ZIMBABWE SCHOOL EXAMINATIONS COUNCIL
General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/1
PAPER 1

NOVEMBER 2021 SESSION 3 hours

Additional materials:
- Answer paper
- Graph paper
- List of Formulae MF7
- Scientific calculator [non - programmable]

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

1 Solve the equation,

$2e^{2x} - 7e^{x} + 6 = 0.$
Give the answer in exact form. [3]

2 Prove the identity

$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta.$ [3]

3 Given that $y = ln[cos(x^2+1)]$, show that $\frac{dy}{dx} = -2x \tan(x^2+1)$. [3]

4 Given that M is inversely proportional to the cube root of $(n-1)$ and that when $n = 9, M = 5,$
find

(a) a formula connecting M and n , [3]

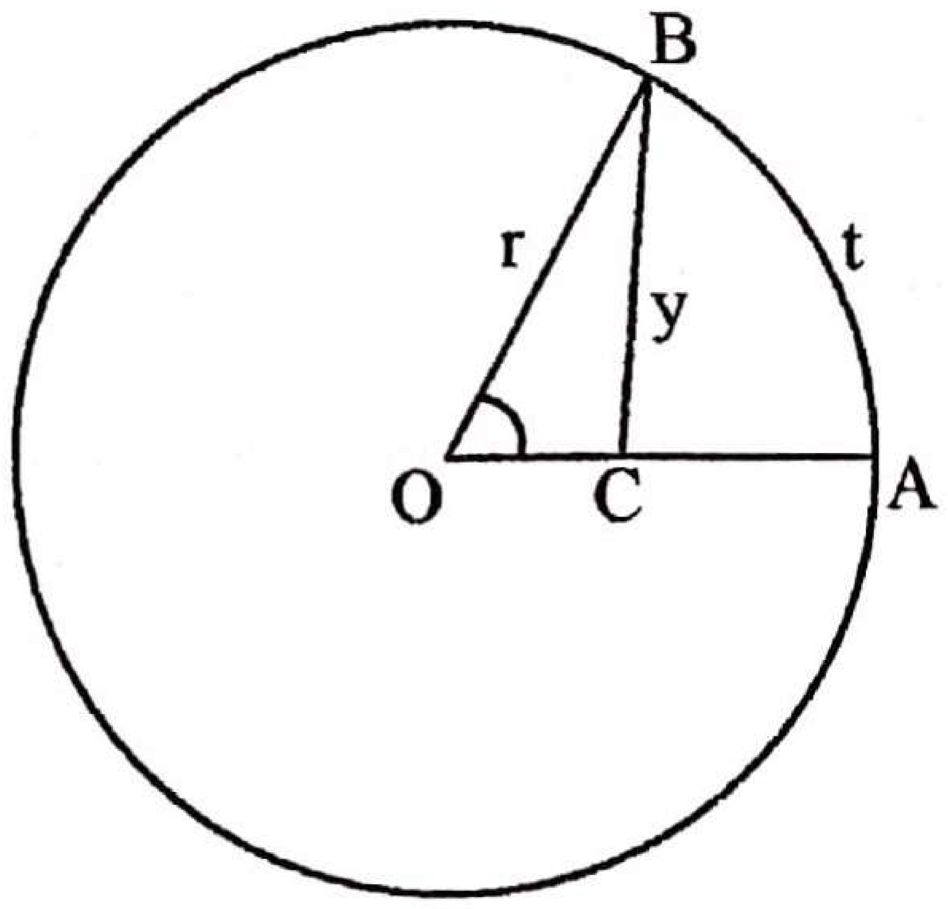
(b) the exact value of n when $M = 25$. [2]

5 (a) Express $2x^2 + 3x + 1$ in the form

$a(x+b)^2 + c$ where a, b and c are constants to be found. [3]

(b) Hence or otherwise solve the equation $2x^2 + 3x + 1 = 0$. [3]

6



In the diagram, A and B are two points on the circumference of a circle centre O and radius r. Angle AOB is θ radians and C is the mid-point of OA. The length of BC is y and the length of the arc BA is t.

