.1 shows the points **P**, **Q** and **R** are on the circumference centre **O**. $O\hat{P}R = 22$ and radius 3,5*cm*.

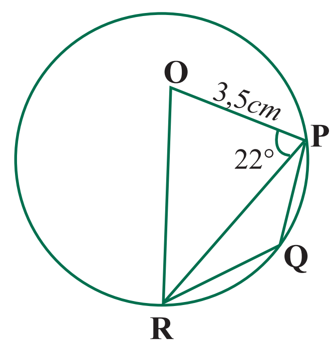


Fig. 1.1

Calculate:

(a) $R\hat{O}P$. [3]

(b) $R\hat{Q}P$. [3]

(c) the area of the sector OAB. [4]

[*use* π *as* $\frac{22}{7}$]

6. **(a)** Factorise completely.

(i) $x^2 + 3x - 10$

(ii) $4x^2 - y^2$ [6]

(b) Solve the simultaneous equations.

$$4x^2 - y^2 = 4$$
$$2x + y = 1$$ [6]

7. Evaluate.

(a) $\log_a 16 \div \log_a 2$ [3]

(b) $\log_{27} 1$ [3]

SPECIMEN EXAMINATION 1

PAPER 2

TIME: 2 hours 30 minutes

Answer **all** questions in **Section A** and any four from **Section B**.

Section A [52 marks]

Answer **all** the questions in this section.

1. Simply $\frac{3}{5}-\frac{1}{10}+\frac{3}{11}\times\frac{1}{17}$. [2]

2. Express 0,0001 in standard form. [2]

3. Find the HCF and LCM of 9; 15 and 18. [4]

4. Express 536_7 as a number in base 3. [3]

5. Factorise completely.

(a) $5x^2 + 15x$ [2]

(b) $2y^2 - 3y + 1$ [2]

6. Simplify $\frac{x-3}{4x+1} - \frac{2}{x+2}$ and express it as a single fraction. [4]

7. (a) Write down the number of degrees in a $\frac{3}{4}$ revolution. [2]

(b) Find the number of sides of a polygon whose interior angles add up to 540°. [3]

(c) Chipo and Tsitsi have 520 sweets to which they shared in the ratio 6:7 respectively.
Find the share of Tsitsi and the difference between Tsitsi's share to Chipo's share. [3]

8. $\xi = \{x : 1 < x \leq 18, x \text{ is an integer}\}$
C = $\{x{:}x$ is a prime number$\}$
D = $\{x{:}x$ is an odd number$\}$.

(a) List all elements of set C. [3]

(b) Find n(D). [3]

(c) Draw a venn diagram to illustrate the information. [4]

9. Evaluate the following.

(a) $2{,}15 \times 10^{-4} - 5{,}1 \times 10^2$ [3]

(b) $(0{,}12 \times 10^2) \div (0{,}6 \times 10^{-7})$ [3]

10. (a) Using a plain paper, draw and label line AB which is 8cm long. [1]

(b) Point PQ is such that area of triangle ABQ = 12cm^2.

(i) Calculate the perpendicular height of the triangle ABQ. [3]

(ii) Construct the possible positions of Q, above AB, which are such that the area of triangle BQ = 12cm^2. [3]

(iii) Draw locus of points which are 2cm from point B. [1]

Section B [48 marks]

Answer any **four** questions in this section.

Each question carries 12 marks.

11. (a) Solve $3x^2 - 1 = 5x$, give your answer correct to 2 significant figures. [5]

(b) In Fig. 1.1, ADC is a straight line. AD = 5cm, DC = 4cm, BD = 3cm and the area of triangle ABD is 4cm^2.

