JUNE 2024 P2 PURE

SUGGESTED MARKING GUIDE

1 a) $y \propto F$

$y = kF$

$0{,}45 = k(6)$

$k = \frac{3}{40}$

$y = \frac{3}{40}F$

b) when $y = 0{,}2$

$0{,}2 = \frac{3}{40}F$

$\frac{0{,}2}{3/40} = F$

$F = \frac{8}{3}$ Newtons

2. (a) $f \circ g = f(x) = x-1$

$f \circ f = f(x-1) = x-1-1 = x-2$

$h \circ f = h(x-1) = x-1+1 = x$

$h \circ h = h(x+1) = x+1+1 = x+2$

$\circ$	g	f	h
g	x	$x-1$	$x+1$
f	$x-1$	$x-2$	x
h	$x+1$	x	$x+2$

b) (i) $e = x$

(ii)

element	x	$x-1$	$x+1$
inverse	x	$x+1$	$x-1$

(iii) It is not closed because $f \circ f$ & $h \circ h \notin S$ hence it is not a group.

3. (a) Shaded area

$\Rightarrow$ Area of Rhombus − Area of Sector OABC

Area of Rhombus

$\Rightarrow r^2 \sin 2\theta$

Area of sector

$\Rightarrow \frac{1}{2} r^2 2\theta$

$= r^2\theta$

shaded area

$\Rightarrow r^2 \sin 2\theta - r^2\theta$

$= r^2(\sin 2\theta - \theta)$ ☐

b) Area $= 8^2\left(\sin 2\times\frac{\pi}{6} - \frac{\pi}{6}\right)$

$= 32\sqrt{3} - \frac{32}{3}\pi$

5. (a) $x^3 + x^2 + 2x + 2 \equiv (x+1)(x^2+2)$

LHS

$x^3 + x^2 + 2x + 2 = x^2(x+1) + 2(x+1)$

$= (x^2+2)(x+1)$

$=$ RHS

(b) $\frac{x^3}{x^3+x^2+2x+2} = 1 - \frac{x^2+2x+2}{x^3+x^2+2x+2}$

$\frac{x^2+2x+2}{(x^2+2)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2}$

$x^2+2x+2 = A(x^2+2) + (Bx+C)(x+1)$

$A = \frac{1}{3}$; $B = \frac{2}{3}$; $C = \frac{4}{3}$

now $\frac{x^3}{x^3+x^2+2x+2} = 1 - \left(\frac{1}{3(x+1)} + \frac{2x+4}{3(x^2+2)}\right)$

