ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

CONFIDENTIAL

MARKING SCHEME

NOVEMBER 2019

STATISTICS 6046/2

1 (a) $\int_0^3 \frac{x}{12}dx + k\int_3^8 (x-8)dx = 1$

$\frac{1}{12}\left[\frac{x^2}{2}\right]_0^3 + k\left[\frac{x^2}{2} - 8x\right]_3^8 = 1$ (Attempt to integrate wt corr lims and equating to 1) M1

$\frac{3^2}{24} + k\left[\frac{8^2}{2} - 8(8) - \frac{3^2}{2} + 8(3)\right] = 1$

$k\left(\frac{-25}{2}\right) = \frac{5}{8}$

$k = \frac{-1}{20} = -0,05$ (c.a.o) A1

(b) $E(X) = \int_0^3 \frac{x^2}{12}dx + \frac{-1}{20}\int_3^8 (x^2 - 8x)dx$

$= \frac{1}{12}\left(\frac{x^3}{3}\right)\Big]_0^3 - \frac{1}{20}\left[\frac{x^3}{3} - \frac{8x^2}{2}\right]_3^8$ attempt to int his $xf(x)$ √M1

$= \frac{1}{36}[3^3 - 0^3] - \frac{1}{20}\left[\frac{8^3}{3} - \frac{8(8)^2}{2} - \frac{3^3}{3} + \frac{8(3)^2}{2}\right]$

$= \frac{27}{36} + \cancel{\frac{121}{60}}\ \frac{35}{12}$

$= \frac{11}{3}\ \cancel{\frac{83}{30}}\ 3\frac{2}{3}$ $3,6\dot{6} = 3,\dot{6}$ A1

(c) $\int_0^3 \frac{x}{12}dx = \frac{x^2}{24}\Big]_0^3 + \frac{-1}{20}\int_3^m (x-8)dx = 0,5$

$= \cancel{\frac{3^2}{24} - \frac{0^2}{24}}$

$= \cancel{\frac{3}{8}}$

OR. equiv.

Median $= \frac{-1}{20}\int_m^8 (x-8)dx = 0.5$ M1

$\frac{-1}{20}\left[\frac{8^2}{2} - 8(8) - \frac{m^2}{2} + 8m\right] = 0,5$ att to int. wt correct limt equated to 0,5

$\frac{m^2}{2} - 8m + 22 = 0$

$m^2 - 16m + 44 = 0$

$m = \frac{-(-16) \pm \sqrt{16^2 - 4(1)(44)}}{2(1)}$ attemp to solve eqn M1

$= 3.528$ A1

[7]

$\frac{3}{8} - \frac{1}{20}\left(\frac{x^2}{2} - 8x\right|_3^m = 0,5$

2 (a) (i) Involves observing desirable characteristics of each and every member of the population. B1 B1

(ii) - ~~flexible~~ ?
- gives information on various aspects of the population
- can get accurate information as every member participates
- unbiased
- can be refered back to in the near future.

Any two B2

(b) (i) Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{139}{100} = \frac{0,25 \times 8 + \ldots + 2,25 \times 21}{100}$ M1

$= 1.39 =$ A1

(ii) Standard deviation $\sqrt{\frac{230.75}{100} - (1.39)^2}$ $\frac{\Sigma x^2 f}{\Sigma f}$. √M1.

$= 0.6127$ A1 [8]

3 (i) (a) $E(X) = 2(0.1) + 3(0.4) + 4(0.1) + 5(0.3) + 6(0.1)$ M1

$= 3.9$ A1

(ii) $Var(X) = E(X^2) - E^2(X)$

$E(X^2) = 2^2(0.1) + 3^2(0.4) + 4^2(0.1) + 5^2(0.3) + 6^2(0.1)$ ~~B1~~

$= 16.7$

correct or working

$\therefore Var(X) = 16.7 - 3.9^2$ his mean √M1

$= 1.49$ A1

(ii) (i) $E(Y) = E(3X - 2)$

$= 3E(X) - 2$

$= 3(3.9) - 2$ his mean √M1

$= 9.7$ A1

(ii) $Var(Y) = Var(3X - 2)$

$= 9Var(X)$

$= 9(1.49)$ his var √M1

$= 13.41$ A1 [8]

