VALENTINE → 1

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

MATHEMATICS 9164/1

PAPER 1

JUNE 2007 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

There is no restriction on the number of questions which you may attempt.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.
Questions are printed in the order of their mark allocations and candidates are advised to attempt questions sequentially.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

1 Solve the inequality $|2x+3| > 7$. [3]

2 Find the coefficient of x^6 in the expansion of $(9+x^2)^{\frac{1}{2}}$ in ascending powers of x. [3]

3 Find the exact value of the solution to the equation $2e^{2x} - 7e^{x} - 4 = 0$. [3]

4 Express $\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$. Hence solve the equation $\cos\theta - 2\sin\theta = 0.2$ for $0 \le \theta \le 360°$. [6]

5 A gas law is given by the equation $\ln p + 1.4\ln v = \ln c$, where c is a constant.

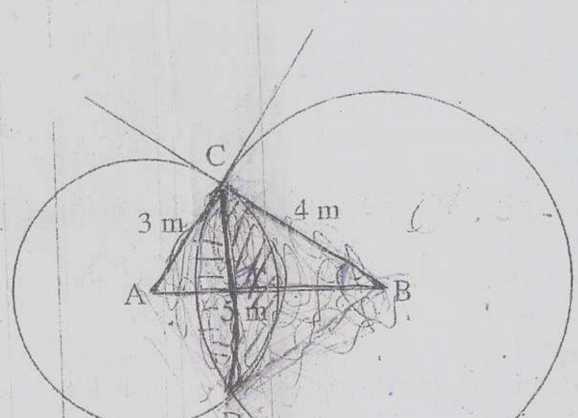
Find

(i) $\dfrac{dv}{dp}$ in terms of v and p, [3]

(ii) the percentage change in v given that p increases by 1% and state whether it is an increase or a decrease. [3]

6 (a) A chord of a circle of radius r subtends an angle of θ radians at the centre of the circle. Show that the area of the minor segment cut off by the chord is $\frac{1}{2}r^2(\theta - \sin\theta)$. [2]

(b)



Two circles of radii 3 m and 4 m have centres at A and B. The centres are 5 m apart. The circles intersect at C and D as shown in the diagram.

(i) Explain why each radius is a tangent to the other circle. [1]

(ii) Calculate the area common to the two circles. [4]

7 Two sequences are defined for $n = 1, 2, 3, \ldots$, as follows.

$$U_n = 10n - 3,$$

$$V_n = 4\left[1-\left(\frac{1}{3}\right)^n\right].$$

(i) Describe the behaviour of each of the sequences as $n \to \infty$. [2]

(ii) Given that $\sum_{n=1}^{N} U_n = 259$, find N. [5]

8 Show that $(x-3)$ is a factor of the function $f(x) = x^3 - 4x^2 - 3x + 18$. [2]

Factorize $f(x)$ completely. [2]

Hence sketch on separate diagrams, the graphs of

(i) $y = f(x)$, showing clearly the coordinates of the points at which the graph meets the axes, [1]

(ii) $y = f(x+3)$, showing clearly the coordinates of the points at which the graph meets the x-axis, [1]

(iii) $y = f(x) - 4$, showing the coordinates of the point at which the graph meets the y-axis. [1]

State, without solving, the number of real roots of the equation $f(x) - 4 = 0$. [1]

9 (a) Taking $x_1 = 1.2$ as a first approximation to a root of the equation

$\frac{x}{10} = \ln x$, use the Newton-Raphson method to find the root correct to

3 decimal places. [4]

(b) The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius.

Show that $V = \frac{1}{6}\pi D^3$, where D is the diameter of the sphere. [1]

Write down an expression for $\frac{dV}{dD}$. [1]

Hence estimate the maximum error in computing the volume of a sphere whose diameter is measured as 16 cm with error 0.01 cm. [2]

[Turn over

A B C

10. The position vectors of the points A, B and C are $\begin{pmatrix}3\\1\\4\end{pmatrix}$, $\begin{pmatrix}7\\3\\0\end{pmatrix}$ and $\begin{pmatrix}15\\7\\-8\end{pmatrix}$ respectively.

(i) Express $\overrightarrow{AC}$ as a column vector and find the angle between $\overrightarrow{OB}$ and $\overrightarrow{AC}$. [4]

(ii) Show that A, B and C are in the same straight line. [3]

(iii) Given that the vector $\begin{pmatrix}5\\-3\\p\end{pmatrix}$ is perpendicular to $\overrightarrow{OA}$, find the value of p. [2]

11 The parametric equations of a curve are

$$x = \sin\phi,$$
$$y = 1 - \cos^2\phi.$$

(i) Show that $\dfrac{dy}{dx} = 2\sin\phi$. [3]

(ii) Find the equations of the tangent and normal to the curve at the point Q, where $\phi = \dfrac{\pi}{6}$. [5] 84

(iii) If the tangent and normal meet the y-axis at points A and B respectively, state the coordinates of A and B. [1]

12 Line l has equation $3y + 2x = 8$ and point A has coordinates (1; 15).
Show that the line from A perpendicular to l has equation $2y - 3x = 27$. [3]

Hence find

(i) the perpendicular distance of A from l, [5]

(ii) the equation of the circle which has centre A and touches l. [3]