$= 1 - \frac{1}{3(x+1)} - \frac{2x+4}{3(x^2+2)}$

x	1	2	3	4	5	6
y/x	-1	1	3	5	7	9

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$y = px^2 + qx$

$\frac{y}{x} = px + q$

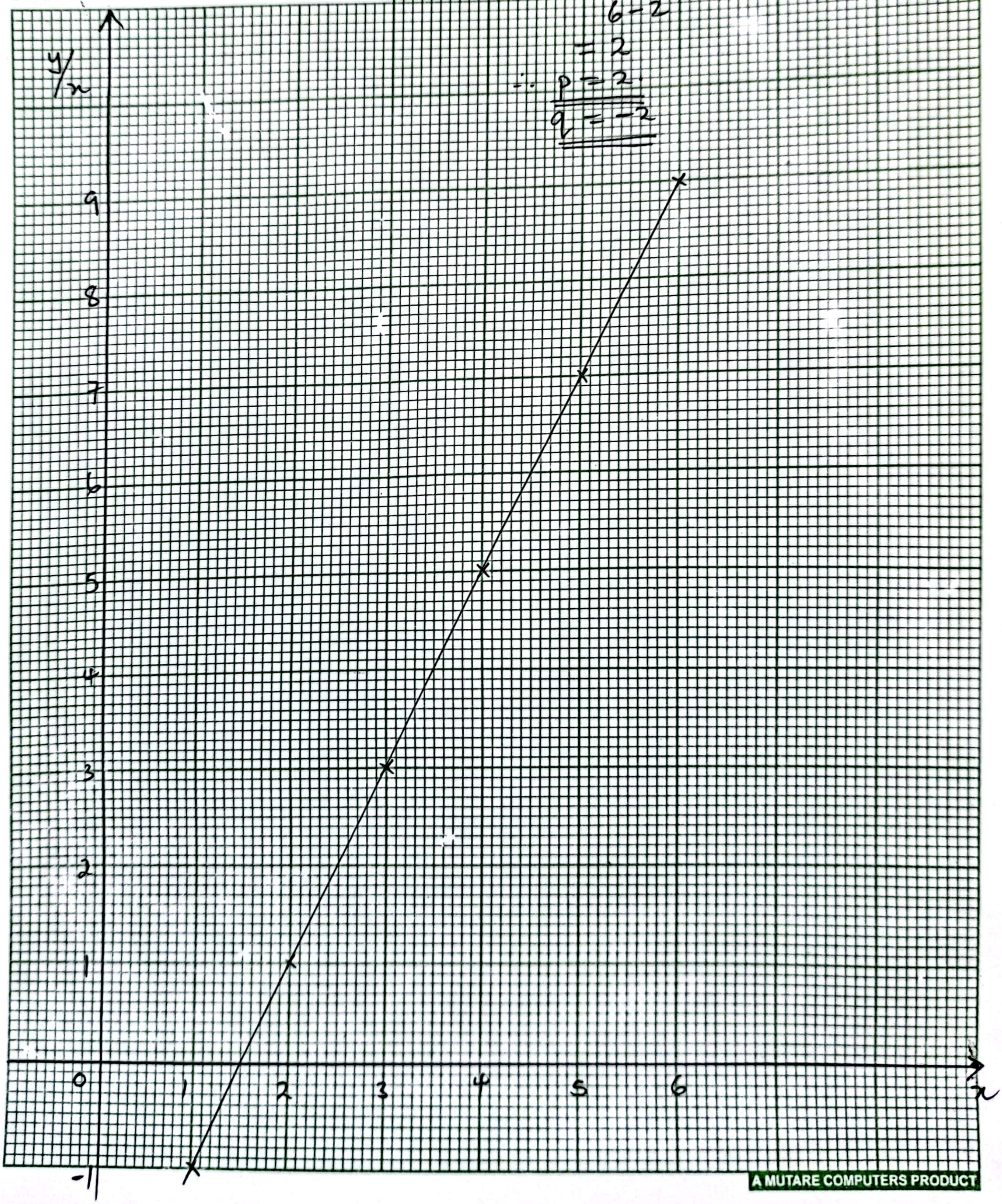
$Y = Mx + c$

Gradient $= \frac{9-1}{6-2}$

$= 2$

$\therefore p = 2$

$q = -2$



6. (a) let $m = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 9 & -6 \\ 15 & -9 \end{pmatrix}$$

$2a + 5b = 9 \quad \cdots (1)$

$2c + 5d = 15 \quad \cdots (2)$

$-a - 4b = -6$

$a + 4b = 6 \quad \cdots (3)$

$-c - 4d = -9$

$c + 4d = 9 \quad \cdots (4)$

Solving (1) & (3)

$a = 2 \; ; b = 1$

Solving (2) & (4)

$c = 5 \; ; d = 1$

$\therefore m = \begin{pmatrix} 2 & 1 \\ 5 & 1 \end{pmatrix}$

b) $c' = \frac{1}{-3}\begin{pmatrix} 1 & -1 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} -6 \\ 15 \end{pmatrix}$

$= \frac{-1}{3}\begin{pmatrix} -21 \\ 45 \end{pmatrix}$

$c' = \begin{pmatrix} 7 \\ -15 \end{pmatrix}$

$|m^{-1}| \; 180$

(c) (i) Area of $ABC = \left|\begin{pmatrix} 2 & 1 \\ 5 & 1 \end{pmatrix}^{-1}\right|(180)$