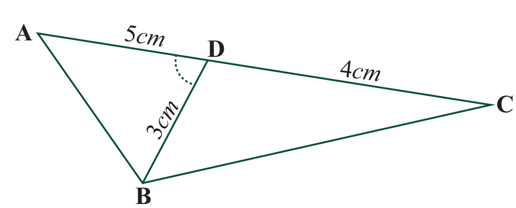


Fig. 1.1

Calculate:

(i) $A\hat{D}B$. [3]

(ii) BC. [4]

12. Answer the whole of this question on a single sheet of graph paper. The table below shows some of the values of the function $y = x^3$.

x	-2	-1.5	-1	0	1	$\frac{3}{2}$	2
y	p	$-3\frac{2}{5}$	-1	0	1	q	8

Find the values of p and q. [2]

(a) Use a scale of 2*cm* to represent 0,5 units on the x-axis and 2*cm* to 2 units on the y-axis. Draw the graph of $y = x^3$ for the range $-2 \leq x \leq 2$.

(b) Use your graph to estimate the value of $\sqrt[3]{-4}$ showing your method clearly. [2]

(c) Draw the graph of $y = 3x - 1$ using the same axes and scales. [2]

(d) Estimate the roots of the equation $x^3 - 3x + 1 = 0$ using your graph. [3]

13. (a) Mrs Mutikani has 300 cows, 104 goats and 76 pigs.

(i) Draw a pie chart to represent the information. [4]

(ii) Two animals are selected at random. Calculate the probability that one is a goat and the other is a cow. Leave your answer as a fraction in its simplest terms. [3]

(b)

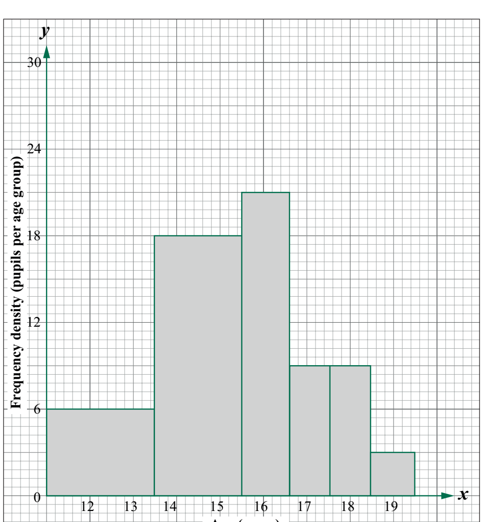


Fig. 1.2

Fig. 1.2 illustrates the distribution of the ages of the learners at a certain school.

(i) State the name of this type of diagram. [1]

(ii) Calculate the number of learners in the 16 to 19 age group. [2]

(iii) In which of the age ranges does the modal value occur?

(iv) In which of the age ranges does the median value occur? [2]

14. (a) List the integer values of x which satisfy all of the inequalities.

$1 + x \leq 19 - 2x < 12$

$11 < 2x + 3 < 19$ [4]

(b)

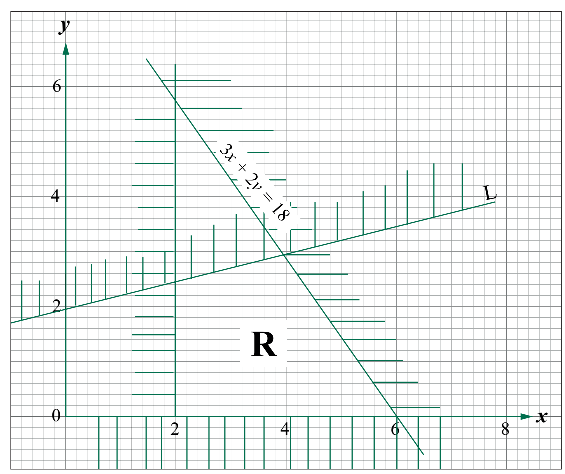


Fig. 1.3

(i) Use Fig. 1.3 to find the equation of the line l and the four inequalities which define the region **R**.

