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Lecture Note on *FEEE (ELECTRICAL ENGINEERING)*

Diploma 1st Semester

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ELECTRIC AND MEGNETIC CIRCUIT

PARAMETERS OF AN ELECTRIC CIRCUIT

Introduction

Fundamentals of different passive circuit elements, active elements, their types i.e. independent and dependent voltage/ current source and the types of signals were introduced in chapter 1. This topic deals with the concept of electric charge, current, voltage and the different electric circuit terminologies and laws for basic analysis of electric circuits.

Current /Voltage and Power/Energy

Current

The fundamental electrical quantity is charge. Its unit is Coulomb and the smallest amount of charge that exists is the negative charge carried by an electron that equals -1.602×10^{-19}C. The other charge carrying particle in an atom is the proton which is positively charged, with magnitude same as that of an electron. The charge of a proton is given as $+1.602\times10^{-19}$C. The electric current consists of the flow of a very large number of these charged particles. The electric current is defined as the time rate of change of the charged particles flowing through a predetermined area. In mathematical form, the current

$$i = \frac{\Delta Q}{\Delta t}$$

where ΔQ is the unit of charge flowing through the predetermined cross sectional area in time Δt. The units of current are called amperes, where 1ampere (A) = 1 $Coulomb/1sec$

Voltage (Potential Difference)

The charge moving in an electric circuit give rise to current. For a charge to be moved say from point **a** to point **b** in a circuit, some work or energy has to be expended. The work done in moving a unit charge from point **a** to **b** is called voltage or potential difference. The unit of voltage is called volt, where 1volt (V) = 1 Joule/1 Coulomb. As in gravitational field the potential difference between two points is independent of the path chosen. As shown in Fig. 4.1 (a), if the voltage of point a is higher than b as indicated as V_{ab} work must be done in moving the charge from point b to a, i.e. energy input to the charge. Similarly if the charge moves from point a back to b , energy is output with $V_{ba} = -V_{ab}$ i.e. voltage drop in going from **a** to **b**. Fig. 4.1 (b) shows the alternate way of representation of voltage difference.

Fig. 4.1: (a) Representation of voltage difference (b) Alternate representation of voltage difference

Power and Energy

Power is defined as the work done per unit time. The power either generated or dissipated by a passive element can be represented by the following equation.

$$Power = \frac{Work}{time} = \frac{Work}{charge} \times \frac{charge}{time} = voltage \times current \text{ or } P = VI$$

The unit of power is joules per second or watts. If both voltage and current remain constant over time t, the energy E transferred is V*I*t joules. Power is also a signed quantity like voltage and current. Power is put in if the current I flow into the positive terminal of V across an element as shown in Fig. 4.2 (a). As per passive sign convention the power dissipated is positive or in other words the element absorb power. In Fig. 4.2 (b) the power is put out if the current I flow out from the positive terminal V of an element, then as per convention the power dissipated is negative i.e. the element delivers power.

Fig. 4.2: (a) Passive sign convention of power dissipated (b) Passive sign convention of power generated

Electric Circuit Terminology

The interconnection of circuit elements is known as electric circuit. The following definition introduces some important parameters of an electric circuit.

Branch

A branch is any portion of a circuit with two terminals connected to it. A branch may consist of more than one circuit elements.

Node

A node is a junction of two or more branches. In practice any connection of two or branch terminals together forms a node. Nodes are very important parameter for circuit analysis. Fig. 4.3 shows a branch and interconnection of branch terminals to form a node.

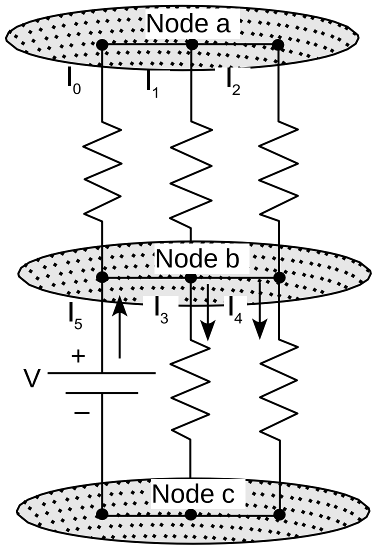


Fig. 4.3: Definition of node

Loop and Meshes

A loop is a closed connection of branches. A mesh is a loop that does not contain any other loop with in it as shown in Fig. 4.4 Meshes are an important aid for analysis of electric circuits.

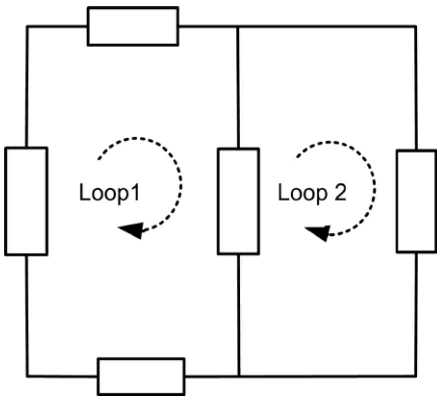


Fig. 4.4: Definition of mesh

Circuit Analysis

The analysis of an electric circuit consists of determining the unknown branch currents and node voltages. For this a set of variables has to identified and after which a set of equations using these variables has to be constructed. The set of equations are constructed by using the two important fundamental laws of circuit analysis namely the Kirchoff's current law and Kirchoff's voltage law.

Kirchoff's Current Law (KCL)

Kirchoff's current law states that since charge cannot be created but must be conserved, the sum of the currents at a node must be zero.

$$\sum_{k=1}^{k=N} i_k = 0 \quad \text{... (4.1)}$$

where, i_k is the individual current flowing through the branches. Consider the node shown in Fig. 4.5 with the current directions. The current entering a node is assumed negative and that leaving a node as positive. Applying KCL law, the resulting equation at node A is given as

$$i_1 + i_2 + (-i_3) + (-i_4) = 0 \quad \text{... (4.2)}$$

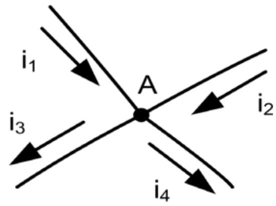


Fig. 4.5: Illustration of KCL

Kirchoff's Voltage Law (KVL)

Kirchoff's voltage law states that no energy is lost or created in an electric circuit. Or in other words, in a closed circuit the sum of all voltages associated with the sources must equal the sum of the load voltages, so that the net voltage around the circuit is zero.

$$\sum_{k=1}^{N} V_k = 0 \quad \text{... (4.3)}$$

where, V_k are the individual voltages across the passive and active elements of the closed loop circuit. To understand the KVL law further, the concept of reference voltage must be understood. Fig. 4.6 shows a circuit with nodes marked as 1 and 2. The corresponding node voltages are V_1 and V_2 respectively. Any one node either node 1 or 2 can be chosen as reference node and the associated node voltage as the reference voltage. For example in the above figure, if node 2 is chosen as the reference node, connected to the negative terminal of the voltage source, then the node1 voltage V_1 is V_s volts above the reference node voltage V_2 . In practice for ease of calculation, the reference voltage is assigned zero volt.

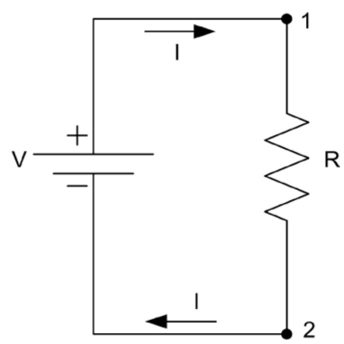


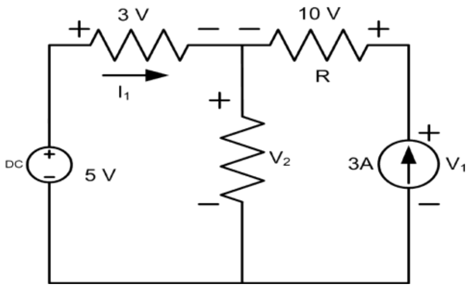
Fig. 4.6: Illustration of reference node

Activity

Prepare a circuit by connecting two bulbs in parallel across a given cell acting as voltage source. Measure the current $\boldsymbol{i}$ drawn from the voltage source and the current $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ drawn by the two lamps. Verify KCL.

Solved Problems

Example 4.1.1: For the circuit shown determine the power absorbed by the resistor R and the power delivered by the current source



Solution: Applying KVL in mesh-1, the KVL equation is
5 - 3 - V_2 = 0, Solving V_2 =2 V.
Now applying KVL in mesh-2, the KVL equation is
V_2 +10 – V_1 = 0.
Putting the value of V_2, the voltage V_1 across the current source of 3A = 12 V
As per passive power sign convention, the power delivered by current source =12× 3= 36 Watt
The power absorbed or dissipated by the resistor R = Voltage drop across R × current flowing through R= 10×3 =30 Watt

PARAMETERS OF A MAGNETIC CIRCUIT

Magnetic Effect of Electric Current

To understand the magnetic field created by a current carrying conductor, consider the case of a long straight conductor carrying current as shown in Fig 4.7. The bold dot shows the current is flowing out of the plane of paper. The current causes a magnetic field to be established in the space surrounding the conductor. The magnetic field force can be felt on a north pole at any point on the magnetic field. The circular closed path around the conductor are known as the line of flux and the magnetic force is tangential to it all points around the line. The direction of flux is given by the right hand rule which states that if the conductor is gripped by the right hand with the thumb point in the direction of current, the direction of flux is towards which the other four fingers would encircle it. The flux lines are denser near the conductor and becomes less dense as we move away from it.

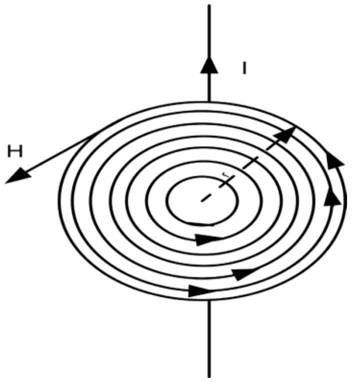


Fig. 4.7: Flux lines surrounding current

Magnetizing force

The magnetizing force or magnetic field intensity 'H' is defined as the current flowing through a conductor per unit length of flux line enclosing the conductor. For a circular flux line with radius 'r' the magnetic field intensity,

$$H = \frac{i}{2\pi r} \frac{A}{m} \qquad \text{... (4.4)}$$

From equation it is seen that the magnetic field intensity is unaffected by the material surrounding the conductor or in other words it is independent of the properties of the materials employed in the construction of magnetic circuits.

