ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

MARKING SCHEME

NOVEMBER 2020 SESSION

STATISTICS **6046/1**

1 (a) $P(I) = \left(\frac{1}{6}\right)\left(\frac{4}{5}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{2}\right)\left(\frac{19}{20}\right)$ M1

$= \frac{103}{120}$ | 0·8583 A1

[2]

(b) $P(F/I) = \frac{\left(\frac{1}{2}\right)\left(\frac{19}{20}\right)}{\frac{103}{120}}$ B1M1 dividing by their (a)

$= \frac{57}{103}$ | 0.5534 A1

[3] (5)

2 (a) Arranging numbers in order of size:

41 42 42 42 43 44 45 46 46 47 48 55

$Q_2 = 44.5$ B1

$Q_1 = 42$ and $Q3 = 46.5$ B1B1

[3]

(b) On graph paper B1B1B1 scale and outlier

[3]

(c) Distribution nearly symmetrical | normal B1 [1] (6)

3 (a) (i) Number of ways of choosing team $= C_4^{11}$

$= \frac{11!}{4!7!}$

$= 330$ B1

[1]

(ii) Number of ways of choosing 3 men and one woman

$= (C_3^6)(C_1^5)$

$= \left(\frac{6!}{3!3!}\right)\left(\frac{5!}{1!4!}\right)$ M1

$= 100$ A1

[2]

(b) When one man is chosen, then 3 women must be chosen

$n(M) = (C_1^6)(C_3^5)$

$= \left(\frac{6!}{5!1!}\right)\left(\frac{5!}{3!2!}\right)$ ~~M1~~

$= 60$ ~~A1~~ B1 soi (6)

$\therefore P(M) = \frac{60}{330} = \frac{2}{11}$ M1A1 ~~[7]~~ /6)

4 $E(X) = 3.5$

$1(0,1) + 2(0,3) + a(0,4) + b(0,2) = 3,5$ ~~B1~~

$2a + b = 14$ (1) B1

$Var(X) = 2.65$

$1^2(0,1) + 2^2(0,3) + a^2(0,4) + b^2(0,2) - 3.5^2 = 2.65$ M1

$2a^2 + b^2 = 68$ (2) A1

$2a^2 + (14 - 2a)^2 = 68$ M1 attempt to sle simultaneously

$(3a - 16)(a - 4) = 0$

$a = 4$ ($a = 5\frac{1}{3}$ not admissible) A1

$b = 6$ A1

[6]

5 (a) $\sum x = 225,\ \sum x^2 = 8875,\ \sum y = 361,$

$\sum y^2 = 22641,\ \sum xy = 12905,\ n = 6$

$m = \frac{6(12905)-(225)(361)}{6(8875)-(225)^2}$ M1 correct subst

$= -1.446$ A1

$y - 60.17 = -1.446(x - 37.5)$ M1A1

$\Rightarrow y = -1.45x + 114.4$ ~~[4]~~

(b) No, because x has been controlled B1B1 ~~[2]~~

[6]

6 (a) $\bar{x} = \frac{745}{18} = 41.4$ | 41.3889 B1

$s^2 = \frac{33951}{18} - 41.4^2$ M1

$=$ ~~172.2066667~~ 173.1265 A1 ~~[3]~~

(b) (i) $\sum x = 41 \times 17 = 697$

Mass of pupil who left $= 745 - 697$ M1

$= 48$ kg A1

(ii) S $= \sqrt{\frac{31647}{17} - 41^2} = 13.4$ M1A1 ~~[2]~~

[7]

7 (a) $P(X = 2) = C^6_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{\cancel{5}\,4}$ M1

$= \cancel{15}\,10 \left(\frac{1}{36}\right)\left(\frac{\cancel{625}\,125}{\cancel{1296}\,216}\right)$

$=$ ~~0.200938786~~ A1

0.16071 | $\frac{625}{3888}$

[~~3~~]

(b) $P(X \geq 4) = 1 - (PX < 4)$ | P(X = 4 or 5) B1

$= 1 - C^6_0\left(\frac{5}{6}\right)^6 - C^6_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{4} - C^6_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{3} - C^6_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{2}$ M1