(ii) Given that $p = 5x + 2y$, find the maximum value of p for integral values of x and y on **R**. [8]

15. (a) If $\overline{OA} = p + 5q$, $\overline{OB} = 7p + 3q$ and $\overline{AB} = 3hp + (h - k)q$, find the values of h and k. [4]

(b)

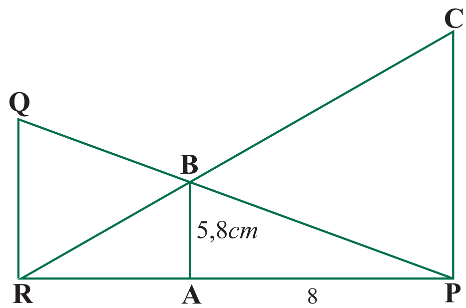


Fig. 1.4

In Fig. 1.4, Δ**QPR** is the image of Δ**PBA** under an enlargement with centre P and scale factor $\frac{3}{2}$, PA = 8*cm* and AB = 5,8*cm*.

(i) Calculate the length of AR and the ratio $\frac{\text{triangle PAB}}{\text{triangle PRQ}}$. [5]

(ii) Δ**PCB** is the image of Δ**QRB** under an enlargement.

For this enlargement, find the centre and the scale factor. [3]

SPECIMEN EXAMINATION 8

PAPER 1

TIME: 2 hours 30 minutes

Answer **all** questions.

1. Express $\frac{5}{16}$ as a:

(a) **(i)** decimal fraction. [1]

(ii) percentage. [1]

(b) Express 0,48 as a:

(i) fraction in its lowest terms. [1]

(ii) number in standard form. [1]

2. Write the next 2 terms in :

(a) 1; 2; 4; 8; 16;... [2]

(b) $\frac{1}{25}; \frac{1}{16}; \frac{1}{9};$......... [2]

(c) 12 ; 9; 6; 3;... [2]

3. Nyamwanza and Mzama are aged 13 years and 15 years respectively. They shared money in the ratio of their ages. If Nyamwanza was given $650, find the total amount shared. [3]

(a) Express $1 \times 2^4 + 2^3 + 1 \times 2$ as a number in base 2. [1]

(b) **(i)** Convert 624_7 to base ten. [1]

(ii) Hence or otherwise, convert 624_7 to base three. [1]

(c) Evaluate $1101_2 + 101_2$, giving your answer in base two. [2]

4. In Fig. 1.1, PQRS is a cyclic quadrilateral in which PQ = PR and QS = RS. PS is the diameter of the circle.

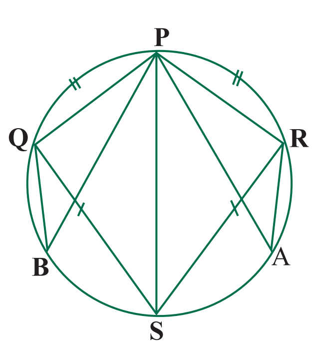


Fig. 1.1

(a) State the special name given to the cyclic quadrilateral PQRS. [1]

(b) Calculate $S\hat{R}P$. [1]

(c) Name other three angles identical to $Q\hat{B}P$. [3]

5. Fig. 1.2 is a venn diagram.

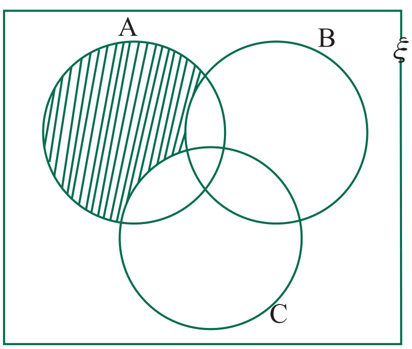


Fig. 1.3

Describe in set notation the shaded region. [2]

6. **(a)** Convert 7,5*m/s* to *km/hr*. [1]

(b) The exchange rate for converting United States dollars to Botswana pula is US$5:P42,10.

