ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

1 - 13

CONFIDENTIAL

MARKING SCHEME

NOVEMBER 2019

MATHEMATICS 6046/1

All 5 questions) $= \frac{C_5^6}{C_5^{10}}$ M1

$= \frac{6}{252}$ A1

$= \frac{1}{42}$ $= 0{,}0238$ ~~A1~~ (2] [2]

It retains the original data B1

Or

Skewness
It shows the shape of the distribution [1]

Median $=$ $\frac{1}{2}(22+1)^{th}$ observation or equiv. method

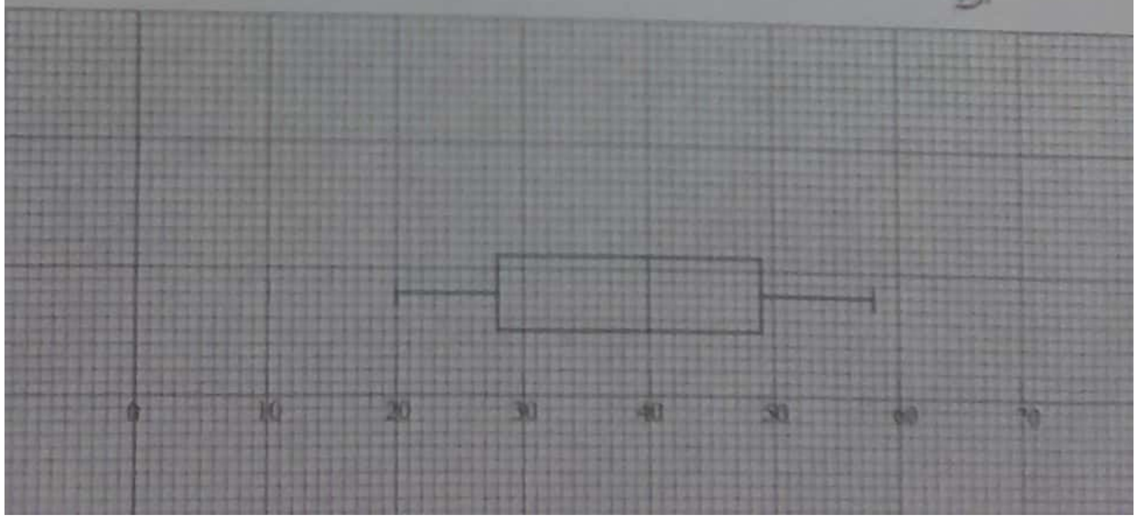
$=$ 11.5^{th} item

$=$ 40 B1

Lower quartile $=$ 28 or 29 B1

Upper quartile $=$ 49 or 49.5 B1

Q1 Q3
(28 ; 49,5)
(29 ; 49) B3



3 (a)

x	f	xf	x^2f
22.5	6	135	3 037.5
28	8	224	6 272
33	7	231	7 623
38	5	190	7 220
43.5	10	435	189 225

or equiv.

$\sum f = 36$ $\sum xf = 1\,215$

$\bar{X} = \frac{\sum xf}{\sum f} = \frac{1\,215}{36}$ ———— M1

$= 33.75$ ———— A1 [2]

$\sum x^2 f = 43\,075$ ———— B1 for $\Sigma x^2 f$ (s.o.i)

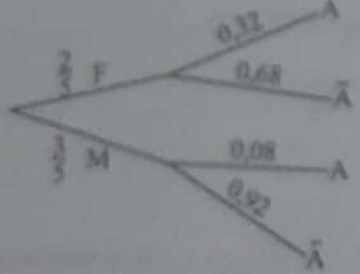
(b) $\sigma = \sqrt{\frac{43\,075}{36} - (33.75)^2}$ ———— M1 for use of formula

$= 7.58058$ ———— A1

[3]

[5]

4



(a) $P(absent) = \frac{2}{5} \times 0.32 + \frac{3}{5} \times 0.08$ ———— M1 Addition of products.