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4 (a) $X \sim Po(2)$

$P(X = 3) = \frac{e^{-2} \cdot 2^3}{3!}$ M1

$= \quad 0.180447$ A1

(b) $X \sim Po(4)$ (s.o.i) B1

$P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$= \quad \frac{e^{-4} \cdot 4^0}{0!} + \frac{e^{-4} \cdot 4^1}{1!} + \frac{e^{-4} \cdot 4^2}{2!} + \frac{e^{-4} \cdot 4^3}{3!}$ his λ not 2. M1

$= \quad e^{-4}\left(1 + 4 + 8 + \frac{64}{6}\right)$

$= \quad 0.43347$ A1

(c) $X \sim Po(6)$

$P(X > 2) = 1 - P(X \le 2)$ (s.o.i) B1

$= \quad 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$

$= \quad 1 - \left[\frac{e^{-6} \cdot 6^0}{0!} + \frac{e^{-6} \cdot 6^1}{1!} + \frac{e^{-6} \cdot 6^2}{2!}\right]$ $\lambda \neq 2$ or 4. ✓ M1

$= \quad 1 - e^{-6}[1 + 6 + 18]$

$= \quad 1 - 0,0619688$

$= \quad 0.9380$ A1

[8]

5 (a) (i) $5 \times 5 \times 5$ M1

$= \quad 125$ (c.a.o) A1

(ii) For the first digits, there are 5 choices and for the second there are 4 choices from the 4 digits