Flux density

The flux density depends on the material properties. The relationship between the magnetic field intensity and flux density is given as

$$B = \mu_0 \mu_r H \ \frac{Wb}{m^2} \ \text{or Tesla} \qquad \text{... (4.5)}$$

The parameter μ is a scalar constant for a particular physical medium and is called the permeability of the medium. The permeability of a material is the product of the permeability of free space μ0 and the relative permeability μ_r. The permeability of free space = $4\pi \times 10^{-7}$. The relative permeability depends on the medium and its magnitude represents the measure of the magnetic properties of the material. The larger the value of permeability, the smaller the current required to produce a large flux density in an electromagnetic structure. The unit of permeability is given as:

$$\frac{Wb}{A\text{-}m}$$

Fig. 4.8 shows the permeability of different types of magnetic materials. The subscript shows whether the material is ferromagnetic, paramagnetic, free space or diamagnetic.

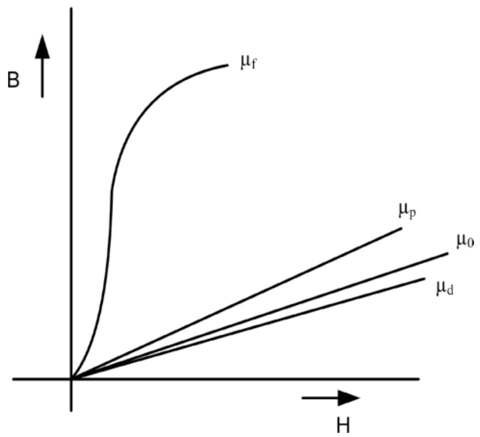


Fig. 4.8: B-H curve of different types of magnetic material

Magnetomotive Force

The magnetic field generated by a single conducting wire is not very strong. The field intensity can be increased by giving shape to a conductor as a tightly wound coil having N turns. The arrangement effectively increases the current linked by the flux lines N fold. The product N.I is called the magneto motive force F and its unit is ampereturns (AT). Fig. 4.8 shows a ferromagnetic material having circular cross section is excited by a coil having N turns and carrying a current I. The flux established in the magnetic core is circular in shape with major portion of the flux lines passing through the core due to its high permeability. The small portion of the flux line that passes through air is known as leakage flux.

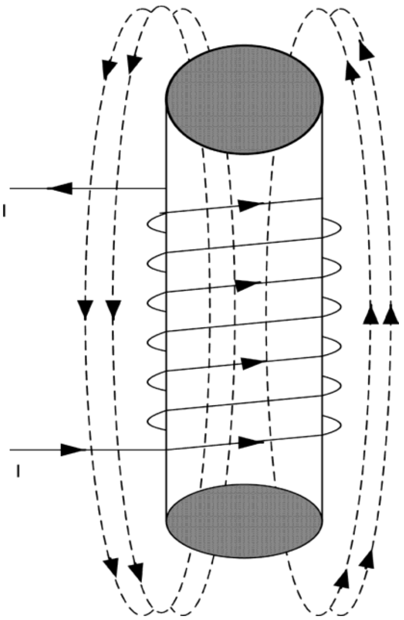


Fig.4.9: Flux

Magnetic Circuits

An electromagnetic core structure with circular cross section area as shown in Fig. 4.10 can be analysed by means of an equivalent magnetic circuit. The flux density for this circuit is given by equation

$$BB \quad \frac{\mu NI}{l} \qquad \text{... (4.6)}$$

where l is the length of the flux mean path. Similarly, the flux density can also be represented as

$$BB \quad \frac{\text{Ø}}{A} \qquad \text{... (4.7)}$$

where A is the cross sectional area of the electromagnetic structure and is perpendicular to the direction of flux lines. Equating the flux density equations, (4.6) and (4.7)

$$NI\,B \quad \text{Ø}\,\frac{l}{\mu A} \qquad \text{... (4.8)}$$

The term $\frac{l}{\mu A}$ is known as the reluctance of the magnetic circuit and is designated the symbol R.

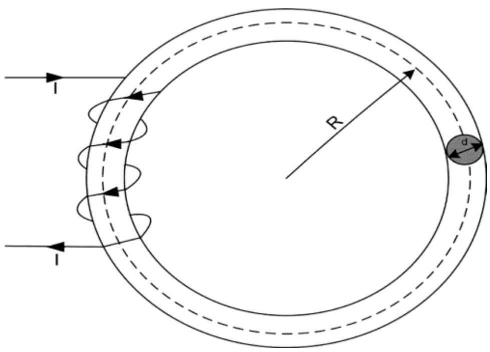


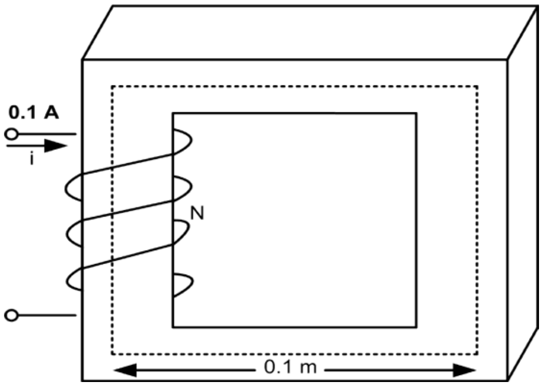
Fig.4.10: Ring of ferromagnetic material with exciting coil

Activity

Prepare a multi turn coil using a single strand bare copper conductor. Mark the two ends of coil as terminal 1 and 2. Connect a 9 V cell with the positive polarity connected to terminal 1 through a slider switch in series and terminal 2 to the negative polarity of the cell. Bring a magnetic compass near to the coil and switch on the slider switch Observe the position of the compass N-S pole needle. Now reverse the polarity of the terminals and observe the position of the N-S pole compass needle. Comment on the observations made.

Solved Problems

Example 4.2.1: Calculate the flux, flux density for the magnetic structure shown, given cross section area A = 0.0001 m^2, N = 500 turns, μ_r = 1000.



Solution: The magneto motive force F = mmf = Ni = 500× 0.1 = 50 AT

The mean length lc of the magnetic core is given as 4 × 0.1 = 0.4 m

The reluctance R = $\frac{lc}{\mu_o \mu_r A} = \frac{0.4}{1000 \times 0.0001 \times 4\pi \times 10^{-7}}$ = 2.865× 106 AT/Wb

The flux $\varphi = \frac{F}{R} = \frac{50}{2.865 \times 10^6} = 1.75 \times 10^{-5}$ Wb

The flux density B = $\frac{\varphi}{A}$ = 0.175 Wb/m^2

ELECTROMAGNETIC INDUCTION

Faraday's law

When the magnetic flux ϕ passing through a surface changes in magnitude an electric field is induced along the outline of the surface. Faraday law states that a time varying flux causes an induced electromotive force or emf, and is given by

$$e = -\frac{d\varphi}{dt} \quad \text{... (4.9)}$$

words if a thin N turn coil is placed along the contour of the surface an emf is induced in it, which is given by

$$e = -N\frac{d\varphi}{dt} = -\frac{d\lambda}{dt} \quad \text{... (4.10)}$$

where $\lambda = N\varphi$ = flux linkage of the coil.

The negative sign means that the induced emf would tend to cause a current to flow in the coil which would oppose the change in flux. This law is known as Lenz's law. The polarity of the induced emf can be determined from physical consideration and therefore the negative sign is omitted from the induced emf equation.

The change in flux linkage of a coil can occur in a variety of ways

Case-I: The flux is constant in value and the coil move relative to it.

Case-II: The coil remains stationary and the flux through it vary in magnitude (flux pulsations)

Case-III: The changes in flux and coil occur together i.e. the coil moving through a varying flux.

In Case-I, the flux cutting rule can be applied, where in the emf induced in a single conductor of length l moving with a velocity v and cutting a stationary magnetic field with flux density B is given by

$$e = Blv \; volts \qquad \text{... (4.11)}$$

where l is in metre, v in metre per sec and B equals weber per square metre. The emf induced is known as dynamically induced emf or motional emf. The motional emf is always associated with electromechanical energy conversion.
The direction of the emf is given by Fleming's right hand rule.

In Case-II, the emf induced in a stationary coil is due to the time varying magnetic field. No motion is involved and there is no energy conversion. The emf so induced is known as statically induced emf or transformer emf and is same as equation 4.9.

In Case-III, both the emf's i.e. the motional emf and transformer emf are induced in the coil.

SELF AND MUTUAL INDUCTANCE

Self-inductance

Self-inductance measures the voltage induced in a coil by the magnetic field created by a current flowing in the same coil. Consider a coil having N turns and carrying a current i as shown in Fig. 4.11.

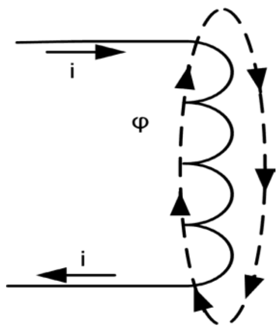


Fig. 4.11: Self Inductance

The coil creates a flux ϕ linking the turns of the coil. Then the emf induced in the coil as per Faradays law is given by

$$e = N\frac{d\varphi}{dt} \text{ volts}$$

$$= N\frac{d\varphi}{di}\frac{di}{dt}$$

$$= \mathrm{L}\frac{di}{dt} \qquad \text{... (4.12)}$$

$$\text{where } L = N\frac{d\varphi}{di}$$

is called the self-inductance of the coil and its unit is H(henrys). For a ferromagnetic material with B-H curve as shown in Fig. 4.8, the curve being nonlinear, the self-inductance corresponds to an incremental value corresponding to incremental change around an operating point on the curve.
Magnetic materials having linear B-H curve, the self-inductance can be expressed as

$$L = \frac{N\varphi}{i} = \frac{\lambda\lambda\lambda}{i} \text{ H}$$

Assuming no flux leakage, the above equation can be rewritten by multiplying numerator and denominator by N, as

$$L = \frac{N^2\varphi}{Ni}$$

Substituting the value of flux ϕ from equation, the above equation can be rewritten as

$$L\, L\, N^2 \mu \frac{A}{l}$$

$$L\, \frac{N^2}{R} \qquad \text{... (4.13)}$$

The unit of self-inductance is Henry and the self-inductance of a coil depends only on the geometry, permeability of the magnetic material and the number of turns of the coil.

Mutual Inductance

When two coils are wound on a common core or are placed close to each other, a part of the flux produced by one coil also links the other coil as shown in Fig. 4.12. There is a magnetic coupling between the two neighbouring coils and this leads to the concept of mutual inductance.