$= 0.176$ ———— A1

[2]

(b) $P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{\frac{2}{5} \times 0.32}{0.176}$ ———— B1 M1 — for correct numerator — for dividing by his p(A)

(both)

5 (a) $\bar{X} \sim N\left(68, \frac{64}{38}\right)$ — B1 for correct parameters s.o.i

$P(\bar{X} < 66) = P\left(Z < \frac{66-68}{\sqrt{\frac{64}{38}}}\right)$ — M1 for standardisation condone interchange of $\bar{X}$ and μ

$= P(Z < -1.541)$

$= 1 - \Phi(1.541)$

$= 1 - 0.9383$

$= 0.0617$ — A1

[3]

(b) $P(67 < \bar{X} < 71)$

$P\left(\frac{67-68}{\sqrt{\frac{64}{38}}} < Z < \frac{71-68}{\sqrt{\frac{64}{38}}}\right)$ ~~Accept use of other tables~~ — M1 for standardisation

$= P(-0.772 < Z < 2.312)$

$= \Phi(0.772) + \Phi(2.312) - 1$ — M1 for correct interpretation

$= 0.7800 + 0.9897 - 1$

$= 0.7697$ — A1 Correct A.

[3]

[6]

6 (a) $\frac{1}{6} + a + \frac{5}{18} + b = 1$ … (i) At least one correct — B1

$\frac{1}{6} + 2a + \frac{5}{6} + 4b = \frac{25}{9}$ … (ii)

$9a + 9b = 5$

$9a + 18b = 8$

$9b = 3$ — M1 Attempting to solve

$b = \frac{1}{3} \qquad a = \frac{2}{9}$ — A1✓ A1✓ for c.A.

[4]

(b) $Var(X) = E(X^2) - [E(X)]^2$

$= \frac{160}{18} - \left(\frac{25}{9}\right)^2$ — B1 ~~BF~~ for the value $8\frac{8}{9} = \frac{160}{18} = E(X^2)$; M1 for correct method

$= \frac{95}{81}$ or equiv. — A1 [3]

$= 1\frac{14}{81}$ / 1.1728.

[7]

7 $P(X . 34) = 0.0238$

$P\left(Z > \frac{34-\mu}{\sigma}\right) = 0.0238$ ——— M1 for standardising and equating to a prob

$\emptyset\left(\frac{34-\mu}{\sigma}\right) = 0.9762$

$\frac{34-\mu}{\sigma} = 1.981$ ——— A1 Correct eqn

$34 - \mu = 1.981\,\sigma$ ....(i)

$P(X < 25) = 0.0163$

$P\left(Z < \frac{25-\mu}{\sigma}\right) = 0.9837$ ——— M1 Standardising and equating to a prob

$\emptyset\left(\frac{25-\mu}{\sigma}\right) = 0.9837$

$\frac{25-\mu}{\sigma} = -2.137$ ——— A1 Correct eqn

$25 - \mu = -2.137\,\sigma$ ....(ii)

$\mu - 25 = 2.137\,\sigma$

$-\mu + 34 = +1.981\sigma$
$9 = 4.118\sigma$ ——— M1 Attempting to solve

$\sigma = 2.186;\ \mu = 29.67$ ——— A1 A1 for each

[7]

[7]

8 (a) Let X be the r.v "the number of cellphone with the application"

$X \sim Bin\left(15\ ;\ \frac{7}{10}\right)$ ——— B1 for parameters s.o.i

$P(X < 13) = 1 - P(X \geq 13)$

$= 1 - [P(X = 13) + P(X = 14) + P(X = 15)]$

$= 1 - \left[C_{13}^{15}\left(\frac{7}{10}\right)^{13}\left(\frac{3}{10}\right)^{2} + C_{14}^{15}\left(\frac{7}{10}\right)^{14}\left(\frac{3}{10}\right) + C_{15}^{15}\left(\frac{7}{10}\right)^{15}\right]$ M1 Correct method interpretation and substitution

$= 0.87317$ ——— A1 c.A.O

[3]

(b) $X \sim Bin\left(60, \frac{7}{10}\right)$

$np = 60 \times \frac{7}{10} = 42$ $\quad npq = 60 \times \frac{7}{10} \times \frac{3}{10} = 12.6$ ——— B1 B1 Correct parameters

$X \sim N(42, 12.6)$

$P(X > 45) \rightarrow P(X > 45.5)$ ——— B1 for c.c

$= \quad P\left(Z > \frac{45.5 - 42}{\sqrt{12.6}}\right)$ ——— M1 for standardisation

$= \quad P(Z > 0.9860)$

$= \quad 1 - \Phi(0.9860)$

$= \quad 0.162 \qquad$ Accept use of other tables ——— A1 C.A.O

[5]

[8]

9 (a) $\int_0^1 kx\,dx + \int_1^3 k(3 - x)\,dx = 1$

$k\left[\frac{x^2}{2}\right]_0^1 + k\left[3x - \frac{x^2}{2}\right]_1^3 = 1$ ——— M1 Attempt to integrate and equating to 1

$\frac{k}{2} + k\left\{\left(9 - \frac{9}{2}\right) - \left(3 - \frac{1}{2}\right)\right\} = 1$ ——— M1 for substitution of correct limits.