(i) Express the exchange rate in the form US$1:*n*Pula. [1]

(ii) Calculate the equivalent of US$16,50 in pula. [2]

7. Solve the simultaneous equations. [3]

$2x - 3y = 17$

$0,3x + 0,4y = 0$

8. Fig. 1.3 shows a quadrilateral ABCD with AB = 11*cm* and CD = 17*cm*.

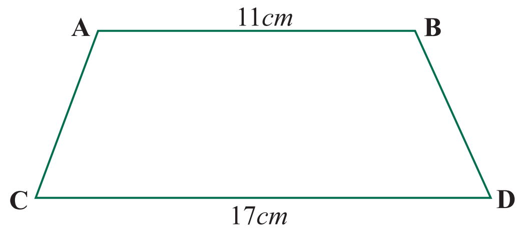


Fig. 1.3

(a) State the special name given to the quadrilateral. [1]

(b) Given that the area of the quadrilateral ABCD is 84cm^2, find the height of the shape. [3]

9. A bag contains 20 balls that are identical in shape and size except for colour. Fifteen are blue and 5 are white.

(a) Calculate the probability of picking a:

(i) yellow ball. [1]

(ii) blue ball. [1]

(b) Two balls are picked at random from the bag. Calculate the probability that :

(i) they are of the same colour. [2]

(ii) ball picked is at least white. [2]

10. Fig. 1.4 is a plane shape PQRS with PR = 13*cm* and RS = 11*cm* and $R\hat{P}Q = 150°$.

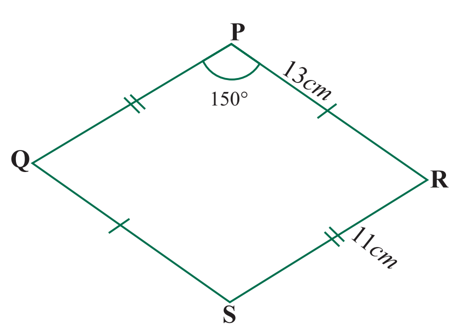


Fig. 1.4

(a) State the special name given to the plane shape PQRS. [1]

(b) State the number of lines of symmetry of the shape. [1]

(c) Find the area of the quadrilateral. [2]

11. It is given that log7 = 0,8451 and log2 = 0,3010. Calculate:

(a) log 14. [2]

(b) $\log 3\frac{1}{2}$. [2]

(c) $\log \frac{1}{64}$. [2]

12. (a) Find the size of a single interior angle of a regular 20 sided polygon.[2]

(b) A regular polygon has an interior angle of 120°.

(i) Find the number of sides of the polygon. [2]

(ii) State the special name of the polygon. [1]

13. P varies jointly as R and as the square root of T. Given that $P = 5$, $R = 2$ and $T = 9$.

(a) Express P in terms of R and T. [1]

(b) Make T the subject of the formula. [2]

(c) Hence or otherwise find T when $P = -2$ and $R = 1$. [1]

14. Fig. 1.5 shows a right-angled triangle ABC, BC is produced to D. AC = 13*cm* and AB = 5*cm*.

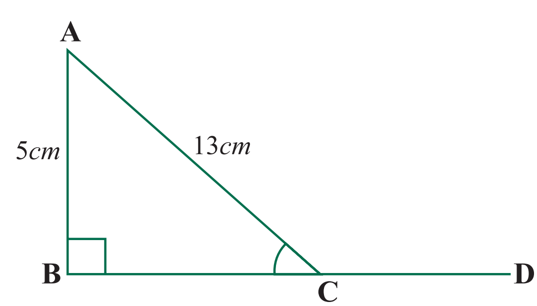


Fig. 1.5

(a) Find BC. [1]

(b) Find:

(i) $\sin A\hat{C}D$. [1]

(ii) $\cos A\hat{C}D$. [1]

(iii) $\tan A\hat{C}D$. [1]

15. The heights in metres of 5 learners at a school are 1,60; 1,10; 1,60; 1,2; 1,8. Find the:

(a) modal height. [1]

(b) median height. [1]

(c) mean height. [2]

16. (a) Factorise completely.