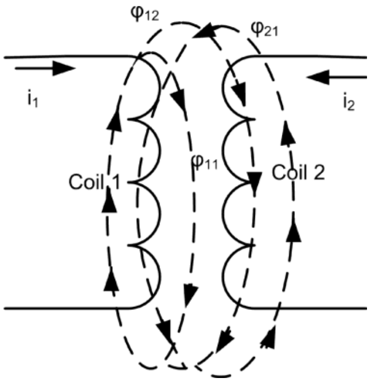


Fig. 4.12: Mutual inductance

Consider the current flowing through coil 1and 2 having the same number of turns N is i_1 and i_2 respectively. The total flux generated by current i_1 in coil 1 is φ_1 . A part of the flux φ_1 linking coil 2 is φ_{12} and similarly a part of the total flux φ_2 generated in coil 2 due to current i_2 linking coil 1 is φ_{21}. Then the mutual inductances of the coils are given as

$$M_{12}\, L\, N \frac{d\varphi_{12}}{dt}\, L\, \frac{\lambda_{12}}{i_1}\ \mathrm{H} \qquad \text{...(4.14)}$$

$$M_{21}\, L\, N \frac{d\varphi_{21}}{dt}\, L\, \frac{\lambda_{21}}{i_2}\ \mathrm{H} \qquad \text{...(4.15)}$$

For a tightly coupled coil there is no flux leakage and hence $M_{12}\, L\, M_{21}\, L\, M$. The unit of mutual inductance is also henry.

Activity

An air cored coil having N turns is made is made from a bare solid conductor of given length and diameter. Measure the self-inductance using LCR meter. Increase the number of turns to 2N, again measure the selfinductance. Compare the measured inductance value and draw the conclusion.

Solved Problems

Example 4.3.1: The total flux at the end of a long bar magnet is 500×10^{-6} Wb. The end of the bar magnet is withdrawn through a 1000 urn coil 1/10 sec. Find the emf generated across the terminals of the coil

Solution: From Faradays law the emf equation $e\, L\, N \frac{d\varphi}{dt}$

Given N =1 000 and $\frac{d\varphi}{dt}\, L\, \frac{500\times10^{-6}}{\frac{1}{10}}\, L\, 5000\times 10^{-6}$

$e\, L\, 1000\times 5000\times 10^{-6}\, L\, 5\ \mathrm{V}$

Example 4.3.2: Find the inductance of a long solenoid of length 1000mm wound uniformly with 3000 turns on a cylindrical iron tube of 60 mm diameter.

Solution: Given N = 3000, l = 1000mm = 1mtr,

Cross sectional area = $\pi \frac{0.060^2}{4} = 2.83\times 10^{-3}\text{m}^2$

The self-inductance $L = N^2\mu \frac{A}{l}$, the relative permeability of paper is 1.

Substituting the value of self-inductance

$$L = 3000\times 3000\times 4\pi \times 10^{-7} \times 1 \times \frac{2.83\times10^{-3}}{1} = 32\text{mH}.$$

ANALOGY BETWEEN ELECTRICAL AND MAGNETIC CIRCUITS

The analysis of a magnetic circuit is analogous to that of a resistive electric circuit. The analogous quantities are listed in Table 4.1.

Table 4.1: Analogy between electrical and magnetic circuits

Electrical quantity	Magnetic quantity
Electrical field intensity E, V/m	Magnetic field intensity H, A-turns/m
Voltage V, Volts	Magneto motive force F, A-turns
Current i, Amp	Magnetic flu x f, Wb
Current density J, A/m^2	Magnetic flux density Wb/m^2
Resistance R,	Reluctance R, A-turns/Wb

The Kirchoff's law for electrical circuits is equally applicable for magnetic circuits. The KVL is interpreted as the magneto-motive force(mmf) of a mesh is equal to the mmf's expended in various parts of the mesh. Similarly, the KCL law is interpreted as the incoming and outgoing fluxes are equal at the junction of magnetic elements. A magnetic circuit as shown in figure can be analysed as a series magnetic circuit with the approximation that the flux density is same in the magnetic core and in air gap. The electrical analogy of the magnetic circuit is shown in Fig. 4.13 consisting of two magnetic elements connected in series having reluctance R_C and R_{ag}. The length of each element corresponds to its mean length.

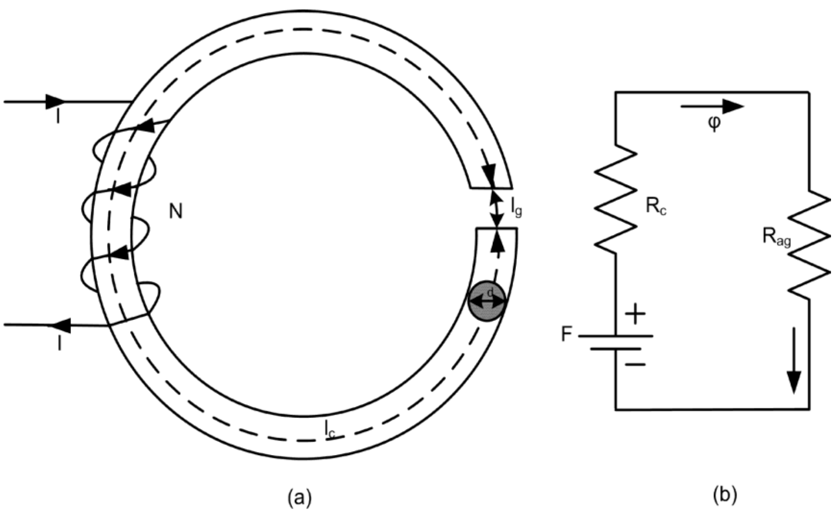


Fig. 4.13: (a) Magnetic core with air gap (b) Electric circuit analogy of the magnetic circuit

The reluctance of the magnetic core with a mean length of R_C where

$$R_C = \frac{l_C}{\mu o \mu r A}$$

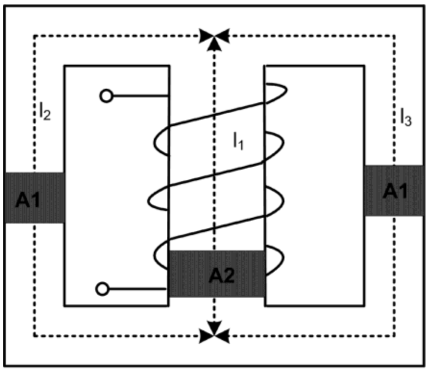
and the reluctance of the air gap is given by

$$R_{ag} = \frac{l_{ag}}{\mu o \mu r A}$$

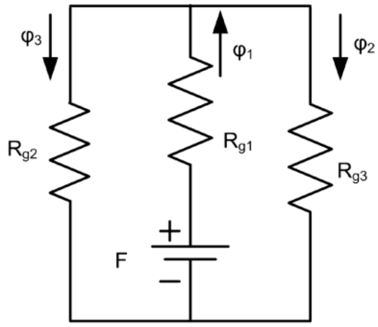
Neglect fringing in the air gap the total reluctance $R = R_C + R_{ag}$
The flux in the circuit is given as

$$\varphi = \frac{F}{R}, \text{ where } F = Ni$$

A parallel magnetic circuit and its electrical analogy is given in Fig.4.14 (a) and (b) respectively



(a)



(b)

Fig. 4.14: (a) Magnetic structure with three limbs (b) Electrical analogy of the magnetic circuit

Activities

Make an electromagnet by wrapping a coil with fixed number of turns on a given iron nail.
Case-I. Connect two 1.5Volt cell in series and connect it across the terminals of the coil through a slider switch. Place some all pins near the coil. Switch ON the slider switch.
Case-II. Repeat the same procedures by replacing the 3 V cell by a 9 V battery. Observe the effect of the electromagnetic force on the all pins for Case-I and II and make necessary conclusions.

Solved Problems

Example 4.3.1: An iron ring with mean circumference of 140 cm and cross sectional area 12 cm^2 is wound with 500 turns of wire. When the exciting current is 2 A, the flux is found to be 1.2 milliweber. Determine relative permeability of iron.

Solution: Given $\phi = \frac{F}{R}$, $F = NI = 500 \times 2 = 1000$ and flux $\varphi = 1.2 \times 10^{-3}$ Wb.

The reluctance $R = \frac{1000}{1.2 \times 10^{-3}} = 833.33 \times 10^3$.

$R = \frac{1}{\mu o \mu l r A}$, The mean length $l = 1.4$ mtr and $A = 12 \times 10^{-4}\, m^2$.

Substituting the above value, we get. $833.33 \times 10^3 = \frac{1.4}{12 \times 10^{-4} \times \mu r \times 4\pi \times 10^{-7}}$

$\mu r = 1114.69$

AC CIRCUIT

ALTERNATING CURRENT FUNDAMENTALS

Introduction

Although alternating quantity has a much wider meaning, it is generally used to mean a sinusoidal quantity. Usually, alternating current (referred to as AC current) or alternating voltage (referred to as AC voltage) is a sinusoidal varying current or voltage. Almost all electrical power supply system's involve sinusoidal AC current, which is derived from sinusoidal AC voltage. A generator is used to produce AC voltage. The voltage generated by utility companies for our home, factories and offices is AC voltage.

Alternating Quantity

An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time, as discussed in unit 1.3 of Unit 1. An alternating voltage or current may not always take the form of a smooth wave such as that shown in Fig. 5.1, yet sine wave is the ideal form and is the accepted standard. The waves deviating from the standard sine wave are termed as distorted waves. In general, however, an alternating current or voltage is one, the direction of which reverses at regularly recurring intervals.

Alternating Voltage and Current

When a coil is rotated in a magnetic field, an alternating electromotive force (e.m.f.) is induced in that coil. The value of e.m.f. induced depends on number of turns in the coil, strength of the magnetic field and the speed of which the coil is rotated in the magnetic field. Consider a conductor rotating in a uniform magnetic field with constant angular velocity of 'ω' radian per second as shown in Fig. 5.2. Its axis of revolution being perpendicular to the magnetic lines of force. As per the different position of conductor such as a, b, c and d, the corresponding value of electromotive force (emf) is shown in Fig. 5.2.