$\Rightarrow \left|-\frac{1}{3}\right| \times 180$

$= \frac{1}{3} \times 180$

$= 60\,u^2$

7. $w(2+3i) = 9 - 6i$

a) $w = \frac{9-6i}{2+3i} \times \frac{2-3i}{2-3i}$

$= -3i$

b) (i) $|w| = 3$

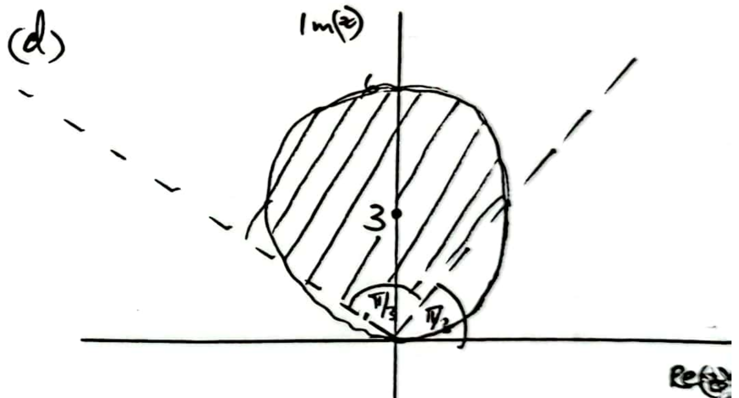
(ii) $\arg w = -\pi/4$

c) $w^3 = \left[3\left(\cos\left(-\pi/4\right) + i\sin\left(\pi/4\right)\right)\right]^3$

$= 3^3\left[\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right]$

$= 27\left(\cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4}\right)$

(d)



8. (a) $\frac{1}{(r+1)(r+3)} = \frac{A}{r+1} + \frac{B}{r+3}$

$\Rightarrow 1 = A(r+3) + B(r+1)$

$A = \frac{1}{2} \qquad B = -\frac{1}{2}$

$\frac{1}{(r+1)(r+3)} = \frac{1}{2(r+1)} - \frac{1}{2(r+3)}$

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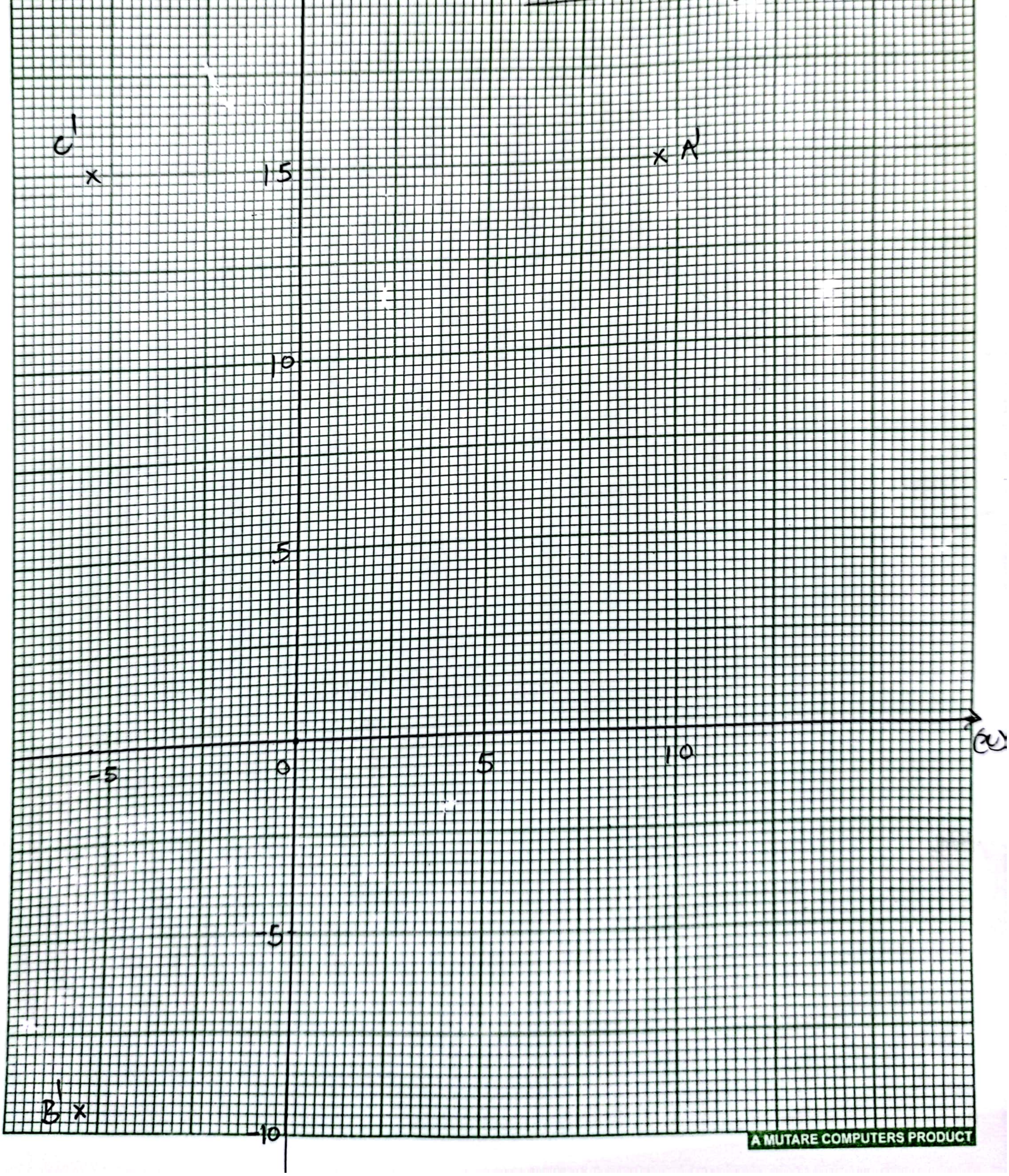
(ii) Area of A'B'C' $= \frac{1}{2} \times 15 \times 24$