(i) $2x^2 - 8$ [1]

(ii) $2x^2 + 9x + 10$ [2]

(b) Hence or otherwise find the:

(i) HCF. [1]

(ii) LCM. [1]

17. A right circular cone has a base diameter of $10cm$ and a slant height of $13cm$.
Calculate the volume of the cone. [3]

[*volume* $= \frac{1}{3}\pi r^2$, *use* π *as* $\frac{22}{7}$]

18. (a) It is given that $f(x) = 5x^2 - 6x + 1$, find:

(i) $f(-1)$. [1]

(ii) the value of x for which $f(x) = 0$. [3]

(b) Evaluate the following.

(i) $\sqrt{63}$ [1]

(ii) $\sqrt{20} + \sqrt{80}$ [2]

19. Solve the equations.

(a) $y^{1\frac{1}{3}} = 81$ [2]

(b) $(x-1)^2 = \frac{1}{4}$ [2]

(c) $\frac{2}{x-2} = \frac{3}{x+2}$ [2]

20. (a) Express the following in standard form.

(i) 312 000 [1]

(ii) 0,000713 [1]

(b) $m = 4{,}4 \times 10^{-2}$ and $n = 0{,}2 \times 10^5$; find:

(i) $m \div n$. [2]

(ii) mn. [2]

(iii) $n - m$. [2]

SPECIMEN EXAMINATION 10

PAPER 2

TIME: 2 hours 30 minutes

Answer **all** questions in **Section A** and any **four** from **Section B.**

Section A [52 marks]

Answer **all** the questions in this section.

1. **(a)** Factorise completely.

(i) $2a^2 - 8b^2$ [2]

(ii) $-16x^2 - 4xy + 3y^2$ [2]

(b) Solve the equations.

(i) $4(2x + 1)(2x + 1) = 9$

(ii) $2a^2 - 9a - 35 = 0$ [6]

2. **(a)** **(i)** Given that $S = \frac{n}{2}[2a + (n - 1)d]$ make d the subject of the formula. [3]

(ii) Hence or otherwise, find d when $n = 2$, $S = 4$ and $a = -1$. [2]

(b)

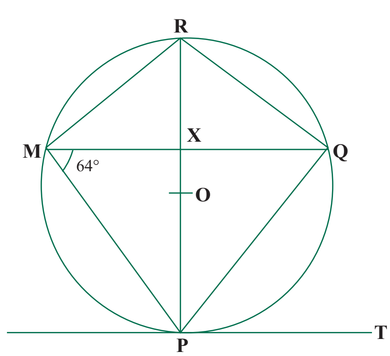


Fig. 1.1

Find:

(i) $P\hat{R}Q$. [1]

(ii) $X\hat{M}R$. [1]

(c) If $\begin{pmatrix} 5 & 4 \\ 3 & 8 \end{pmatrix} - 2\begin{pmatrix} -2 & m \\ 12 & 7 \end{pmatrix} = \begin{pmatrix} -2 & -5 \\ n & 6 \end{pmatrix}$, find the values of m and n. [3]

3. **(a)** D is partly constant and partly varies as H.

(i) Express D in terms of H and constants c and k. [2]

(ii) Given that $D = 80$, $H = 10$ and when $D = 60$, $H = 6$. Find the value of c and the value of k. [2]

(iii) Find the value of H when $D = 90$. [2]

(b) **(i)** Express 334 to a number in base 2. [2]

(ii) Given that $v^2 = u^2 + 2$ make u the subject of the formula. [2]

4. **(a)** **(i)** Solve $x - 1 < 3x - 4 \leq 2x + 2$. [3]

(ii) Illustrate the solution set on a number line. [2]

(b) If $f(k) = 3k^2 + 5k$. Find:

(i) $f(-2)$. [2]

(ii) the value of k for which k is an integer in $f(k) = -2$. [3]

(c) Evaluate $\log_3 27 - \log_{10}\sqrt{10}$. [2]

5. Answer the whole of this question on a sheet of plain paper. Use ruler and compasses only and show clearly all construction lines and arcs.