13 Express $2x^2 - 6x + 3$ in the form $a(x+b)^2 + c$. [3]

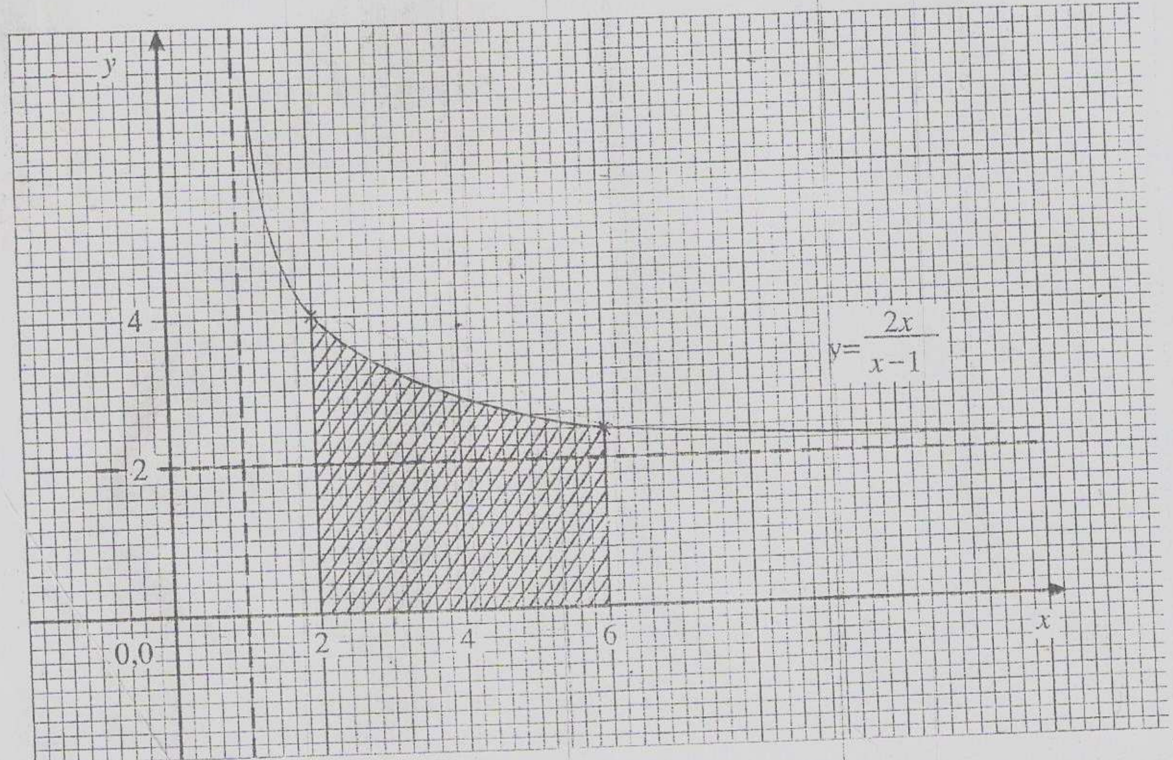
(a) Hence state

(i) the coordinates of the turning point of the graph of $y = 2x^2 - 6x + 3$. [2]

(ii) a sequence of three transformations by which the graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 6x + 3$. [3]

(b) Find the values of k for which the equation $2x^2 - 6x + 3 = kx + 1$ has distinct real roots. [4]

14 (a) Use the substitution $u = 2x + 3$ to find $\int \frac{4x}{(2x+3)^3}\,dx$. [5]



(b) The graph shown above is part of the curve $y = \frac{2x}{x-1}$ for $x > 1$.

(i) Use the trapezium rule with 4 intervals to estimate the area of the shaded region, giving your answer to 2 decimal places. [3]

(ii) Explain why the shaded area is greater than $\frac{6}{2} + \frac{8}{3} + \frac{10}{4} + \frac{12}{5}$. [1]

(iii) Show that $\frac{2x}{x-1} \equiv 2 + \frac{2}{x-1}$ and hence find the exact area of the region. [5]

15 Given that $f(x) = xe^{3x}$, $f'(x) = 3xe^{3x} + e^{3x}$ and $f''(x) = 3e^{3x}(3x+2)$.

(i) Find $f'''(x)$, $f^{iv}(x)$ and hence write down the Maclaurin series of $f(x)$ up to the term in x^4. [6]

(ii) Find by intergration the area in the first quadrant bounded by $f(x)$, the x-axis and the line $x = 1$, giving your answer correct to 3 decimal places. [4]

(iii) Use Maclaurin series obtained in (i) to estimate the area and state the absolute error. [4]

$\ln p =$

$\ln p + 1{,}4 \ln v = \ln c$

$\frac{1}{p} + \frac{1{,}4}{v}\frac{dv}{dp} = 0$

(i) $\frac{1{,}4}{v}\frac{dv}{dp} = -\frac{1}{p}$ $\quad f(x) = xe^{3x}$. $\quad f'(x) = 3xe^{3x} + e^{3x}$

$\frac{dv}{dp} = \frac{v}{-1{,}4p} = -\frac{5v}{7p}$ $\quad f'''(x) = 9xe^{3x} + e^{3x} + 3e^{3x}$

$= 9xe^{3x} + 4e^{3x}$

(ii) $\ln v^{7/5} = \ln\left(\frac{c}{p}\right)$

$v^{7/5} = \frac{c}{p}$

$v_1^{7/5} = \frac{c}{1{,}01p}$ $\quad x[e^{3x}] + e^{3x}$

$\left(\frac{v_1}{v}\right)^{7/5} = \frac{c}{1{,}01p} \times \frac{p}{c}$

$\frac{v_1}{v} = \left(\frac{1}{1{,}01}\right)^{5/7}$

$3e^{3x} + 9$

$v_1 = 0{,}9929v$

$9e^{3x} + 6e^{3x}$

v is decreased by 0,71%.

2

$9e^{3x} + 6e^{3x}$

$3(3e^{3x}) + 2(3e^{3x})$

$3e$

$9xe^{3x} + 6e^{3x} + 2(3e^{3x})$

$9\,3x(3e^{3x})$

$3e^{3x}(3x)$