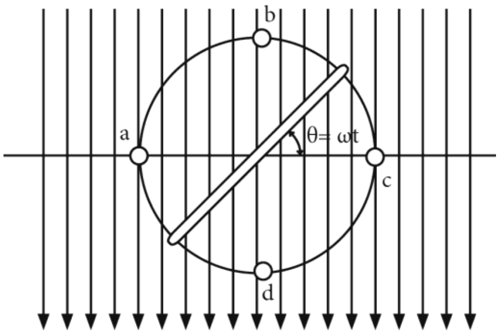


Fig. 5.1: EMF generated in a Coil rotating in a magnetic field

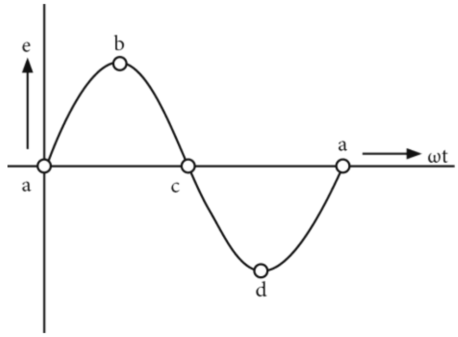


Fig. 5.2: Sinusoidal wave

At point a and point c, the conductor moves parallel to magnetic field. Hence, e.m.f. induced is zero. While at point b and d, the conductor moves in a direction perpendicular to the magnetic field. Hence, e.m.f. induced is maximum. In one complete revolution of a conductor, one complete cycle of e.m.f. is obtained. As the direction of e.m.f. is reversed at points a and c such e.m.f. is known as alternating e.m.f. or alternating voltage. When a coil with induced alternating e.m.f. is connected to external circuit, the alternating current starts flowing. The waveform of this alternating current is similar to waveform of an alternating voltage. Also, e.m.f. varies as sine function. The curve traced is sine curve and, hence, it is known as sinusoidal e.m.f.

5.1.1 Important terms related with an alternating quantity

Some of the important terms related with an alternating quantity, that should be understood are cycle, frequency, periodic time, amplitude, angular velocity, rms value, average value, form factor, peak factor, impedance, phase angle, and power factor; They are described in brief:

1. Cycle

One complete set of positive and negative values of alternating quantity is known as cycle. Hence, each diagram of Fig. 5.3 represents one complete cycle. A cycle may also be sometimes specified in terms of angular measure. In that case, one complete cycle is said to spread over 360° or 2π radians.

2. Frequency (f)

It is the number of cycles that occur in one second. The unit for frequency is Hz or cycles/sec. For example, 50 Hz is 50 cycles in one second.

3. Time Period or Periodic time (T)

It is the time taken in seconds to complete one cycle of an alternating quantity. It is denoted by T. The relationship between frequency and time period can be derived as follows:

Time taken to complete f cycles = 1 second

Time taken to complete 1 cycle = 1/f second

$$T = 1/f \quad ...(5.1)$$

4. Amplitude

It is the maximum value, positive or negative, attained by an alternating quantity. It is also called as maximum or peak value.

5. Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called its average value. The average value of an alternating quantity over a cycle is zero. Therefore, it is defined over half a cycle. It is defined as that value of steady current which transfers the same electric charge as transferred by that alternating current.

Average value of a sinusoidal current, $iiI_m sin\ nt$ ...(5.2)

$$I_{av}\ i\ \frac{1}{\pi}\int_0^{\pi} i\ ddntd$$

$$I_{av}\ i\ \frac{1}{\pi}\int_0^{\pi} I_m \sin\ ntd\ dntd$$

$$I_{av}\ i\ \frac{2I_m}{\pi}\ i\ 0.6371_m$$ ...(5.3)

∴ Sinusoidal alternating quantity, Average value = 0.637 x Maximum value.

6. R.M.S. Value

The R.M.S. (Root Mean Square) value of an alternating current is that value of steady current or direct current, which when flowing through a given circuit for a given time produces the same amount of heat as is produced by the alternating current flowing through the same circuit for the same time. It is also known as Effective Value. The value of an alternating current measured by ammeter is the R.M.S. value of the current. For sinusoidal alternating current;

$$iiI_m sin\ nt$$

$$I_{\text{rms}}\ i\ \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 ddntd}$$

$$I_{\text{rms}}\ i\ \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2{}_m sin^2 nt\ ddntd}$$

$$I_{\text{rms}}\ i\ \frac{I_{\text{m}}}{\sqrt{2}}\ i\ 0.707 I_m$$ ...(5.4)

∴ R.M.S. value of a sinusoidal waveform = 0.707 × maximum value

7. Form Factor

The ratio of R.M.S. value of current to its average value is called form factor.

Form Factor = R. M.S. value/ Average value

$i\,0.707\ \text{I}_m / 0.637\ \text{I}_m\ i 1.11$ ...(5.5)

Thus, for sinusoidal waveform, the value of form factor is 1.11.

8. Peak Factor

The ratio of maximum value to R.M.S. value is called peak factor.

Peak Factor $i\ \frac{Maximum\ value}{R.M.S.\ value}$ ...(5.6)

For sinusoidal wave form, the value of peak factor is 1.414.

9. Angular Velocity

Angular frequency is defined as the number of radians covered in one second(i.e. the angle covered by the rotating coil). The unit of angular frequency is rad/sec.

$$ni\frac{2\pi}{T}\ i\ 2\pi\text{f}$$ ...(5.7)

10. Instantaneous Value

It is the value of the quantity at any instant.

$$v = V_m \sin\theta = V_m \sin\omega t = V_m \sin 2\pi f t = V_m \sin\frac{2\pi}{T}t \quad ...(5.8)$$

where V_m = Maximum value of voltage,
f = Frequency in Hz, and t = Time in seconds

From the relationship expressed by equations 5.8,

i. the maximum value or peak value or amplitude of an alternating voltage is given by the coefficient of the sine of the time angle.
ii. the frequency f is given by the coefficient of time divided by 2π.

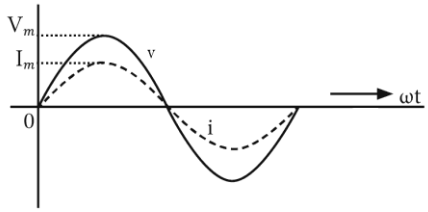


Fig. 5.3: Alternating Voltage and Current

For example, if the equation of an alternating voltage is given by v = 20 sin 31
then its maximum value is 20 V and its frequency is f = 314/2π = 50 Hz.
If the current is in phase with the above voltage and I_m is the maximum value of current then equation for instantaneous value of alternating current is

$$i = I_m \sin 2\pi ft = I_m \sin \omega t = I_m \sin\frac{2\pi}{T}t \quad ...(5.9)$$

Comparison of AC system and DC system

A DC waveform is graphically shown in Fig 5.4. Some of the important advantages of AC system over DC system are as follows:

1. AC voltages can be efficiently stepped up/down using transformer.
2. As compared to DC motors, AC motors are cheaper and simpler in construction.
3. Switchgear for AC system is simpler than DC system.
4. Easy and cheaper generation and transmission, up to the break even distance.

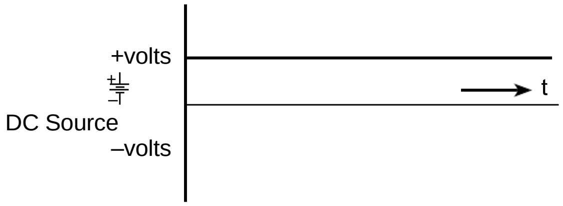


Fig. 5.4: DC Voltage

Single-phase and three-phase AC

AC can be single-phase or three-phase. Single-phase AC (Fig. 5.3) is used for small electrical demands such as in the home. Three-phase AC shown in Fig. 5.5 is used where heavy load i.e. large amount of power is required in commercial and industrial facilities. Three-phase is a continuous series of three overlapping AC cycles. Each wave represents a phase, and is offset by 120 degrees.

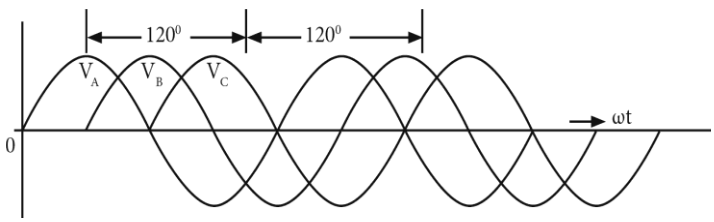


Fig. 5.5: Three-Phase Sine Wave

5.1.2 Phase, Phase Difference and Power Factor

During a cycle, the alternating current or voltage passes through various values. Starting from zero, it rises to a maximum and, then gradually reduces to zero. Then rises in the reverse direction, becomes maximum and finally, comes back to zero again. All alternating quantities go through these various stages. These various stages are termed as various phases in electrical engineering. By phase of an alternating quantity is meant the fraction of the time period of that alternating quantity, which has elapsed since the quantity last passed through the zero position of reference. Two alternating waves will be said to be in phase when they reach their maximum and zero values at the same time. Their maximum values may be different in magnitude. The actual phase at any particular instance of time is not that significant. However, the angle or time difference is important. The different quantities may be two different voltages or two different currents or a voltage and a current. The relative difference between two alternating quantiles is called 'phase difference' and, is expressed in terms of 'phase angle'.

Phase of I_m is π/2 rad or T/4 sec
Phase of $-I_m$ is 3π/2 rad or 3T/4 sec

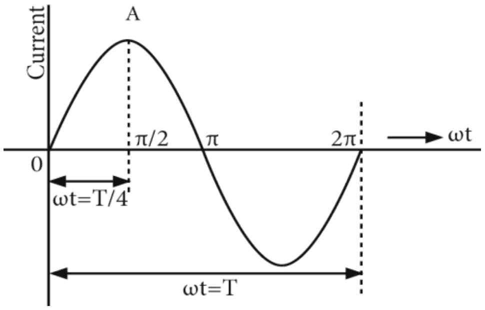


Fig. 5.6: Sine Wave with phase angle

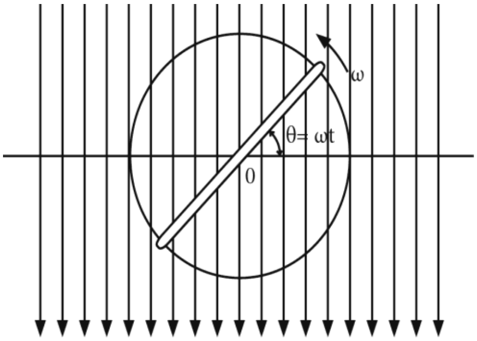
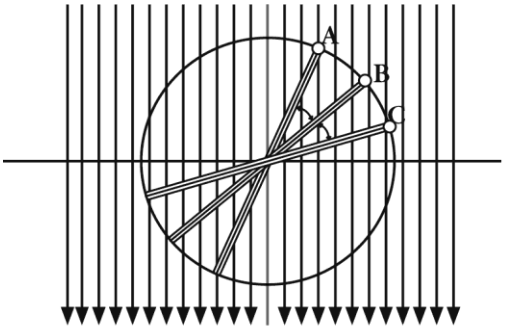


Fig. 5.7: Phase of rotating coil

For example, the phase of current at point A is T/4 second, where T is time period or expressed in terms of angle, it is π/2 radians (Fig. 5.6). Similarly, the phase of the rotating coil at the instant shown in Fig. 5.7 is θ that is equal to ωt which is therefore, called its phase angle.