(a) Construct on a single diagram :

(i) quadrilateral ABCD in which AB = 9*cm*, BC = 6*cm*, AD = 7,3*cm*, DC = 5,5*cm* and $B\hat{A}D = 45°$ [5]

(ii) the locus of points equidistant from A and B. [2]

(iii) the locus of points 5,7*cm* from B. [1]

(b) Measure and write down the size of $A\hat{D}C$. [1]

(c) Point P is inside the quadrilateral and is such that it is equidistant from A and B and is 5,7*cm* from B. Measure and write down the distance of P from D. [1]

Section B [48 marks]

Answer any **four** questions in this section.

Each question carries 12 marks.

6.

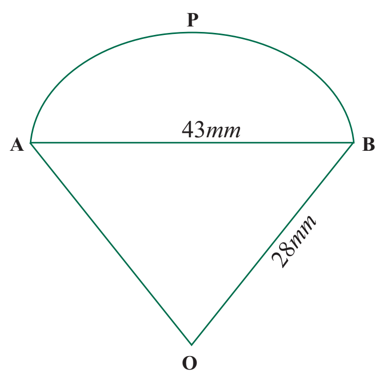


Fig. 1.2

In Fig. 1.2, OAPB is a sector of a circle centre O and radius 28*mm*. AB = 43*mm*. Calculate the:

(a) angle AOB. [3]

(b) area of:

(i) sector OAPB. [3]

(ii) triangle AOB. [3]

(iii) the segment APB. [3]

7. Answer the whole of this question on a sheet of graph paper.

The table below shows the marks of 100 students in a Mathematics test marked out of 100.

Mark (*x*)	**Frequency(*f*)**	**Frequency density**
$0 < x \leq 10$	3	0,3
$10 < x \leq 20$	7	0,7
$20 < x \leq 30$	18	1,8
$30 < x \leq 40$	19	p
$40 < x \leq 50$	30	3,0
$50 < x \leq 75$	15	0,6
$75 < x \leq 90$	5	0,3
$90 < x \leq 100$	3	q

(a) State the modal class of the distribution given above. [1]

(b) Calculate an estimate of the mean mark. [3]

(c) Find the values of p and q. [2]

(d) Using a scale of 2*cm* to represent 10 marks on the x-axis and 2*cm* to represent 1 unit on the y-axis, draw a histogram to show this information.[4]

(e) Two students, were chosen at random, calculate the probability that they both had a mark above 60.

8. **(a)** A teacher has the exact amount to buy 66 notebooks at $2,20 each.

(i) How much money does she have altogether? [1]

(ii) If she decides to buy note books at $3,30 each, how many will she be able to buy? [2]

(b) The value of a new house increases by 20% during the first year. In the second year and subsequent years its value increases by 10% of its value at the beginning of that year. If at the beginning of 1990, the value of a new house was $60 000, calculate the value of the house at the beginning of:

(i) 1991.

(ii) 1993. [5]

(c) A map is drawn to a scale of 1:4000.

(i) Two villages are 1,8*km* apart. Calculate, in centimetres, the distance between them on the map. [2]

(ii) A sports field is represented by a rectangle 3*cm* by 1,2*cm*. Find the actual area, in square metres of the sports field. [2]

9. **(a)** Solve $-21 < 8x - 5 \leq 75$ and illustrate the solution on a number line. [3]

(b) A farmer grows tomatoes and beans on his 60 hectare farm. It costs \$80 to plant a hectare of tomatoes and \$50 to plant a hectare of beans. He has only \$4000 to meet the expenses. It takes 24 hours to plant a hectare of tomatoes and 16 hours to plant a hectare of beans. The farmer has at most 1224 hours for planting. Taking t to represent the number of hectares under tomatoes and b to represent the number of hectares under beans, write down three inequalities, other than $t \geq 0$ and $b \geq 0$, which satisfy the above conditions. [3]

(c) Answer this part of the question on a single sheet of graph paper.