$= 1 - \left(\frac{5}{6}\right)^6 - 6\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5 - 15\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4 - 20\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^3$

$=$ ~~1 – 0.991298011~~

$=$ ~~0.008701989~~ 0.00334 | $\frac{13}{3888}$ A1

[~~3~~]

(c) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= C^6_0\left(\frac{5}{6}\right)^{6} + C^6_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{5} + C^6_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{4}$ M1

$= \left(\frac{5}{6}\right)^6 + 6\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5 + 15\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^4$

$=$ ~~0.9377143347~~ 0.9645 A1 ~~[2]~~

[7]

8 (a) $P(X = 4) = (0{,}6)(0{,}4)^3$ M1

$= 0{,}0384$ A1 [~~2~~]

(b) $E(X) = \frac{1}{0{,}6}$

$= \frac{5}{3}$ | 1.6667 B1

$Var(X) = \frac{0{,}4}{(0{,}6)^2}$ M1

$= \frac{2}{3}$ $\frac{10}{9}$ | 1.111 A1 [~~3~~]

(c) P(X ≥ 3) = P(X = 3) + P(X > 3) / P(X > 2) B1

= $(0,6)(0,4)^2 + (0,4)^3$ 0·42 M1

= 0,16 0·16 A1 ~~[3]~~ 4
[8]

9 (a) $C\int_0^{\frac{5}{2}}(2x - x^2)\,dx = c\left[x^2 - \frac{x^3}{3}\right]_0^{\frac{5}{2}}$ M1

= $c\left[\frac{25}{4} - \frac{125}{24}\right]$ M1

= $\frac{25c}{24} > 0$ A1 [3]

(b) (i) $\frac{25c}{24} = 1$ M1

c = $\frac{24}{25}$ A1 [2]

(ii) E(X) = $\frac{24}{25}\int_0^{\frac{5}{2}}(2x^2 - x^3)\,dx$

= $\frac{24}{25}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^{\frac{5}{2}}$ M1

= $\frac{24}{25}\left[\frac{125}{12} - \frac{625}{64}\right]$ M1

= $\cancel{\frac{3}{16}}$ $\frac{5}{8}$ / 0·625 A1 [3]
[8]

10 (a) P(2 < X < 5) = $P\left(\frac{2-3}{3} < \frac{X-3}{3} < \frac{5-3}{3}\right)$ M1 standardising

= $P\left(-\frac{1}{3} < Z < \frac{2}{3}\right)$

= 0,7477 + 0,6304 – 1 M1

= ~~0,3779~~ 0·3781 A1

(b) P(X > 0) = $P\left(Z > \frac{0-3}{3}\right)$ M1

= P(Z > – 1)

= Φ(1)

= 0,8413 A1

(c) $P(|X-3|>6|= P(X<3)+P(X>9)$ B1 interpretation

$= P\left(Z<\frac{-3-3}{3}\right)+P\left(Z>\frac{9-3}{3}\right)$ M1

$= P(Z<-2)+P(Z>2)$

$= 2[1-\Phi(2)]$

$= 2[1-0{,}9772]$ ~~A1~~ B1 table value

$= 0{,}0456$ A1 [~~4~~] [9]

11 (a) Number of years used = 8 B1

(b) Sales for 1985:

$\hat{y} = 284 + 14.4\,(11)$ M1

$=$ ~~537,6~~ 442.4 / 442 400 A1

(c) Annual increase in sales = 14.4 / 14 400 B1

(d) $\hat{y} = \frac{284}{12} + \frac{14.4}{12}\cdot\frac{x}{12}$ M1

$= 23{,}67 + 0{,}1x$

To move origin to 15 July 1980, add 36,5 increments of 0,1 to 23,67:

$\hat{y} = 23{,}67 + 36.5(0{,}1) + 0{,}1x$ M1

$= 27.32 + 0{,}1x$ A1

Sales for September 1980:

$\hat{y} = 27.32 + 0{,}1(2{,}5)$ M1

$= 27.57$ A1

[9]