(a) Express y^2 in terms of r and θ. [2]

(b) Hence or otherwise show that if θ is small then $y^2 \approx \frac{1}{4}r^2 + \frac{1}{2}t^2$. [4]

7 If $A = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$, prove by induction that $A^n = \begin{pmatrix} 2n+1 & -n \\ 4n & 1-2n \end{pmatrix}$, where n is a positive integer. [7]

8 (a) Solve the equation $\frac{2^x+1}{2^x-1} = 5$

giving the answer correct to 3. significant figures. [3]

(b) Solve the inequality $|x-3| > 2|3x+1|$. [5]

9 Given that x is so small that x^4 and higher powers can be neglected,

(a) show that

$$\frac{e^x}{1+x} \approx 1 + \frac{x^2}{2} - \frac{x^3}{3},$$ [6]

(b) use the expansion to evaluate

$\frac{e^{0,01}}{1,01}$ correct to 7 decimal places. [2]

10 (a) Given that $f(x) = 2 - |x|$ and $g(x) = \frac{1}{3}x + 1$,
sketch on the same axes the graphs of $y = f(x)$ and $y = g(x)$, [3]

(b) Find the co-ordinates of the points of intersection of $f(x)$ and $g(x)$. [5]

11 (a) Express $\frac{2x}{(x+2)(x-2)(x-1)}$, in partial fractions. [4]

(b) Solve the inequality $\frac{2x}{(x+2)(x-2)(x-1)} < 0$. [4]

12 If $p = -4 + 3i$ and $q = -1 + \sqrt{3}i$,

(a) calculate the modulus of,

(i) p, [2]

(ii) q. [2]

(b) find

(i) the argument of q, [2]

(ii) pq^2 in the form $a + bi$, [3]

(iii) $\frac{p}{q}$ in the form $a + bi$. [3]

13 The curve $y = 10 - \frac{5}{x}$ and the line $y + x = 6$ intersect at two points P and Q.

Find the

(a) coordinates of P and Q, [5]

(b) coordinates of the midpoint of PQ, [2]

(c) equation of the perpendicular bisector of the line with P and Q as end points. [3]

14 The curve $y = x^2 + px + q$, where p and q are constants, has a turning point at $(-1; -5)$

Find the

(a) values of p and q, [5]

(b) equations of the tangent and the normal in the form $ay + bx + c = 0$, at the point where the curve cuts the y-axis. [5]

15 **(a)** A point $(1; 2)$ is mapped onto point $(7; 2)$ by a shear parallel to the x-axis. Find the shear factor. [3]

(b) A triangle with vertices A $(-3; 1)$, B $(3; 1)$ and C $(3; 5)$ is transformed by $M = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$.

(i) Find the area of triangle ABC. [2]

(ii) Find the co-ordinates of A_1 B_1 and C_1 the images of A, B and C under transformation M. [4]

(iii) Hence or otherwise find the area of the triangle $A_1B_1C_1$. [2]

16 The gradient function of a curve is directly proportional to $20 - y$. If $y = 0$, $x = 0$ and the gradient is 1

(a) Show that $\frac{dy}{dx} = 0{,}05\,(20 - y)$. [3]

(b) Solve the differential equation giving y in terms of x. [6]

(c) Find y when $x = 10$ giving the answer correct to 2 decimal places. [2]

(d) Briefly describe what happens to y as x becomes very large. [1]

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS 6042/2

PAPER 2

NOVEMBER 2021 SESSION 3 hours

Additional materials:
Answer paper
Graph paper
List of Formulae MF 7
Scientific calculator (Non-programmable)

TIME 3 hours

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre number and Candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and any **five** questions from Section B.

If a numerical answer cannot be given exactly, and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 120.

The use of a non-programmable scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (40 marks).

Answer **all** questions in this section.

1 Solve the inequality

$|2x - 3| < x - 5.$ [4]

2 **(a)** Prove the identity
$cosec\ 2\theta + cot\ 2\theta \equiv cot\ \theta.$ [3]

(b) Hence or otherwise solve the equation
$cosec\ 2\theta + cot\ 2\theta = 0{,}5$, for $0 \leq \theta \leq 2\pi$, giving the answer to 2d. p [4]

3 **(a)** Find the value of x given that $3^{x+2} = \frac{1}{81}$. [3]

(b) Solve the equation.
$3(2^{2x}) - 7(2^x) + 2 = 0$ [5]

4 Given that $-m, n$ and 1 are any consecutive terms of a geometric progression and 1, n and m are the first three terms of an arithmetic progression,

(a) show that $n^2 + 2n = 1$, [5]

(b) hence or otherwise find the exact value of m if n is positive. [4]

5 **(a)** Show that the set $\{0,1,2,3\}$, forms a group under addition modulo 4. [7]

(b) Show that group is abelian. [2]

(c) Write down the subgroups of the group. [2]

(d) State the order of the group. [1]

Section B (80 marks)

Answer any **five** questions from this section. Each question carries **16** marks.