The points with coordinates $(x; y)$ satisfy the following inequalities.

$y \geq 0$, $\quad x + y \leq 10$

$5x + 2y \geq 0$ $\quad 2y \leq x$

(i) Using a scale of $2cm$ to represent 2 units on each axis, construct accurately on graph paper and indicate, by shading the unwanted region, the region in which the points $(x; y)$ must lie. [4]

(ii) Use the graph to estimate the maximum value of $x + 3y$. [2]

ANSWERS FOR ALL EXERCISES AND EXAMINATIONS

Topic 1: Real Numbers

Exercise 1.1

1.

2	252
2	126
3	63
3	21
7	7
	1

$252 = 2^2 \times 3^2 \times 7$

2. Prime numbers between 30 and 40 = 31 and 37

3. Common factors of 30 and 36 = 1; 2; 3 and 6

4. The smallest number which can make it a perfect square is multiplying by those prime numbers to make all powers be even numbers.

These are $2 \times 7 = 14$

5. 2

Exercise 1.2

1. **(a)** The largest integer is the same as HCF.

$294 = 2 \times 3 \times 7^2$

$784 = 2^4 \times 7^2$

$\text{HCF} = 2 \times 7^2$

$= 98$

(b) $\sqrt{784} = \sqrt{2^4 \times 7^2}$

$= 2^{\frac{4}{2}} \times 7^{\frac{2}{2}}$

$= 2^2 \times 7$

$= 4 \times 7$

$= 28$

Express the number as a product of its prime factors in index form, then divided the power by 2, and the result is the square root.

2. $12 = 2^2 \times 3$

$18 = 2 \times 3^2$

$\text{LCM} = 2^2 \times 3^2$

$= 4 \times 9$

$= 36$

3. **(a)** $198 = 2 \times 3^2 \times 11.$

Multiply by 2×11 to make it a perfect square, that is $k = 22$

(b) $198 = 2 \times 3^2 \times 11$

$90 = 2 \times 3^2 \times 5$

$\text{HCF} = 2 \times 3^2$

$= 2 \times 9$

$= 18$

Exercise 1.3

1. **(a)** $\sqrt{(5p + p)^2}$

$= \sqrt{(6p)^2}$

$= 6p$

(b) $\sqrt{50} = \sqrt{25 \times 2}$

$= \sqrt{25} \times \sqrt{2}$

$= 5 \times \sqrt{2}$

$= 5\sqrt{2}$

2. **(a)** $\sqrt{3} + \sqrt{12}$

$= \sqrt{3} + \sqrt{4 \times 3}$

$= \sqrt{3} + \sqrt{4} \times \sqrt{3}$

$= \sqrt{3} + 2 \times \sqrt{3}$

$= \sqrt{3} + 2\sqrt{3}$

$= \mathbf{3\sqrt{3}}$

(b) $3\sqrt{2} + 5\sqrt{2}$

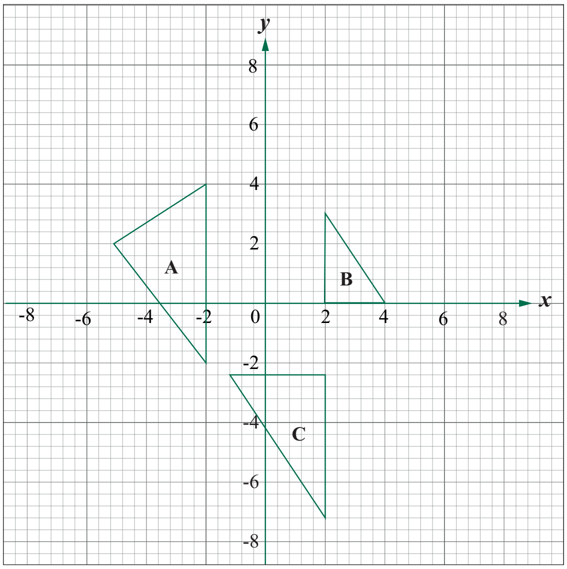
$= \mathbf{8\sqrt{2}}$

3. $\sqrt{0.0081} = \sqrt{\dfrac{81}{10\,000}}$

$= \dfrac{\sqrt{81}}{\sqrt{10\,000}}$

$= \mathbf{\dfrac{9}{100}}$

9. **(a)** and **(b)** are illustrated on the graph below.