$5 \times 4 \times 3 = {}^5P_3$ M1

$= \quad 60$ A1

(b) (i) $P(T) = 0.8 \times 0.9 + 0.2 \times 0.6$ M1

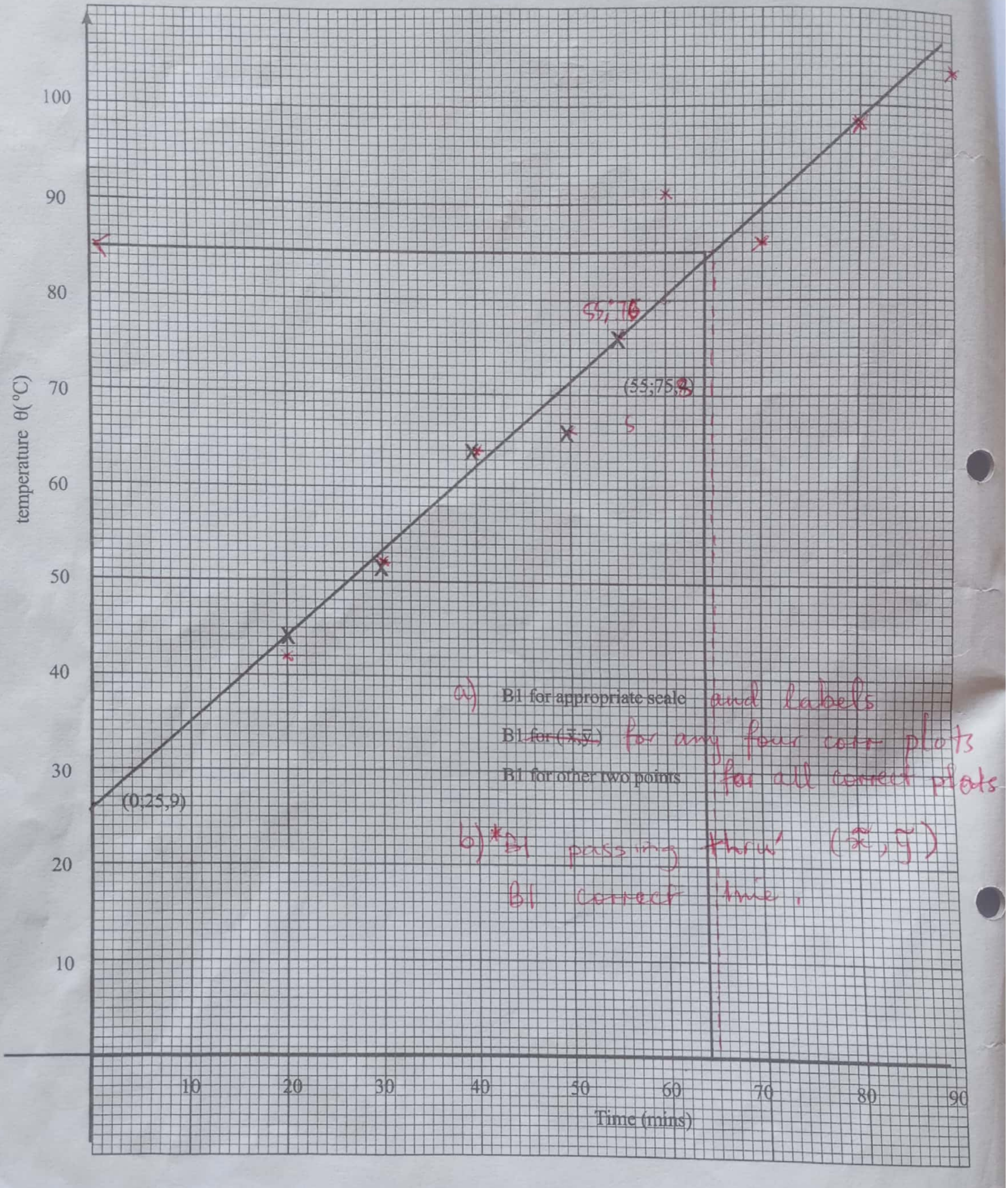
$= \quad 0.84 = \frac{21}{25}$ A1

(ii) $P(F/T) = \frac{0.2 \times 0.6}{0.84}$ $\frac{0.12}{0.84}$ num B1 M1 or his denom

$= \quad 0.14285 = 1/7$ A1

[9]

6 (a)



a) B1 for appropriate ~~scale~~ and labels

B1 ~~for ($\bar{x}$,$\bar{y}$)~~ for any four corr plots

B1 ~~for other two points~~ for all correct plots

b)* B1 passing thru' ($\bar{x}$, $\bar{y}$)

B1 correct line.

(b) $M = \frac{8(36\,940) - 440(603)}{8\left(28\,400 - \frac{440^2}{8}\right)}$ $\frac{8(36\,940) - 440(603)}{8(28\,400) - 440^2}$ M1

$=$ 0.8988 A1

$\theta - 75.75 = 0.8988(t - 55)$ his grad. √M1 A1

$\theta = 0.8988t + 25.94$

(c) $\theta = 84,36$ 85 ± 1 from graph √M1 A1

(d) Not possible (outside range) B1

(e) $r = \frac{8(36\,940) - 440(603)}{\left(\sqrt{8(28\,400) - 440^2}\right)\left(\sqrt{(49\,017)8 - 603^2}\right)}$ M1

$=$ 0.9755 A1

Comment – very Strong positive
Linear correlation correlation (B1 B1)

[16]

7 (a) $\bar{X} = \frac{196}{100}$ $\frac{0 \times 18 + \ldots + 5 \times 9}{100}$ M1

$=$ 1.96 A1

(b) H_0 : X follows a poisson distribution B1 for H_0

H_1 : X does not follow poisson ×.

Expected frequencies

X	$P(X=x)\times 100$	Expected	
0	$e^{-1.96}\times 1.9^{0}\times 100$	14.09	M1 A1
1	$\frac{e^{-1.96}\times 1.96^{1}}{1!}\times 100$	27.61	Any one correct
2	$\frac{e^{-1.96}\times 1.96^{2}}{2!}\times 100$	27.06	
3	$\frac{e^{-1.96}\times 1.96^{3}}{3!}\times 100$	17.68	~~A1~~ (Any 4 correct)
4	$\frac{e^{-1.96}\times 1.96^{4}}{4!}\times 100$	8.66	A1 (All correct
5	$1-P(X\le 4)\times 100$	4.90	

Revised table

X	0	1	2	3	4
$E(X)$	14.09	27.61	27.06	17.68	13.56

B1 for pooling

$V = 5 - 2$ B1

$= 3$

Reject H_0 when $X^2_{cal} > 7.815$ B1 (table value)

$X^2_{cal} = \sum \frac{(O-E)^2}{E}$

O	E	$\frac{(O-E)^2}{E}$	
18	14.09	1.09	M1 A1
25	27.61	0.247	(Any one correct)
25	27.06	0.157	
16	17.68	0.160	
15	13.56	0.153	A1 (all correct
100	100	1.807 1.7 – 1.9	M1 A1