$= \frac{1}{2} \times 15 \times 24$ $= \cancel{\frac{4/38}{2}}$

$= \underline{180\ u^2}$ $= \cancel{2/17/5/ u^2}$



b

8b) $\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)}$

$$= \sum_{r=1}^{n} \frac{1}{2}\left(\frac{1}{r+1} - \frac{1}{r+3}\right)$$

$$= \frac{1}{2} \sum_{r=1}^{n} \frac{1}{r+1} - \frac{1}{r+3}$$

$$= \frac{1}{2}\Big[\frac{1}{2} - \cancel{\frac{1}{4}}$$

$$+ \frac{1}{3} - \cancel{\frac{1}{5}}$$

$$+ \cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}$$

$$+ \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}$$

$$+ \vdots$$

$$+ \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n+1}}$$

$$+ \cancel{\frac{1}{n}} - \frac{1}{n+2}$$

$$+ \cancel{\frac{1}{n+1}} - \frac{1}{n+3} \Big]$$

$$= \frac{1}{2}\left[\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}\right]$$

$$= \frac{1}{2}\left[\frac{5}{6} - \frac{n+2+n+3}{(n+2)(n+3)}\right]$$

$$= \frac{1}{2}\left[\frac{5}{6} - \frac{2n+5}{(n+2)(n+3)}\right]$$

$$= \frac{5}{12} - \frac{2n+5}{2(n+2)(n+3)}$$

(ii) $\sum_{r=1}^{\infty} = \frac{1}{2}\left[\frac{1}{2}+\frac{1}{3}\right] = \frac{5}{12}$

c) $\sum_{r=12}^{25} \frac{1}{(r+1)(r+3)}$

$$= \sum_{r=1}^{25} \frac{1}{(r+1)(r+3)} - \sum_{r=1}^{11} \frac{1}{(r+1)(r+3)}$$

$$= \frac{5}{12} - \frac{2(25)+5}{2(25+2)(25+3)} - \left[\frac{5}{12} - \frac{2(11)+5}{2(11+2)(11+3)}\right]$$

$$= + \frac{743}{19656}$$

9. $m: r = \begin{pmatrix}1\\2\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\5\end{pmatrix} = \begin{pmatrix}1+\lambda\\2-\lambda\\-1+5\lambda\end{pmatrix}$