Phase Difference

Consider three similar single-turn coils displaced from each other by angles and rotating in a uniform magnetic field having the same angular velocity as shown in Fig. 5.8 (a).



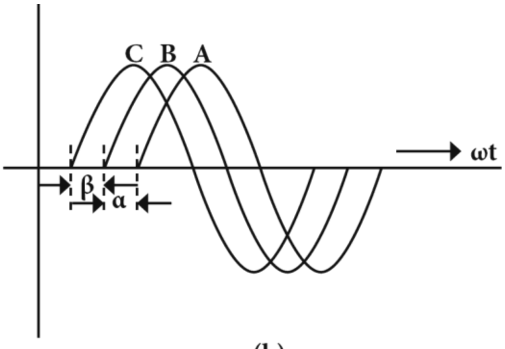


Fig. 5.8: Phase Difference

In this case, the value of induced e.m.fs. in the three coils are the same, but there is one important difference. The e.m.fs. in these coils do not reach their maximum or zero values simultaneously but one after another. The three sinusoidal waves are shown in Fig. 5.8 (b). It is seen that curves B and C are displaced from curve A at angles β and (α+ β) respectively. Hence, it means that phase difference between A and B is β and between B and C is α, but between A and C is (α+ β). The statement, however, does not give indication as to which e.m.f. reaches its maximum value first. This deficiency is supplied by using the terms 'lag' or 'lead'. A plus (+) sign when used in connection with phase difference denotes 'lead' whereas a minus (–) sign denotes 'lag'.

A leading alternating quantity is one which reaches its maximum (or zero) value earlier as compared to the other quantity. Similarly, a lagging alternating quantity is one which reaches its maximum or zero value later than the other quantity. For example, in Fig. 5.8 (b), B lags behind A by angle β and C lags behind A by (α+ β) because they reach their maximum values later.

The three equations for the instantaneous induced e.m.fs are (Eq. 5.10 a, b and c)

$e_A = E_m \sin \omega t$ ...reference quantity (Eq. 5.10b)

$e_B = E_m \sin (\omega t - \beta)$ ...(Eq. 5.10a)

$e_C = E_m \sin [\omega t - (\alpha + \beta)]$ ...(Eq. 5.10c)

Phasor

A phasor is a vector that is used to represent a sinusoidal function. It rotates about the origin with an angular speed ω in anti-clock wise direction. Electrical quantities like current and voltage are represented by means of phasor with the length representing the magnitude and the arrow representing the direction. The vertical component of phasors represents the quantities that are sinusoidal varying for a given equation. Here, the magnitude of the phasors represents the peak or maximum value of the current (I_m) and voltage (V_m). From Fig. 5.6 and Fig. 5.7, the relation between a phasor and the sinusoidal representation of the function with respect to time can be observed. The projection of the phasor on the vertical axis represents the value of the quantity. For example, in the case of a current or a voltage phasor, the projection of the phasor on the vertical axis, given by $I_m \sin \omega t$ and $V_m \sin \omega t$ respectively, gives the value of the current or the voltage at that instant.

A.C in Pure Resistors, Inductors and Capacitors

Pure Resistive Circuit

From Ohm's law, I =V/R or V= I R

When an alternating voltage V is applied across a pure resistance R as shown in Fig. 5.9, the instantaneous value of current flowing through the resistance in given by $i = I_m \sin 2\pi f t$.

Putting the value of V in terms of maximum voltage and $I_m = V_m/R$,

$$v = V_m \sin 2\pi f t.$$

From the expressions of v and i, we see that the quantities can be represented as shown in Fig. 5.10. From the phasor diagram of a pure resistive circuit as shown in Fig. 5.11, the phasors for the voltage and the current are in the same direction for all instances, the phase angle between the voltage and the current is zero, that is the phase difference is zero. Hence, the value of power factor or cos ø is unity, i.e. one.

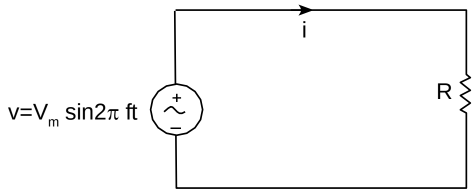


Fig. 5.9: Pure Resistive Circuit with AC Source

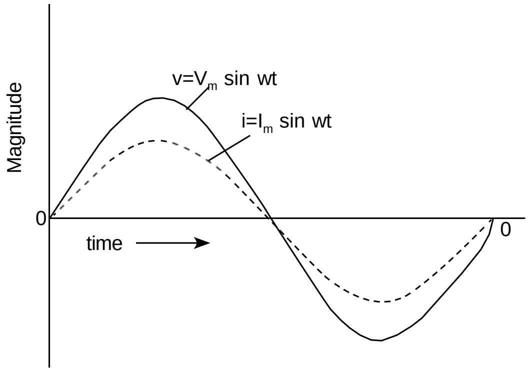


Fig. 5.10: Response of a pure resistive circuit to AC voltage input

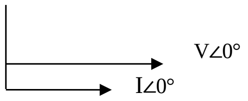


Fig. 5.11: Phasor diagram

Pure Inductive Circuit

An AC voltage is connected to a pure inductive coil as shown in Fig. 5.12. It results in e.m.f. getting induced in the coil due to self-inductance. This e.m.f. is dependent on the rate of change of current which flows through the coil. Due to this opposition, the current lags behind the applied voltage by an angle of $\pi/2$ or $90°$.

Let $v = V_m \sin 2\pi f t$

∴ the value of current $i = I_m \sin (2\pi f t - \pi/2)$

$= - I_m \cos 2\pi ft$

This is shown in Fig. 5.13.

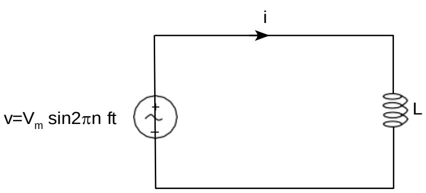


Fig. 5.12: Response of a to a ac voltage input

Also, the value of current $I_m = V_m/\omega L$

where "ωL" is known as "Inductive Reactance" and is denoted by X_L,

L is the value of inductance in Henry and ω = 2πfL.

X_L is measured in ohms.

In this circuit, the phase difference between voltage and current is 90° as shown in Fig. 5.14. Hence, the value of power factor or cos ø is zero.

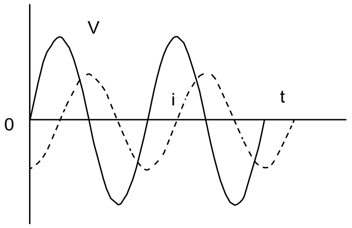


Fig. 5.13: Pure inductive circuit to AC

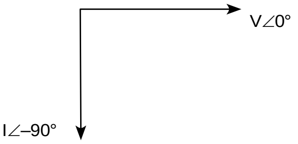


Fig. 5.14: Phasor Diagram

Pure Capacitive Circuit

When an alternating voltage is applied to a pure capacitance as in Fig. 5.15, the process of charging and discharging begins. It is charged in one direction and, then in the opposite direction. This results in flow of current. Due to charging and discharging process, the current leads the applied voltage by an angle π /2 as shown in Fig. 5.16.

Let $v = V_m \sin 2\pi ft$

then the value of current $i = I_m \sin (2\pi ft + \pi/2)$

Also, the value of current $I_m = V_m/(1/\omega C)$

where $1/\omega C$ plays the role of resistance and is known as "Capacitive Reactance". It is denoted by X_C and is measured in Ohms.

C is the value of capacitance in Farads and $\omega = 2\pi ft$. X_c is measured in ohms.

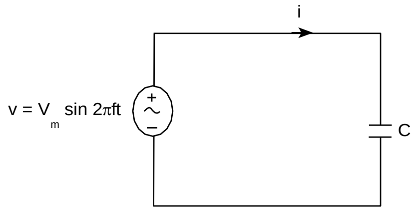


Fig. 5.15: Pure Capacitive Circui with AC

In this circuit, the phase difference between voltage and current is 90° as shown in Fig. 5.17. Hence, the value of power factor or cos ø is zero.

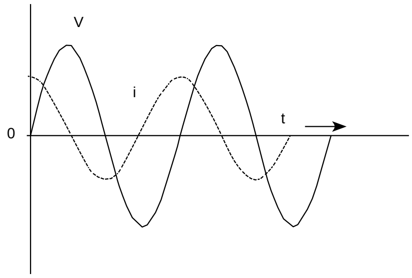


Fig. 5.16: Response of a pure capacitive circuit to AC voltage input

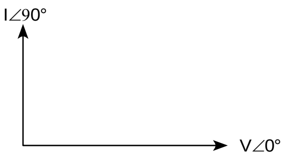


Fig. 5.17: Phasor Diagram a pure capacitive circuit

Applications

- Almost every home in the world is powered by AC. DC is generally not used for these purposes due to more power lost to heat compared to AC, higher risks of producing a fire, higher costs, and issues with converting high voltage to low voltage using transformers.
- With AC, it is also feasible to build electric generators, and power distribution systems that are far more efficient than DC, and so it is seen that AC is mainly used across the world in high power applications.
- AC is also more popular when it comes to powering electric motors, a device that converts electric energy into mechanical energy.
- Most of the household appliances that are used rely on AC like refrigerators, fans, air-conditioners, ovens, toasters, etc.
- The three basic linear passive components: the resistor (R), the capacitor (C), and the inductor (L) are used in number of AC applications on its own and may be combined as RC circuit, the RL circuit, the LC circuit, and the RLC circuit, with the acronyms indicating which components are used. These circuits, among them, depicts a large number of important types of behaviour that are fundamental to many electrical and analog electronic applications.

Solved Problem

Example 5.1.1: An alternating current i is given by $i = 141.4 \sin 314t$

Find i) The maximum value ii) Frequency iii) Time Period iv) The instantaneous value when t =5 ms .