$X^2_{cal} = 1.807$

$1.807 < 7.815$ M1

∴ Do not reject H_0 or equiv. A1

Distribution of weeds follow a poisson distribution

[16]

8 (a) $P(H < 0.65)$

$= \quad P\left(Z < \frac{0.65-0.5}{0.15}\right)$ (Standardising) M1

$= \quad P(Z < +1{,}0)$ M1

$= \quad \emptyset(1)$ ~~M1~~

$= \quad 0.8413$ A1

(b) $E_1 + E_2 + E_3 + E_4 \sim N(4(0.2); \cancel{16}\,4(0.07)^2)$

$N(0.8; \cancel{0.0784})$ 0,0196 B1 B1 s.o.i.

$\therefore P(E_1 + E_2 + E_3 + E_4 < 0.9)$

$= \quad P\left(Z < \frac{0.9-0.8}{\sqrt{0.0\cancel{784}\,196}}\right)$ (Standardising) M1

$= \quad P(Z < 0.71429)$

$= \quad \cancel{\emptyset(0.3571)}$

$= \quad \cancel{0.6394}$ (c.a.o) A1

0,7611

(c) $P(E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + E_7 > H_1)$

$= \quad P(7E - H) > 0$

$= \quad 7E - H \sim N(7(0.2) - (0.5); 7(0.07)^2 + (0.15^2))$

$\pm$ 0,0568

$N(0.9; \cancel{0.3901})$ B1 B1 s.o.i

$\therefore P\left(\frac{Z>0-0.9}{\sqrt{0.3901}}\right)$ $P\left(\frac{0-0.9}{\sqrt{0,0568}}\right)$ (Standardising) M1

$= \quad P(Z > -1.441)$ $P(z > -3,770)$

$= \quad \emptyset(+1.441)$

$= \quad 0.0748$ ≈ 1 (c.a.o) SR $= 1$ A1

accept 0,9986 from 2,999

(d) $P(H < 3E)$

$= \quad P(H - 3E < 0)$

$= \quad \therefore H - 3E \sim N(0.5 - 3(0.2; 0.15^2 + 3^2(0.07)^2))$

$N(\overset{+}{-}0.1; 0.0666)$ B1 B1 s.o.i

$= P\left(Z < \frac{0+0.1}{\sqrt{0.0666}}\right)$ (Standardising) M1

$= \quad P(Z < 0.3875)$

$= \quad \emptyset(0.3875)$ M1

$= \quad 0.6510$ A1

[16]

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9 (a) (i) 1. $\bar{x} = \frac{738.5}{75} + 5$ M1

$= 9.85 + 5$

$= 14.84$ A1

2. $\hat{\delta}^2 = \frac{1}{74}\left(18723 - \frac{738,5^2}{75}\right)$ B1 M1

Simplifying

$= 154.879$ A1

(ii) $98\%\ \text{CI} = \bar{x} \pm Z_{\frac{\alpha}{2}} \frac{\hat{\delta}}{\sqrt{n}}$

$= 14.84667 \pm 2.326 \frac{\sqrt{154.879}}{\sqrt{75}}$ (table value) B1 M1

$= (11.50414; 18.1892)$ all A1 ~~A1~~

(b) (i) $\hat{\delta}^2 = \frac{n}{n-1} s^2$

$= \frac{100}{99}(0.2846)$ M1

$= 0.287474$ A1

(ii) $X \sim N(\mu; 0.28474)$

$H_0: \mu = 2kg$

$H_1: \mu < 2kg$ both B1

$X \sim N(2; 0.28474)$

One tailed test

Level 10%

Reject H_0 when $Z_{cal} < -1.282$ $\pm 1,282$ table value B1

$Z_{cal} = \frac{(1.98-2)\sqrt{99}}{\sqrt{0.28474}}$ M1

$= -0,3729$ 0,3748 A1

$-0.37\cancel{29}\,48 > -1.282$ M1

Do not Reject H_0 / accept H_0

Company is not packing packs of sugar less than 2 kg A1

[16]