6 Functions h and g are defined by

$h: x \longmapsto \frac{1}{2}x - 4, \qquad x \in \mathbb{R}$
$g: x \longmapsto \frac{32}{4-x^2} \qquad x \in \mathbb{R}\ x \neq k.$

(a) Find

(i) the possible values of k, [3]

(ii) the values of x for which $hg(x) = 0$, [3]

(iii) $h^{-1}(x)$. [3]

(b) On the same axes sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$ showing clearly the relationship between the graphs. [3]

(c) Describe completely the sequence of transformation which map the graph $y = \frac{1}{x^2}$ onto $y = \frac{32}{4-x^2}$. [4]

7 A curve has the equation $y = x^2 - xy$ and passes through the point $P\left(1; \frac{1}{2}\right)$

(a) Find the equation of the tangent to the curve at P. [6]

(b) Hence or otherwise show that the equation of the normal to the curve at P intersects the curve again at $\left(-\frac{11}{14}; \frac{121}{42}\right)$. [8]

(c) Find the distance between the two points $\left(1; \frac{1}{2}\right)$ and $\left(-\frac{11}{14}; \frac{121}{42}\right)$. [2]

8 The vector equations of the lines n and m, and the plane π_1, are

$$\boldsymbol{r} = \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix},$$

$$\boldsymbol{r} = \begin{pmatrix} 0 \\ 10 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \text{ and}$$

$$\boldsymbol{r} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = -1$$ respectively where μ and λ are parameters.

The line m intersects the plane π_1, at a point P.

(a) Show that the line n lies in the plane π_1. [2]

(b) (i) Find the coordinates of point P. [4]

(ii) Hence or otherwise find the vector equation of the plane π_2 passing through point P and perpendicular to the line n. [3]

(c) (i) Find the point of intersection of the line n and the plane π_2 [4]

(ii) Hence or otherwise show that the vector equation of the line through point P which lies in plane π_1 and is perpendicular to line n is; $r = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ [3]

9 **(a)** (i) Find the first 3 terms of the Maclaurin series in the expansion of $ln\ (5 + x)$. [3]

(ii) Write down the first 3 terms of the Maclaurin series in the expansion of $ln\ (5 - x)$. [1]

(iii) Hence or otherwise show that when x is small $ln\left(\frac{5+x}{5-x}\right) \approx \frac{2x}{5}$. [2]

(b) (i) Prove by mathematical induction that

$$\frac{d^n}{dx^n}(x^m) = \frac{m!}{(m-n)!}.x^{m-n}$$

for all $n \in Z^+$ [8]

(ii) Hence find $\frac{d^4}{dx^4}(x^5)$. [2]

10 Given the complex numbers
$z_1 = 2$ and $z_2 = 2 + 2i$,

(a) (i) express z_2 in the form $r(Cos\theta + isin\theta)$, [3]

(ii) hence or otherwise, express the complex number $\frac{z_1}{z_2}$ in polar form. [3]

(b) Describe fully the locus represented by Re $\frac{z+3}{z-3} = 0$ given that z is a complex number $x + iy$. [4]

(c) By using de Moivres theorem or otherwise, find the roots of the equation, $z^4 + 4 = 0$ [6]

11 It is given that $\mathbf{A} = \begin{pmatrix} 3 & -1 & 2 \\ 1 & -1 & 0 \\ -2 & 4 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$.

(a) A line with the equation $y = 2x - 3$ is transformed under the transformation matrix B. Show that the equation of its image is $y - 6x + 24 = 0$. [5]

(b) Evaluate $\boldsymbol{A}^{-1}$. [7]

(c) Hence or otherwise solve the following simultaneous equations. [4]

$$3x - y + 2z = 1$$
$$x - y = 2$$
$$-2x + 4y - 3z = 3$$

12 (a) A learner used the value of π as $\frac{22}{7}$ to calculate the volume of a sphere of radius 8 cm.

Find correct to 3 decimal places the

(i) absolute error involved,

(ii) percentage error involved. [5]

(b) (i) Sketch on the same axes the graphs of $y = \ln 2x$ and $y = x - 1$ and state the number of real roots for the equation $\ln 2x = x - 1$.

(ii) Show that a root of the equation $\ln 2x = x - 1$ lies between 0.2 and 0.3.

(iii) Using 0.2 as the first approximation, use the Newton Raphson method twice to obtain an approximation to the root correct to 3 decimal places. [11]