12 (a) $P(X<20) = 10\int_{10}^{20}\frac{dx}{x^2}$

$= 10\left[-\frac{1}{x}\right]_{10}^{20}$

$= 10\left[\frac{1}{10}-\frac{1}{20}\right]$ M1

$= \frac{1}{2}$ A1 [~~2~~]

(b) For $x > 10$, $F(x) = 10\int_{10}^{x} \frac{dt}{t^2}$

$= 10\left[-\frac{1}{t}\right]_{10}^{x}$ M1

$= 1 - \frac{10}{x}$ A1

For $x \leq 10$, $F(x) = 0$

$\therefore F(x) = \begin{cases} 1 - \frac{10}{x}, & x > 10 \\ 0 & , x \leq 10 \end{cases}$ B1 ~~[3]~~

(c) $Y \sim \text{Bin}\left(6, \frac{1}{2}\right)$ B1

$P(Y \geq 3) = 1 - P(Y < 3)$ B1

$= 1 - C_0^6\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^6 - C_1^6\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^5 - C_2^6\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4$ M1

$= 1 - \left(\frac{1}{2}\right)^6 - 6\left(\frac{1}{2}\right)^6 - 15\left(\frac{1}{2}\right)^6$

$= 1 - 0{,}34375$

$= 0{,}65625$ A1 ~~[4]~~

[9]

13 (a) $P(A > 65) = P\left(Z > \frac{65-75}{6}\right)$ M1

$= P(Z > -1.667)$

$= 0{,}9522$ A1

(b) $M = A_1 + A_2 + A_3 - (B_1 + B_2 + B_3) \sim N(30, 183)$ B1B1

$P(M > 0) = P\left(Z > \frac{0-30}{\sqrt{183}}\right)$ M1

$= P(Z > -2.218)$

$= 0{,}9867$ A1

(c) $H = A - 1.15\,B \sim N(0.25, 69.0625)$ B1B1

$P(H > 0) = P\left(Z > \frac{0-0.25}{\sqrt{69.0625}}\right)$ M1

$= P(Z > -0{,}03008)$

$= \Phi(0{,}03008)$

$= 0{,}5120$ A1

[10]

14 (a) X ~ Bin(500,0.4)

np = 200 > 5 and nq = 300 > 5

X ~ N(200,120) B1B1

P(X = 190) = P(189.5 < X < 190.5) B1

= $P\left(\frac{189.5-200}{\sqrt{120}} < Z < \frac{190,5-200}{120}\right)$ M1

= P(–0,9585 < Z < – 0,8672)

= Φ(0.9585) - Φ(0.8672)

= 0,8312 – 0.8070

= 0,0242 A1

(b) P(180 < X < 210) = P(179.5 < X < 210.5) B1

= $\mathrm{P}\left(\frac{179.5-200}{\sqrt{120}} < \mathrm{Z} < \frac{210.5-200}{\sqrt{120}}\right)$ M1

= P(–1.871 < Z < 0.9585)

= Φ(1.871) + Φ(0.9585) – 1

= 0,9694 + 0.8312 – 1

= 0,8006 A1

(c) P(X > 180) = P(X > 180.5) B1

= $\mathrm{P}\left(Z > \frac{180.5-200}{\sqrt{120}}\right)$ M1

= P(Z > – 1.780)

= Φ(1.780)

= 0,9625 A1

[11]

15 H$_0$: Sample population has Poisson distribution B1 SO1
Ha: Sample population has some other distribution

$\alpha = 0{,}01$

df = 8 – 1 – 0 = 7 B1 SO2

Reject H0 if $\mathcal{X}^2_{cal} > 18{,}48$ B1

$E_0 = 600 \times \frac{e^{-2.5}(2.5)^{\circ}}{0!} = 49.2$ M1A1

M1A1

Bi	Ei	$(O-E)^2/E$
34	49.3 49.251	4.723 4.7226
131	123	0.504
160	153.9	0.242
136	128.3	0.467
72	80.2	0.838
37	40.1	0.240
22	16.7	1.682
8	8.5	0.029

A1A1

$\mathcal{X}^2_{cal} = 8.725$ M1A1

Since $\mathcal{X}^2_{cal} = 8.725 < 18.48$, we do not reject H0 and conclude that the sample population has a Poisson distribution. M1A1 [13]