(c) translation vector = $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(d) (–10;2), (–4;2) and (–4;4)

10. **(a)** **(i)** $\overline{BC} = \boldsymbol{b} - \boldsymbol{a}$

(ii) $\overline{BN} = \frac{1}{3}(\boldsymbol{b} - \boldsymbol{a})$

(iii) $\overline{AN} = \frac{1}{3}(\boldsymbol{b} + 2\boldsymbol{a})$

(iv) $\overline{BM} = \frac{1}{2}\boldsymbol{b} - \boldsymbol{a}$

(b) $\overline{BX} = \frac{h}{2}(\boldsymbol{b} - \boldsymbol{a})$ and $\overline{AX} = \overline{AB} + \overline{BX}$

$= a + \frac{h}{2}(\boldsymbol{b} - \boldsymbol{a})$

(c) $\overline{AX} = \frac{k}{3}(\boldsymbol{b} + 2\boldsymbol{a})$

(d) $\overline{AX} = \mathbf{AX}$

$\frac{k}{3}\boldsymbol{b} + \frac{2}{3}ka = \boldsymbol{a} + \frac{1}{2}h\boldsymbol{b} - \frac{1}{2}ha$

$k = \frac{3}{2}h$----------(i)

$4k = 6 - 3h$-----(ii)

$\therefore k = 1$ and $h = \frac{2}{3}$

SPECIMEN EXAMINATION 7

PAPER 1

SECTION A

1. **(a)** 32 **(b)** $\frac{1}{27}$
(c) –4

2. **(a)** $(x - 1)(x + 7)$
(b) $-2xy(3x - y)(3x + y)$

3. **(a)** $7{,}9958 \times 10^2$ **(b)** $8{,}0042 \times 10^2$
(c) $3{,}36 \times 10^2$ **(d)** $5{,}5 \times 10^{-4}$

4. **(a)** US$1,00:8R **(b)** US$65,00

5. **(a)** $\begin{pmatrix} -5 \\ 6 \end{pmatrix}$ **(b)** 5 units

6. **(a)** 5,34*cm* and 8,74*cm*
(b) 45,412 5cm^2

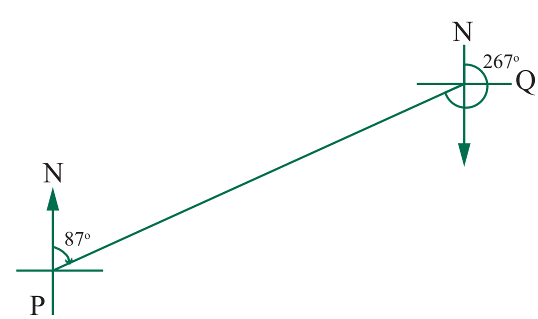
7. **(a)** **(i)** $y = -\frac{1}{2}x + 1$
(ii) $x = \frac{2}{3}$
(b) $y = -\frac{1}{2}x$-1

8. 25

9. **(a)** $P = \frac{135R}{Z^3}$ **(b)** P = 45

10. $x = \frac{2}{7}$ and $y = \frac{6}{7}$

11. **(a)** **(i)** 87°
(ii)



(b) $\sqrt{12}$; π; $\sqrt{13}$; $\sqrt{2}$

12. **(a)** 20 metres **(b)** 6cm^2

13. $\frac{3a - 32}{(a + 1)(a - 6)}$