$l: r = \begin{pmatrix}3\\2\\-1\end{pmatrix} + t\begin{pmatrix}-4\\1\\1\end{pmatrix} = \begin{pmatrix}3-4t\\2+t\\-1+t\end{pmatrix}$

$\Pi_1: r \cdot \begin{pmatrix}3\\-2\\-1\end{pmatrix} = 0$

10 a) $x = 2 + 4\cos\theta$

$y = 3 + 4\sin\theta$

$x - 2 = 4\cos\theta$

$(x-2)^2 = 16\cos^2\theta \quad \ldots (1)$

$(y-3) = 4\sin\theta$

$(y-3)^2 = 16\sin^2\theta \quad \ldots (2)$

adding (1) and (2)

$(x-2)^2 + (y-3)^2 = 16\cos^2\theta + 16\sin^2\theta$

$(x-2)^2 + (y-3)^2 = 16(\cos^2\theta + \sin^2\theta)$

$(x-2)^2 + (y-3)^2 = 4^2$

which is a circle $C(2,3); r = 4$

b) $C(2,3)$; $r = 4$

c) $\frac{dx}{d\theta} = -4\sin\theta$

$= -4\sin\pi/4$

$= -2\sqrt{2}$

$\frac{dy}{d\theta} = 4\cos\theta$

$= 4\cos\pi/4$

$= 2\sqrt{2}$

$\frac{dy}{dx} = \frac{2\sqrt{2}}{-2\sqrt{2}}$

$= -1$

when $\theta = \pi/4$

$x = 2 + 4\cos\pi/4 = 2 + 2\sqrt{2}$

$y = 3 + 4\sin\pi/4 = 3 + 2\sqrt{2}$

$\frac{y - (3+2\sqrt{2})}{x - (2+2\sqrt{2})} = -1$

$y - 3 - 2\sqrt{2} = -1(x - 2 - 2\sqrt{2})$

$y - 3 - 2\sqrt{2} = -x + 2 + 2\sqrt{2}$

$y = -x + 5 + 4\sqrt{2}$

(d) on meeting the x-axis $y = 0$

$0 = -x + 5 + 4\sqrt{2}$

$x = 5 + 4\sqrt{2}$

it meets the x-axis at $(5 + 4\sqrt{2}; 0)$

11. $f(x) = x^2 + 2x \; ; \; -4 \le x \le -1$

a) $f(x) = (x+1)^2 - 1$

b) (i) let $y = (x+1)^2 - 1$

$y + 1 = (x+1)^2$

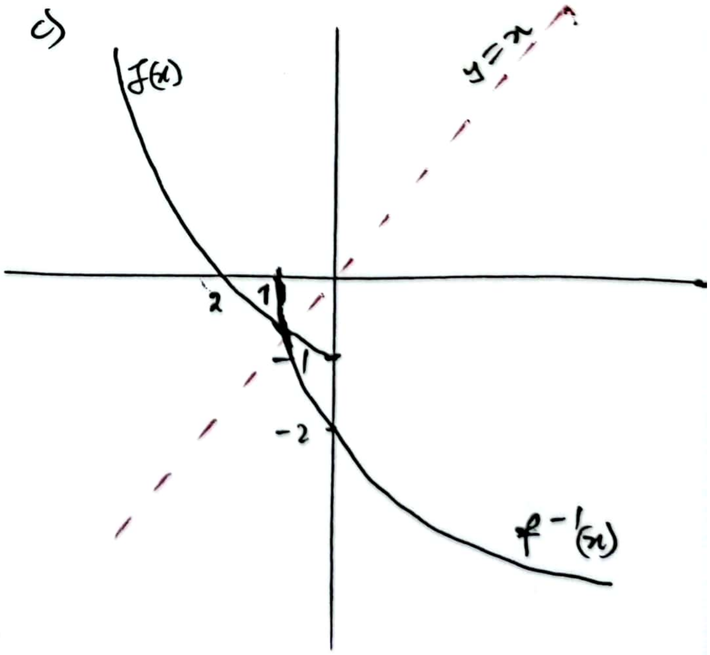
$\sqrt{y+1} = x + 1$

$-1 \pm \sqrt{y+1} = x$

$f^{-1}(x) = -1 + \sqrt{x+1} \; ; \; x \geq -1$

(ii) $x \geq -1$

c)