Solution:

Given, $i = 141.4 \sin 314t$

$i = I_m \sin \omega t$

i. Maximum value $I_m = 141.4$ A

ii. $\omega = 314$ rad/sec

$f = \omega/2\pi = 50$ Hz

iii. $T = 1/f = 0.02$ sec

iv. $I = 141.4 \sin(314 \times 0.005) = 3.87$ A

AC SERIES AND PARALLEL CIRCUITS

Introduction

In electrical engineering, when practical circuits are analysed, they normally consist of two or more of the elements of resistance, inductance and capacitance. Hence there are a number of situations in which calculation of different components related to AC series circuit is required. For studying the performance of AC machines, the knowledge of AC series circuit is quite essential. In this section, AC series circuits consisting the combinations of resistance, inductance and capacitance are described.

Resistance - Inductance (R-L) circuit

In the previous section, AC voltage applied across circuits consisting of pure resistance, inductance and capacitance in turn was explained. However, in a series circuit when AC voltage is applied across the combination of the two i.e. a circuit consisting of pure resistance R and pure inductance 'L' in series as shown in Fig.5.18, the current flowing in R and L will be same and therefore, will have same instantaneous value as also the R.M.S. and maximum value. 'i' is taken as reference, for the solution of the series circuit.

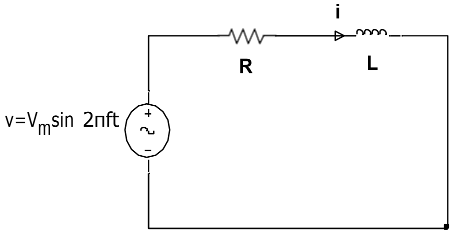


Fig. 5.18: R.L. Series circuit

Let $i = I_m \sin \omega t$ be the expression for current flowing. This will cause voltage drops across R and L.
The instantaneous voltage drop across 'R' is

$$V_R = i\,R = I_m \sin \omega t\, R \qquad ...(5.11)$$

and, the instantaneous voltage drop across 'L' is

$$V_L = L\,\frac{di}{dt} = L\,\frac{d}{dt}\,(I_m \sin \omega t)dt$$

$$= (I_m \cos \omega t)\,\omega L \qquad ...(5.12)$$

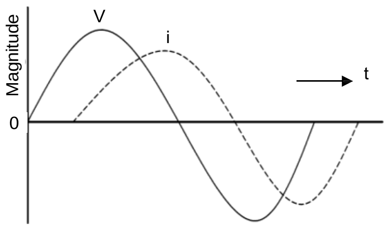


Fig. 5.19: Response of a R-L series circuit to AC input

Then, the total instantaneous value of the supply voltage is

$$v = V_R + V_L$$
$$= I_m R \sin \omega t + I_m \omega L \cos \omega t$$
$$= I_m (R \sin \omega t + \omega L \cos \omega t]$$

Substituting

$R = Z \cos \vartheta$ and $\omega L = X_L = Z \sin \vartheta$; ...(5.13)

where Z is called impedance of the circuit.

$$V = I_m [Z \cos \vartheta \sin \omega t + Z \sin \vartheta \cos \omega t]$$
$$= I_m Z [\cos \vartheta \sin \omega t + \sin \vartheta \cos \omega t]$$
$$= I_m Z[\sin (\omega t + \vartheta)]$$
$$= V_m \sin (\omega t + \vartheta) \quad ...(5.14)$$

Thus, the voltage leads over the current by an angle θ, this also means that, the current in an inductive circuit lag over the voltage by an angle θ.

The value of θ in terms of known parameters be found out by taking ratio of

$$Z \sin \theta / Z \cos \theta = \tan \theta$$
$$\text{or } \theta = \tan^{-1} \omega L / R \quad ...(5.15)$$

It is a function of ' ω ' the frequency. The value of Z in terms of given parameters is

$$R = Z \cos \theta \; ; \; \omega L = Z \sin \theta.$$

Squaring and adding it,

$$R^2 + \omega^2 L^2 = Z^2 (\cos^2 \theta + \sin^2 \theta) = Z^2$$

$$Z = \sqrt{R^2 + \omega^2 L^2} \quad ...(5.16)$$

Fig. 5.19 shows variation of voltages and current across an R-L circuit.
Total power of the circuit, $P = VI \cos \theta$
where V and I are the r.m.s. values of voltage and current.

Resistance - Capacitance (R-C) circuit

On comparing RC circuit as shown in circuit Fig. 5.20 with R-L circuit, it is seen

Impedance $Z = \sqrt{R^2 + (1/\omega^2 C^2)}$ ...(5.17)

$I_m = V_m/Z$; $\tan \theta = X_C/R$. or $\theta = \tan^{-1} 1/(\omega CR)$ ...(5.18)

$\cos \theta = R/Z$

$i = I_m \sin (2\pi ft + ø)$ ...(5.19)

because current leads the voltage by an angle ø.
Total power of the circuit, $P = VI \cos ø$

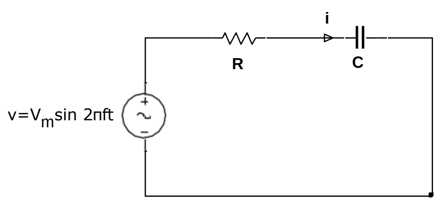


Fig. 5.20: R-C series circuit

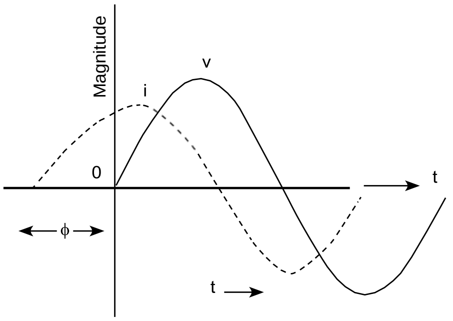


Fig. 5.21: Response of a R-C series circuit to AC voltage input

Resistance, Inductance and Capacitance Circuit (R.L.C. Circuit)

A pure resistance of R ohms ,pure inductive reactance of X_L ohm and pure capacitive reactance of X_C ohms are connected in series across AC voltage, $v = V_m \sin 2\pi ft$ as shown in Fig. 5.22.
In this case, the current is opposed by total combined resistance which is known as impedance. The impedance consists of resistance R and net reactance X.

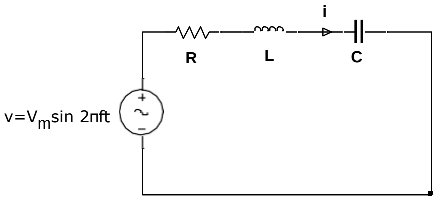


Fig. 5.22: R-L-C series circuit

The net reactance is given by $X = X_L - X_C$, when $X_L > X_C$
or $X = X_C - X_L$, when $X_C > X_L$

Thus, the impedance $Z = \sqrt{R^2 + X^2}$ or $Z = \sqrt{R^2 + (X_L - X_C)^2}$ or $Z = \sqrt{R^2 + (X_C - X_L)^2}$ . ...(5.20)

The value of current $I_m = \frac{V_m}{Z}$; value of power factor $\cos \phi = \frac{R}{Z}$; and phase angle $\phi = \cos^{-1} \frac{R}{Z}$.

Impedance Triangle

Impedance Triangle is a right angled triangle whose base, perpendicular and hypotenuse represents Resistance 'R', Reactance 'X' and Impedance 'Z' respectively as shown in Fig. 5.23. It is basically a geometrical representation of circuit impedance.

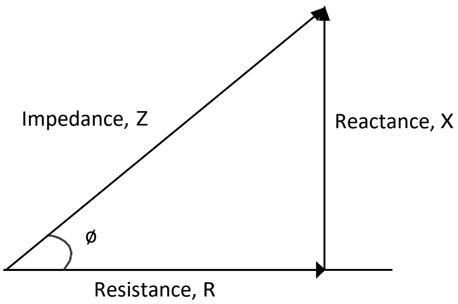


Fig. 5.23: Impedance Triangle

Parallel RLC Circuit

Consider a RLC circuit in which resistor, inductor and capacitor are connected in parallel to each other. This parallel combination is supplied by voltage supply, V as shown in Fig 5.24. In series RLC circuit, the current flowing through all the three components i.e the resistor, inductor and capacitor remains the same, but in parallel circuit, the voltage across each element remains the same and the current gets divided in each component depending upon the impedance of each component. That is why parallel RLC circuit is said to have dual relationship with series RLC circuit.
The total current is drawn from the supply is equal to the vector sum of the resistive, inductive and capacitive current, not the mathematical sum of the three individual branch currents, as the current flowing in resistor, inductor and capacitor are not in same phase with each other; so they cannot be added arithmetically.

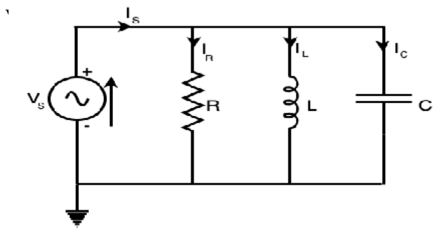


Fig. 5.24: R-L-C parallel circuit

Apply Kirchhoff's current law, which states that the sum of currents entering a junction or node, is equal to the sum of current leaving that node,

$$I_S^2 = I_R^2 + (I_L - I_C)^2$$

Phasor Diagram of Parallel RLC Circuit

Let V be the supply voltage;
I_S, the total source current;
I_R, the current flowing through the resistor;
I_C, the current flowing through the capacitor;
I_L, the current flowing through the inductor; and
θ, the phase angle difference between supply voltage and current.

For drawing the phasor diagram of parallel RLC circuit, voltage is taken as reference since voltage across each element remains the same and all the other currents i.e. I_R, I_C, I_L are drawn relative to the voltage vector. In case of resistor, voltage and current are in same phase; current vector I_R is drawn in same phase and direction to voltage. In case of capacitor, current leads the voltage by 90^0, so drawing I_C vector leading voltage vector, V by 90^0. For inductor, current vector I_L lags voltage by 90^0 so drawing I_L lagging voltage vector, V by 90^0. The resultant of I_R, I_C and I_L i.e. current I_S at a phase angle difference of θ with respect to voltage vector, V as shown in Fig. 5.25(a).

