For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

PURE MATHEMATICS

6042/2

PAPER 2

3 hours

JUNE 2024 SESSION

Additional materials:
Answer booklet
List of Formulae MF 7
Scientific calculator [Non-programmable]

ZC

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions in Section A and any **five** questions from Section B.

If a numerical answer cannot be given exactly and the accuracy required is not specified in the question, then in the case of an angle it should be given correct to the nearest degree, and in other cases it should be given correct to 2 significant figures.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

This question paper consists of 6 printed pages and 2 blank pages.

[Turn over

Section A (40 marks).

*Answer **all** questions in this section.*

1 The extension y, of an elastic string varies directly as the magnitude of force F, exerted in extending it. If $F = 6$ Newtons and $y = 0.45$m

(a) Express F as a function of y. [4]

(b) Hence or otherwise find F when $y = 0.2m$. [2]

2 Set S contains functions $g(x)$, $f(x)$ and $h(x)$ which are defined by

$g: x \rightarrow x$
$f: x \rightarrow x - 1$
$h: x \rightarrow x + 1$

A Cayley table is drawn for set S under function composition (°).

$\circ$	x	$x-1$	$x+1$
x	x	$x-1$	$x+1$
$x-1$			x
$x+1$	$x+1$		

(a) Copy and complete the table. [2]

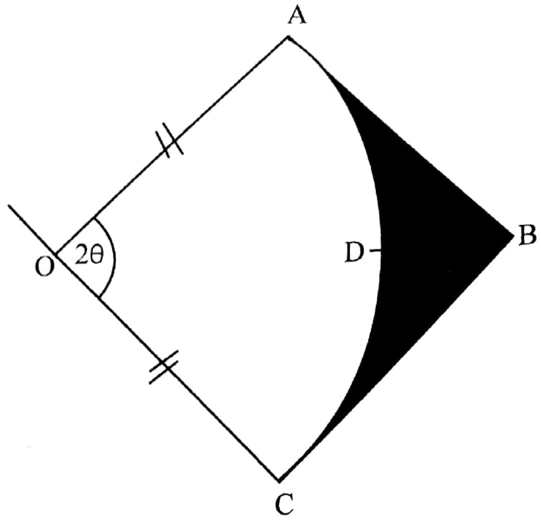
(b) Use the table to state,

(i) the identity element of the set, [1]

(ii) the inverse of each element of the set, [3]

(iii) why set S is not a group under function composition. [2]

3



In the diagram OABC is a rhombus. ADC is an arc of a circle centre O, radius r and subtends an angle 2θ at 0. The shaded region is bonded by the arc ADC and sides AB and CB.

(a) Show that the area of the shaded region is $r^2(Sin2\theta - \theta)$. [6]

(b) Given that $r = 8cm$ and $\theta = \pi/_6$ radians, find the exact area of the shaded region. [2]

4 An experiment is performed to find corresponding values of x and y. Results were recorded in the table below.

x	1	2	3	4	5	6
y	-1	2	9	20	35	54

It is suggested that the variables x and y are connected by the equation $y = px^2 + qx$ where p and q are constants.

(a) By plotting $\frac{y}{x}$ against x draw a graph for the given data. [5]

(b) Use the graph to find the values of p and q. [4]

5 **(a)** Show that $x^3 + x^2 + 2x + 2 \equiv (x + 1)(x^2 + 2)$ [2]

(b) Hence or otherwise express $\frac{x^3}{x^3+x^2+2x+2}$ in partial fractions. [7]

[Turn over

Section B (80 marks)

*Answer any **five** questions from this section. Each question carries **16** marks.*

6 Matrix M maps A(2;5) and B(-1;-4) onto $A^1(9;15)$ and $B^1(-6;-9)$, respectively.

(a) Find Matrix M [6]

(b) Point $C^1(-6;15)$ is the image of point C under the transformation with Matrix M.

(i) Sketch Point A^1, B^1 and C^1 on a cartesian plane. [1]

(ii) Hence calculate area of triangle $A^1B^1C^1$. [2]

(c) Find the

(i) area of triangle ABC [4]

(ii) co-ordinates of C [3]

7 Given that a complex number W is such that $W(2+3i) = 9-6i$.

(a) Express W in the form $x + iy$. [4]

(b) Hence or otherwise state

(i) $|W|$,

(ii) arg W. [2]

(c) Using DeMoivre's theorem or otherwise find W^3 in trigonometric form. [4]

(d) On an Argand diagram, shade the region which satisfy the loci $|Z - W| \leq 3$ and $-\frac{2\pi}{3} < argZ \leq -\pi/_3$ simultaneously. [6]

8 (a) Express $\frac{1}{(r+1)(r+3)}$ in partial fractions. [5]

(b) Hence or otherwise

(i) show that

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+3)} = \frac{5}{12} - \frac{2n+5}{2(n+2)(n+3)},$$

[6]

(ii) state the sum to infinity. [1]

(c) Evaluate

$$\sum_{r=12}^{25} \frac{1}{(r+1)(r+3)}.$$

[4]

9 Lines m and l have equations.

$\bar{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix}$ and $\bar{r} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} + t \begin{pmatrix} -4 \\ 1 \\ 1 \end{pmatrix}$ respectively.

Plane π_1 has equation

$\bar{r} \cdot \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and point P has coordinates (-3; 1; 2).

(a) Show that

(i) lines m and l are perpendicular to each other, [3]

(ii) point P does not lie on Plane π_1. [2]

(b) Find the equation of the

(i) plane π_2 that contains line m and point P, [7]

(ii) line of intersection of plane π_2 and π_1. [4]

10 **(a)** Show that the curve represented by the parametric equations $x = 2 + 4\,Cos\,\theta$ and $y = 3 + 4\,Sin\,\theta$ where θ is the parameter is a circle. [4]

(b) Hence state the coordinates of the centre and the radius of the circle. [2]

(c) Find the equation of the tangent to the curve where $\theta = \frac{\pi}{4}$ in the form $y = mx + c$ where m and c are exact. [8]

(d) Find the coordinates of the point where the tangent in **(c)** above meets the x-axis. [2]

11 The function $f(x)$ is defined as $f(x) = x^2 + 2x$ for $x \in R, -4 \leq x \leq -1$.

(a) Express $f(x)$ in the form $(x+b)^2 + c$ where b and c are constants. [3]

(b) Find **(i)** $f^{-1}(x)$, [2]

(ii) the domain $f^{-1}(x)$. [1]

[Turn over

(c) Sketch on the same axis the graphs of $y = f^{-1}(x)$ and $y = f(x)$ showing clearly the relationship between the two graphs. [4]

(d) Prove by induction that for all $n \geq 1$: $1 + 4 + 7 \cdots (3n - 2) = \frac{n(3n-1)}{2}$. [6]

12 (a) By sketching graphs of $y = x^2 - 4$ and $y = \frac{1}{x}$, determine the number of roots of the equation $x^2 - 4 = \frac{1}{x}$. [6]

(b) Show

(i) by calculations that one of the roots of the equation $x^2 - 4 = \frac{1}{x}$ lies between 2 and 2.5. [4]

(ii) that the roots of the equation $x^2 - 4 = \frac{1}{x}$ satisfies the equation

$$x = \sqrt{\frac{1}{x} + 4}$$ [2]

(c) Hence use the iterate formula $x_{n+1} = \sqrt{\frac{1}{x_n} + 4}$ taking $x_1 = 2.1$ as the first approximation to estimate the root of the equation to 3 decimal places. [4]