11(d)

let P_n denotes

$$1+4+7+\cdots(3n-2) = \frac{n(3n-1)}{2}$$

when $n=1$

$P_1(LHS) \Rightarrow 1$; $P_1(RHS) = \frac{1(3\cdot1-1)}{2} = 1$

$P_1(LHS) = P_1(RHS) = 1$

hence statement is true when $n=1$

Assuming P_n is true when $n=k$

ie $P_n \Rightarrow 1+4+7+\cdots 3k-2 = \frac{k(3k-1)}{2}$

the when $n=k+1$

$$P_{k+1} \Rightarrow 1+4+7+\cdots+(3k-2)+3(k+1)-2$$
$$= \frac{(k+1)(3(k+1)-1)}{2}$$
$$= \frac{(k+1)(3k+2)}{2}$$

Proof.

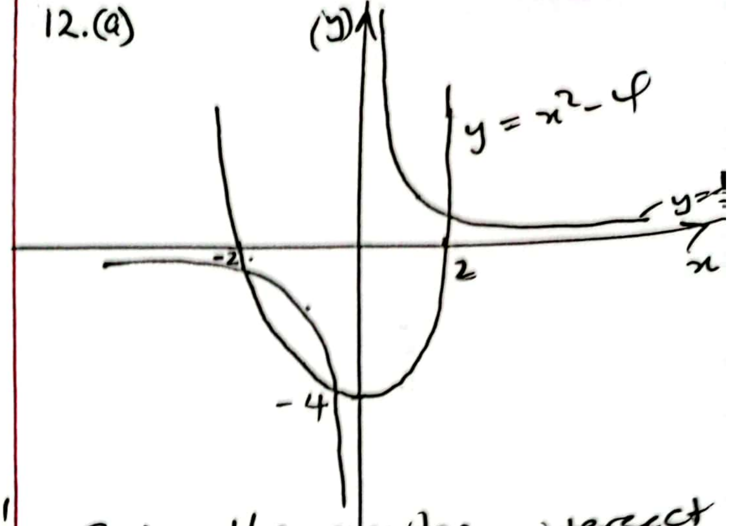
$1+4+7+\ldots+(3k-2)+3(k+1)-2$

$$= \frac{k(3k-1)}{2} + 3(k+1) - 2$$
$$= \frac{3k^2-k}{2} + 3k+1$$
$$= \frac{3k^2-k+6k+2}{2}$$
$$= \frac{3k^2+5k+2}{2}$$
$$= \frac{3k^2+3k+2k+2}{2}$$
$$= \frac{3k(k+1)+2(k+1)}{2}$$
$$= \frac{(3k+2)(k+1)}{2}$$

hence P_n is true when $n=k+1$.

since P_n is true when $n=1, k$ & $k+1$ it is true if $n \geq 1$

12.(a)



Since the graphs intersect thrice there are 3 roots to the equation $x^2-4 = \frac{1}{x}$

b) (i) $f(x) = x^2 - 4 - \frac{1}{x}$

$f(2) = 2^2 - 4 - \frac{1}{2} = -0,5$ (-ve)

$f(2,5) = 2,5^2 - 4 - \frac{1}{2,5} = 1,85$ (+ve)

Since there is a sign change a root lies between $x=2$ & $x=2,5$

(ii) $x^2 - 4 = \frac{1}{x}$

$$x^2 = \frac{1}{x} + 4$$
$$x = \sqrt{\frac{1}{x}+4}$$

hence it satisfies

c) $x_{n+1} = \sqrt{\frac{1}{x_n}+4}$

$x_1 = 2,1$

$x_2 = \sqrt{\frac{1}{2,1}+4} = 2,1157009$

$x_3 = 2,114865$

$x_4 = 2,1149$

$\therefore$ the root is $x = 2,115$

Binaz 0779912288