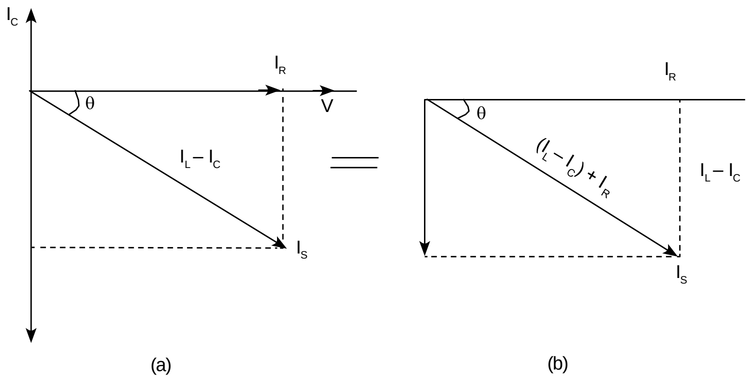


Fig. 5.25: Phasor Diagram of Parallel RLC Circuit

Simplifying the phasor diagram, simplified phasor diagram is obtained as shown in right hand side in Fig 5.25(b). On the phasor diagram of parallel RLC circuit, applying Pythagoras theorem,

$$I_S^2 = I_R^2 + (I_L - I_C)^2$$

Since $I_R = V/R$, $I_C = V/X_C$, and $I_L = V/X_L$, substituting the value of I_R, I_C, I_L in above equation,

$$I_S = \left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2$$

On simplifying, Admittance, $Y = \frac{1}{Z} = \frac{I_S}{V} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$ ...(5.21)

As shown above in the equation of impedance, Z of a parallel RLC circuit each element has reciprocal of impedance (1/Z) i.e. admittance, Y. For solving parallel RLC circuit, it is convenient if admittance of each branch is found and the total admittance of the circuit can be found by simply adding each branch's admittance.

Resonance in Series RLC Circuit

Resonance in AC circuit implies a special frequency determined by the values of resistance, inductance and capacitance.

Inductive reactance $X_L = \omega L$ or $X_L = 2\pi f\,L$
and capacitive reactance $X_C = \frac{1}{\omega C}$ or $X_C = \frac{1}{2\pi f\,C}$;
where f is the frequency, L is inductance in Henry and C is the capacitance in Farad.

At certain value of frequency, it may happen that, the value of $X_L = X_C$. At this stage, net reactance will be zero. The series circuit in which, net reactance becomes zero; is said to be in "Electrical resonance." The frequency at which this happens is known as "resonant frequency" and can be derived using equation $X_L = X_C$.

Its value is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$ ...(5.22)

At resonant frequency, it is seen that, the value of Z = R ohm and current is opposed by resistance only and, hence, it is characterized by minimum impedance, maximum current and zero phase.

Resonance in Parallel RLC Circuit

Like series RLC circuit, parallel RLC circuit also resonates at particular frequency called resonance frequency i.e. there occurs a frequency at which inductive reactance becomes equal to capacitive reactance but unlike series RLC circuit, in parallel RLC circuit the impedance becomes maximum and the circuit behaves like purely resistive circuit leading to unity electrical power factor of the circuit.

Solved Problems

Example 5.2.1: A series circuit consists of a resistance of 6Ω and an inductive reactance of 8Ω. A potential difference of 141.4 V (r.m.s.) is applied to it. At a certain instant, the applied voltage is + 100 V, and is increasing. Calculate at this instant, (i) the current (ii) the voltage drop across the resistance and (iii) voltage drop across inductive reactance.

Solution: $Z = R + jX = 6 + j8 = 10 \angle 53.1°$

It shows that current lags behind the applied voltage by 53.1°. Let V be taken as the reference quantity.

Then v = (141.4 × 2) sin t = 200 sin t; i = (V_m/Z sin t) −30° = 20 sin (t −53.1°).

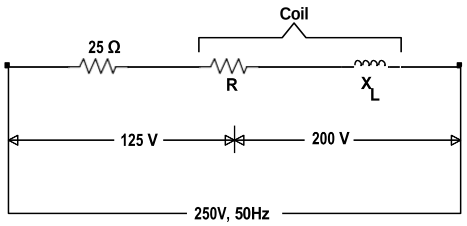
i. When the voltage is + 100 V and increasing; 100 = 200 sin t; sin t = 0.5; ωt = 30°. At this instant, the current is given by i = 20 sin (30° −53.1°) = −20 sin 23.1° = −7.847 A.

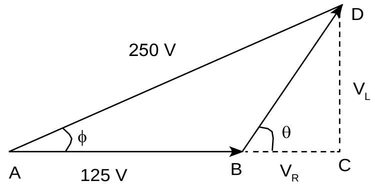
ii. Voltage drop across resistor = iR = −7.847 × 6 = −47 V.

iii. Let us first find the equation of the voltage drop V_L across the inductive reactance. Maximum value of the voltage drop = $I_m X_L$ = 20 × 8 = 160 V. It leads the current by 90°. Since current itself lags the applied voltage by 53.1°, the reactive voltage drop across the applied voltage by (90° − 53.1°) = 36.9°. Hence, the equation of this voltage drop at the instant when ωt = 30° is

$$V_L = 160 \sin(30° + 36.9°) = 160 \sin 66.9° = 147.2\ V.$$

Example 5.2.2: A current of 5 A flows through a non-inductive resistance in series with a choking coil, when supplied at 250-V, 50-Hz. If the voltage across the resistance is 125 V and across the coil 200 V, calculate (a) impedance, reactance and resistance of the coil (b) the power absorbed by the coil and (c) the total power.





Solution:

I = 5 A

As seen from the vector diagram drawn,

$BC^2 + CD^2 = 200^2$ ...(i)

$(125 + BC)^2 + CD^2 = 250^2$ ...(ii)

Subtracting Eq. (i) from (ii), we get, $(125 + BC)^2 - BC^2 = 250^2 - 200^2$

$\therefore$ BC = 27.5 V; CD = 200 − 27.5 = 198.1 V

a. Coil impedance = 200/5 = 40 Ω

$V_R = IR = BC$ or

5R = 27.5

$\therefore$ R = 27.5/5 = 5.5Ω

Also $V_L = I \cdot X_L = CD = 198.1$

$\therefore X_L = 198.1/5 = 39.62\ \Omega$

or $X_L = 40 - 5.5 = 39.62\ \Omega$

b. Power absorbed by the coil = Voltage across coil × current × cos θ

= 200 × 5 × 27.5/200 =137.5W

Also $P = I^2 R = 5^2 \times 5.5 = 137.5W$

c. Total power = VI cos ϕ = 250 × 5 × AC/AD = 250 × 5 × 152.5/250 = 762.5 W

The power may also be calculated by using I^2R formula.

Series resistance = 125/5 = 25 Ω

Total circuit resistance = 25 + 5.5 = 30.5 Ω

$\therefore$ Total power = $5^2 \times 30.5 = 762.5$ W

Applications

The applications of RC & RLC circuits include the following:

- RF Amplifiers
- Filtering Circuits
- Oscillator Circuits
- Processing of Signal
- Magnification of Current or Voltage
- Frequency, Amplitude Modulation Circuit
- Radio Wave Transmitters

RL combination is comparatively expensive , hence it is found in very less appliances e.g. choke of tube light, power supplies, etc. LC circuits and RLC Circuitbehave as electronic resonators, which are a key component in many applications like Oscillators, Filters, Tuners, Mixers, Contactless cards, Graphics tablets, electronic article surveillance (security tags), etc.

AC POWER AND THREE PHASE CIRCUIT

Introduction

In the earlier units, single phase circuit was discussed. Now a days, three phase systems are the most commonly used systems. Most of the electrical machines are operating on three-phase system. Not only that, the complete generation, transmission distribution as well as utilization of electrical energy is based on three phase system. Three phase motors are used in a floor mill and in most of the industries. The switches used normally in the house are double pole type while, the switches used in industries are of 'triple pole' type because three phase supply is commonly used in industries. Hence it is necessary to learn about basics of three-phase system.

Advantages of Three Phase System

Compared to single phase system, there are certain advantages in case of three phase system which are as follow:

i. The power in three phase system is about three times than that of single phase system.
ii. Compared to only one voltage available in single-phase system, the values of voltages in three-phase system are two phase and line voltages in case of star connection.
iii. Large amount of power can be transmitted in three-phase system compared to power transmitted in single phase system.
iv. Power factor of motors operated on three-phase system is higher, than the p.f. of single phase motors for same output and speed.
v. Three phase currents can produce rotating magnetic field (which is required for operation of AC motors) while single phase supply can produce only pulsating field.

Star and Delta Connection

The three phase circuits can be connected in two ways.

i. Star connection

In star connection as shown in Fig. 5.26, three ends of a coil or resistance are shorted together to make point N. This junction acts as a neutral point. Remaining three ends named as R, Y and B are the supply terminals.

ii. Delta connection

In delta connection two ends, one from one coil or resistance and, other from other coil or resistance are joined together. Thus, it forms three junctions as shown in Fig. 5.27. Three junctions named as R, Y and B are the supply terminals.

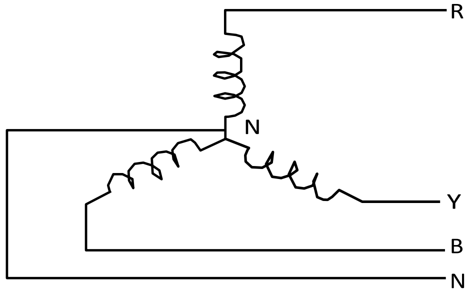


Fig. 5.26: Star Connection

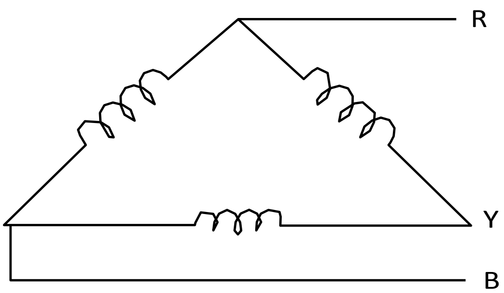


Fig. 5.27: Delta Connection

Relationship between Line and Phase Values of Voltages and Currents

In case of three phase connection, the voltage between two outer conductors or lines is called 'Line voltage'. It is denoted by V_L. The voltage across each coil or phase is called 'phase voltage'. It is denoted by V_p. Similarly, the current flowing in outer conductor or line is called 'Line current'. It is denoted by IL. The current flowing in a coil or phase is called 'phase current'. It is denoted by I_p. All these are shown in Fig. 5.28 and Fig 5.29 , and it will help in finding the relation between V_L and V_p, I_L and I_p in case of star and delta connections.