(b) $X \sim Bin\left(60 \quad \frac{7}{10}\right)$

$np = 60 \times \frac{7}{10} = 42$ ✓ $\quad npq = 60 \times \frac{7}{10} \times \frac{3}{10} = 12.6$ ✓ — B1B1 Correct parameters

$X \sim N(42 : 12.6)$

$P(X > 45) \rightarrow P(X > \underline{45.5})$ ——— B1 for c.c

$= \quad P\left(Z > \frac{45.5-42}{\sqrt{12.6}}\right)$ ——— √M1 for standardisation

$= \quad P(Z > 0.9860)$

$= \quad 1 - \Phi(0.9860)$

$= \quad 0.162$ Accept use of other tables ——— A1 C.A.O

[5]

[8]

9 (a) $\int_0^1 kx\,dx + \int_1^3 k(3-x)\,dx = 1$

$k\left[\frac{x^2}{2}\right]_0^1 + k\left[3x - \frac{x^2}{2}\right]_1^3 = 1$ ——— M1 Attempt to integrate and equating to 1

$\frac{k}{2} + k\left[\left(9 - \frac{9}{2}\right) - \left(3 - \frac{1}{2}\right)\right] = 1$ ——— M1 for substitute to correct limit.

$k = \frac{2}{5}$ ——— A1 [3]

(b) Let median be m

$\int_0^1 kx\,dx + \int_1^m k(3-x)\,dx = \frac{1}{2}$

$\frac{2}{5}\int_m^3 (3-x)\,dx = \frac{1}{2}$ ——— B1

$\Rightarrow \quad \left[3x - \frac{x^2}{2}\right]_m^3 = \frac{5}{4}$ ——— M1 attempt to integrate

$2m^2 - 12m + 13 = 0$ ——— A1 Correct eqn in m?

$m = \frac{12 \pm \sqrt{12^2 - 4(2)(13)}}{4}$ ——— M1 Attempting to solve (his)

$m = 1.419$ ——— A1 C.A.O

[5]

m = 1.419 or 4.–
Take [illegible] value.

[8]

10 Let X be the distance travelled by John and let Y be the distance travelled by Peter

$X \sim N(106 \;;\; 7^2)$

$Y \sim N(32;\;\; 5^2)$

(a) $E(X - 3Y) = E(X) - 3E(Y)$

$= 106 - 3(32) = \pm 10$ ✓ B1 for 10 Condone sign

$Var(X - 3Y) = Var(X) + 9\,Var(Y)$

$= 49 + 9(25) = 274$ ✓ B1 for 274

$P(X - 3Y > 0) = P\left(Z > \frac{0-10}{\sqrt{274}}\right)$ M1 for standardisation

$= P(Z > -0.604)$

$= \Phi(0.604)$

$= 0.727$ A1 CAO

(b) $E(X + Y) = E(X) + E(Y)$

$= 106 + 32 = 138$ ✓ B1 for 138

$Var(X + Y) = Var(X) + Var(Y)$

$= 49 + 25 = 74$ ✓ B1 for 74

$X + Y \sim N(138;\;\; 74)$

$P(X + Y > 150) = P\left(Z > \frac{150-138}{\sqrt{74}}\right)$ M1 for standardising

$= P(Z > 1.395)$

$= 1 - \Phi(1.395)$

$= 1 - 0.9185$

$= 0.0815$ A1 CAO

[8]

(c) (i) $r = \frac{10(27\,268)-(398)(664)}{\sqrt{16\,836}\ \sqrt{5904}}$ ———— M1 correct substitution

$= 0.8433$ ———— A1 C.A.O [2]

(ii) Strong positive linear correlation ~~B2~~ B1 B1 for the 3 words [2]

[11]

13 1. $\frac{\binom{6}{5}}{\binom{10}{5}}$ M1 $= \frac{6}{252} = \frac{1}{42} = 0{,}0238$ A1

O	E	$\frac{(O-E)^2}{E}$
67	61.43 61	0.50504 0,59
128	133.57 134	0.23227 0,27
26	28.04 28	0.14842 0,14
63	60.96 61	0.06827 0,066
16	19.53 20	0.63804 0,8
46	42.47 42	0.29340 0,38