10 (a) $1-\phi(-2)=0.15$

$\frac{\mu-1.2}{\delta}=\phi^{-1}(0.85)$ (For standardising) M1

$\frac{\mu-1.2}{\delta}=1.036$

$\mu-1.036\delta=1.2$ (i) (Correct equation) A1

$1-\phi(t)=0.10$

$\frac{1,6-\mu}{\delta}=\phi^{-1}(0.9)$ (For standardising) M1

$1,6-\mu=1.282\delta$

$1.282\delta+\mu=1.6$ (ii) (Correct equation) A1

$\mu-1.036\delta=1.2$

$1.282\delta+\mu=1.6$

$-2.31\delta=-0.4$ (attempt to solve simultaneously) M1

$\delta=0.17256$ (c.a.o) A1

$\mu=1.036(0.17256)+1.2$

$=\quad 1.3788$

$=\quad 1.379$ (c.a.o) A1

[7]

(b) $\bar{X}\pm Z_{\frac{\alpha}{2}}\delta$

$=\quad 1.379\pm0.674(0.17256)$ soi B1 M1

$=\quad 1.2624;\ 1.4956$

$Q_1=1.2624$ A1

$Q_3=1.4956$ A1

[4]

(c) Interquartile range

$= \quad 1.4956 - 1.2624$ (subtraction of his Q_3 and Q_1) M1

$= \quad 0.2332$ A1

(d) $p(|h - \mu| < 0.1)$

$= \quad p(-0.1 < h - \mu < 0.1)$ B1 s.o.i

$= \quad p\left(\frac{-0.1}{0.1756} < z < \frac{0.1}{0.1756}\right)$ his σ √M1

$= \quad p(-0.5795 < z < 0.5795$ Standardising ~~M1~~

$= \quad 2\phi(0.5795) - 1$

$= \quad 2(0.7188) - 1$

$= \quad 0.4376$ (c.a.o) A1

[3]

[16]

11 (a) (i) 1. $X \sim B(15; 06)$ (Identifying binomial) or without parameters B1 s.o.i

$P(X = 4) = C_4^{15}(0.6)^4(0.4)^{11}$ M1

$= \quad 0.007419811$

$= \quad 0.0074$ (c.a.o) A1

2. $P(X < 13) = 1 - P(X \geq 13)$ B1 s.o.i

$= \quad 1 - [P(X = 13) + P(X = 14) + P(X = 15)]$ ~~M1~~

$= 1 - \left[C_{13}^{15}(0.6)^{13}(0.4)^2 + C_{14}^{15}(0.6)^{14}(0.4)^1 + C_{15}^{15}(0.6)^{15}0.4^0\right]$ M1