M1 A1 A1 for E — Correct method; Any one correct; for all

M1 —— Correct method

A1 A1 for $\frac{(O-E)^2}{E}$ — for all correct; any one correct

H_0: there is no relationship between the size of the commuter omnibus and the number of fatal accidents. ——— B1 for Ho

$X^2 = \frac{\Sigma(O-E)^2}{E} = 1.885$ ———— M1 A1 —— for summation; C.A.O (1,485 − 2,2482)

$X^2_{1\%}(2) = 9.21$ ———— B1 9,21

Since $1.885 < 9.21$. Do not reject H_0 and conclude that there is no relationship. ✓ M1 —— Comparison; A1 —— Conclusion [12]

[12]

14 (a) $\bar{X} = \frac{\Sigma xf}{\Sigma f} = 2.5$ ✓ ———— B1

Let X be the number of buses passing a road block

$X \sim Po(2.5)$

H_0: X can follow a Poisson distribution ———— B1 for correct Ho

H_1: X is not distributed in this way

Expected frequencies

$P(X = 0) = e^{-2.5}\frac{(2.5)^0}{0!} = 0.082084998624 \times 100$

$= 8.2085$

12
12
28
26
20

0778087878

Best

① 390 71560

$P(X = 1) = e^{-2.5}(2.5) = 0.20521249656 \times 100$

$= 20.5212$

$P(X = 2) = 0.2565156 \times 100$

$= 25.65156$

$P(X = 3) = 0.2137630 \times 100$

$= 21.3763$

$P(X = 4) = 0.133602 \times 100$

$= 13.3602$

$P(X = 5) = 0.0668009 \times 100$

$= 6.68009$

$P(X > 6) = 0.0420210$

$= 4.2021$

Correct method

M1 A1 — Anyone correct

A1 — for all correct

$P(x>6) = 1 - (P(x \le 6))$

O	E	$\frac{(O-E)^2}{E}$
5	8.21	1.2551
23	20.52	0.2997
23	25.65	0.2738
25	21.38	0.6129
14	13.36	0.03066
10	10.88	0.07118

for correct method E

M1 A1 — for all correct

B1 — Pooling!

$P(X = 1) = e^{-2.5}(2.5) = 0.20521249656 \times 100$

$= \quad 20.5212$

$P(X = 2) = 0.2565156 \times 100$

$= \quad 25.65156$

$P(X = 3) = 0.2137630 \times 100$

$= \quad 21.3763$

$P(X = 4) = 0.133602 \times 100$

$= \quad 13.3602$

$P(X = 5) = 0.0668009 \times 100$

$= \quad 6.68009$

$P(X > 6) = 0.0420210$

$= \quad 4.2021$

Correct method

M1 A1 — anyone correct

A1 — for all correct

$P(x>6) = 1-(P(x \leq 6))$

O	E	$\frac{(O-E)^2}{E}$
5	8.21	1.2551
23	20.52	0.2997
23	25.65	0.2738
25	21.38	0.6129
14	13.36	0.03066
10	10.88	0.07118

for correct method E

M1 A1 — for all correct

B1 — Pooling!

15 (a) (i) $\bar{X} = \frac{215}{72} + 20 = 22.986$ /2 — M1 A1 — C.A.O — Correct method

(ii) $\hat{\sigma}^2 = \frac{n}{n-1}s^2 = \frac{72}{71}\left[\frac{\sum(x-20)^2}{72} - \left(\frac{\sum x-20}{72}\right)^2\right]$

$= \frac{72}{71}\left[\frac{3234}{72} - \left(\frac{215}{72}\right)^2\right]$ — M1 Correct substitution

$\frac{1}{71}\left(3234 - \frac{215^2}{72}\right) =$ 36.507 — A1 C.A.O

[4]

(b) $\bar{x} \pm 1.645\frac{s}{n}$ — B1 table value

$22.986 \pm 1.645\sqrt{\frac{36.507}{72}}$ — M1 Correct method

(21.815 ; 24.157) — A1 for both correct

[3] up to 2 d.p.

(c) H_0 $\mu = 24.5$

H_1 $\mu < 24.5$ — B1 for both correct

Test statistics $Z = \frac{(22.986 - 24.5)\sqrt{72}}{\sqrt{36.507}}$ — M1 Correct method

$Z = -2.126$ — A1 Condone sign

$Z_{0.05} = -1.645$ — B1 table value Condone sign

Since $-2.126 < -1.645$ We reject H_0 — M1 Correct comparison

$\therefore$ there is significance evidence of less time. — A1 — Conclusion

[6]

[13]