$= \quad 1 - 0{,}027114$

$= \quad 0.972886$ A1

$= \quad \cancel{0.9279}$ (c.a.o) ~~A1~~

(ii) $X \sim N(120; 48)$ S.O.i B1 B1 normal both parameters

$P(X > 150) = P(X > 150.5)$ c. (correction) B1 S.O.i

$= P\left(z > \frac{150.5-120}{\sqrt{48}}\right)$ M1

$= P(Z > 4.40)$ $1 - \Phi(4.40)$ MA1 ~~M1~~

$= \approx 0$ $1 - 0{,}9986$ A1

$\approx 0{,}0014$

(b) (i) Variance $= \frac{q}{p^2}$

$= \frac{0.75}{0.25^2}$ correct parameters M1

$= 12$ A1

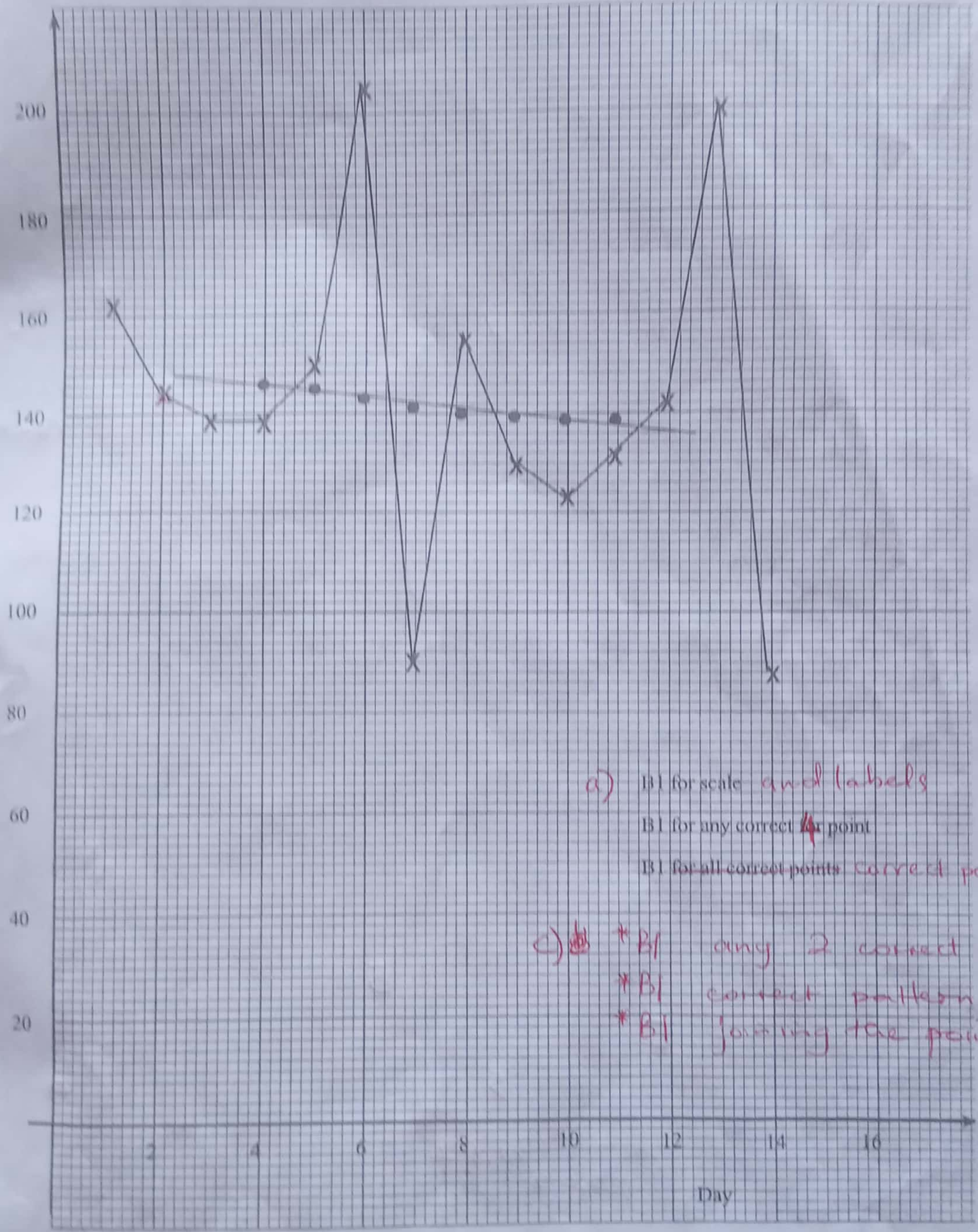
(ii) $P(X > 3) = q^3$ or equiv.

$= (0.75)^3$ correct par. M1

$= 0.421875$ A1

[16]

12 (a)



a) B1 for scale and labels

B1 for any correct 4 point

B1 ~~for all correct points~~ correct pattern

c) * B1 any 2 correct plots

* B1 correct pattern
* B1 joining the points

(b)

Week	Day	Sales	7 point moving average
	1	162	
	2	143	
	3	138	
	4	138	146
	5	149	145
	6	204	143
	7	90	141
	8	155	140
	9	130	140
	10	123	140
	11	132	139
	12	142	
	13	200	
	14	88	

M1 A1
(any one correct)

A1 any 3

A1 (all correct)
centred

(d) Regression line

Day x	4	5	6	7	8	9	10	11
Sales y	146	145	143	141	140	140	140	139

$$M = \frac{8(84\,538) - 60(1\,133.7)}{8(492) - 60^2}$$ M1

$= \quad -1.16548$ A1

$y - 141{,}7125 = -1.16548(x - 7.5)$ √M1 A1

$y = -1.16548x + 150.454$

Monday 3rd week = Day 1~~3~~5

$\therefore x = 1\cancel{3}5$

$\therefore$ Sales $= -1.16548\,(1\cancel{3}5) + 150.454$ M1

$=$ ~~130.64~~ 132,97

$\therefore$ Sales $=$ ~~130.64~~ 132,97 $+ 14.2$ √M1

$=$ ~~144.84~~ \$147.17 A1

[16]

Do not accept 150