Star connection

In star connection as shown in Fig. 5.28, it is seen that line current is equal to phase current

i.e. $I_L = I_p$. ...(5.23)

Regarding voltage, Line voltage is equal to $\sqrt{3}$ times the phase voltage

i.e. $V_L = \sqrt{3}\, V_p$ or $V_p = V_L / \sqrt{3}$ ...(5.24)

Also, phase current $I_p = \dfrac{\text{Phase voltage } (V_p)}{\text{Impedance per phase (Z)}}$ ...(5.25)

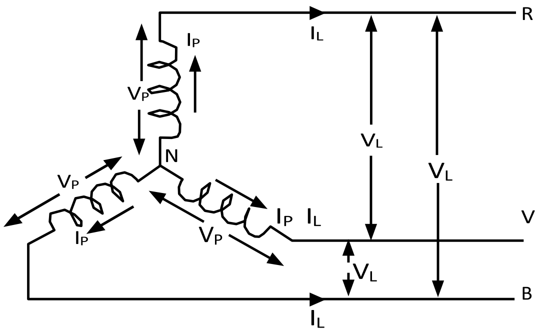


Fig.5.28: Star connection with Voltage and Current

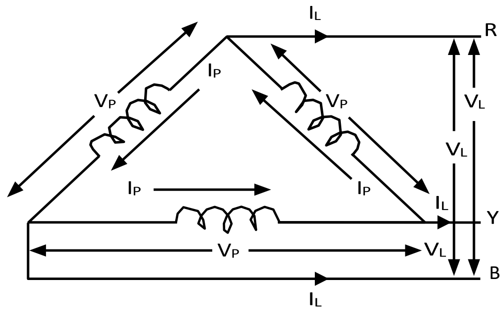


Fig.5.29: Delta connection with Voltage and Current

Delta connection

For delta connection as shown in Fig. 5.29, it is seen that line voltage equal to phase voltage
i.e. $V_L = V_p$. ...(5.26)
Line current is equal to $\sqrt{3}$ times the phase current
i.e. $I_L = \sqrt{3}\, I_p$ or $I_p = I_L/\sqrt{3}$. ...(5.27)

Also, phase current $I_p = \dfrac{\text{Phase voltage oV}_p)}{\text{Impedance per phase oZp}}$ ...(5.28)

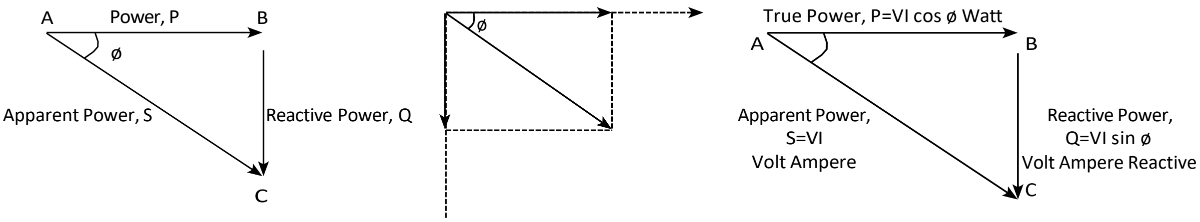
Electric Power

Power is the rate at which work is performed, or the rate at which energy is expended. Work is often expressed in joules. In electrical terms, one joule of work is accomplished when a voltage of one volt causes one coulomb of electrons to pass through a circuit. When this amount of work is accomplished in one second, it is equal to one watt. Most of the times, electrical equipment are rated in watts. A watt is the basic unit of power. One watt is also defined as the amount of work that is accomplished when a voltage of one volt causes one ampere of current to pass through a circuit. This relationship between power, voltage, and current is expressed by the following formula:

Power = Volts × Amperes
or
P = V × I
In terms of other Ohm's Law components, the formula for power can be represented in two other ways as follows:
$P = I^2 R$ or $P = V^2/R$ ...(5.29)
where P is power in watts or volt-amperes (VA), V is voltage in volts, I is current in amperes, R is resistance in ohms.

Power Triangle

Power Triangle is the representation of a right angled triangle whose sides represent the active, reactive and apparent power. Base, perpendicular and hypotenuse of this right angled triangle denotes the active, reactive and apparent power respectively. When each component of the current that is the active component (I cos ϕ) or the reactive component (I sin ϕ) is multiplied by the voltage V, a power triangle is obtained shown in the Fig 5.30.



(a) (b) (c)

Fig. 5.30: Power Triangle

Fig 5.30 (a) shows a power triangle. Side AB, BC and AC represents P, Q and S respectively. The power triangle is obtained from the phasor diagram shown at Fig 5.30 (b). The power which is actually consumed or utilized in an AC Circuit is called true power or active power or real power. The unit is MW. The power which flows back and forth means it moves in both the direction in the circuit or can be reacted upon it is called Reactive Power. The reactive power is measured in kilovolt-ampere reactive (KVAR) or MVAR. The product of root mean square (RMS) value of voltage and current is known as apparent power. This power is measured in KVA or MVA. ...(5.30)

The relationship between these quantities is explained by graphical representation called Power Triangle shown in Fig 5.30(c).

- When an active component of current is multiplied by the circuit voltage V, it results in active power. It is this power which produces torque in the motor, heat in the heater, etc. This power is measured by the wattmeter.
- When the reactive component of the current is multiplied by the circuit voltage, it gives reactive power. This power determines the power factor, and it flows back and forth in the circuit.
- When the circuit current is multiplied by the circuit voltage, it results in apparent power.
- From the power triangle shown above the power, the factor may be determined by taking the ratio of true power to the apparent power.

$$\text{power factor} = \frac{Active\ Power}{Apparent\ power} = \frac{KW}{KVA}$$

Basically, power means the product of voltage and current, but in AC circuit except for pure resistive circuit there is usually a phase difference between voltage and current and thus VI does not give real or true power in the circuit.

∴ True power, $P = VI \cos ø$. ...(5.31)

For a pure inductance or a pure capacitance, the power consumed in the circuit is zero, as phase angle is 900. However, in the case of pure resistive circuit the power consumed which is given by P = VI watts where V and I are the r.m.s. values of voltage and current.

5.1.3 Power in Three Phase Connection

The power consumed in each phase for star and delta connections is $V_p\, I_p \cos\Phi$. The total power in the circuit is the sum of the three phase powers.

∴ Total power consumed is given by $W = 3V_p\, I_p \cos ø$ ...(5.32)

Now $I_p = I_L$; $V_p = V_L/\sqrt{3}$ for star connection and $V_p = V_L$; $I_p = I_L/\sqrt{3}$ for delta connection

Converting these phase values of V_p and I_p into line values i.e. V_L and I_L, the above expression for total power in both star as well as delta connection becomes

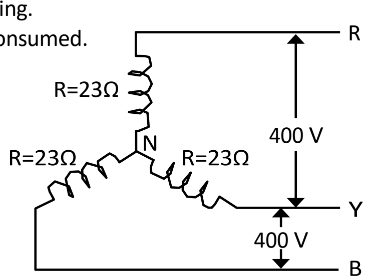
$W = \sqrt{3}\ V_L I_L \cos ø.$ ...(5.33)

Activity

Each batch will visit a nearby sub-station or industry and observe the arrangements for the 3- phase power supply and the power factor improvement. Each batch will prepare a brief report based on their observation.

Solved Problems

Example 5.3.1: Observe the circuit shown with the given data and determine the following.
(a) Phase current, (b) line current (c) power factor of each phase, and (d) Total power consumed.



Solution : It is seen that line voltage V_L = 400 Volts
Resistance per phase R = 23 ohms
Now, phase voltage $V_p = 400/\sqrt{3} = 230$ volts
Current, $I_P = V_p/R = 230/23 = 10$ amps
Now, calculating line value of current from phase value
$I_L = \sqrt{3} \times I_P = 17.3$ amps.

TRANSFORMER AND MECHINES

TRANSFORMER

Introduction

One of the most important advantages of an alternating current over direct current is the extreme ease with which the transformation from a low voltage to high voltage or vice versa can be accomplished with the help of transformers. The transformer is a static device (with no rotating parts) which transfers electrical energy from one alternating current circuit to another with the desired change in voltage or current level and without any change in frequency. The high-voltage long-distance transmission with the help of transformers has made possible the utilization of electrical energy generated in one geographical region to load centres located in other region.

The transformers are designed to operate either on single-phase or on three-phase supply and accordingly are known as single-phase or three-phase transformers. The discussion in this unit is confined to the single-phase transformers only. The three-phase transformers, however, work on similar principle as single-phase transformers.

Parts of a Transformer

A transformer mainly consists of the following parts: The first part consists of the limbs, yokes and clamping structure forming the magnetic circuit and the second part i.e. the electrical circuit consists of the primary winding, the secondary winding and insulation. With the increase in the size (capacity) and the operating voltage, there are several other parts such as tank body, bushings, conservator, breather, explosion vent, Buchholz relay, tapping switches etc. Fig. 6.1 shows the constructional details of transformer.

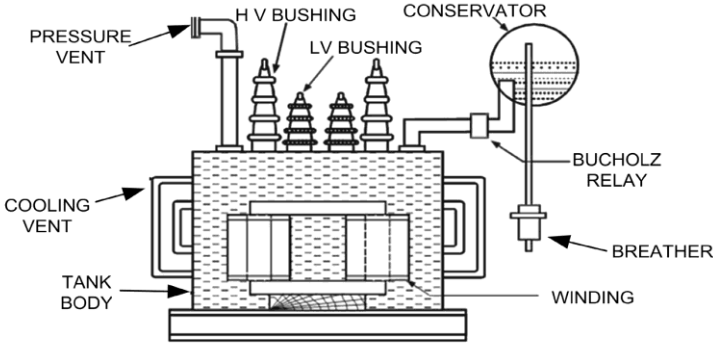


Fig. 6.1: Front view of a transformer

a. Core and Windings: The core of a transformer is made up of magnetic material and is used to provide the path of low reluctance for the flux. The lesser the reluctance of the magnetic circuit, the stronger is the field. The material actually used for the core is high grade silicon steel in the form of laminations about 0.35 to 0.5 mm thick. These laminations are varnished or coated with enamel to insulate them from each other.

The coils forming the primary and the secondary winding are former wound using well insulated copper conductor in the form of round wire or strip. These coils are then placed around the limbs of the core. These windings are insulated from each other and the core using cylinders of insulating material such as press board or Bakelite.

In the elementary transformer, the primary and secondary windings are shown on separate limbs of the core for simplicity. However, if such an arrangement is used in actual practice, all the flux produced by the primary winding will not link with the secondary winding as some of the flux will leak out through air. Such flux is known as leakage flux. More the value of the leakage flux, poorer is the performance of the transformer. Therefore, to reduce this leakage flux, the primary winding and the secondary winding are placed together on the same limb in actual transformer. These windings are either cylindrical in form or sandwich type as shown in Fig. 6.2.

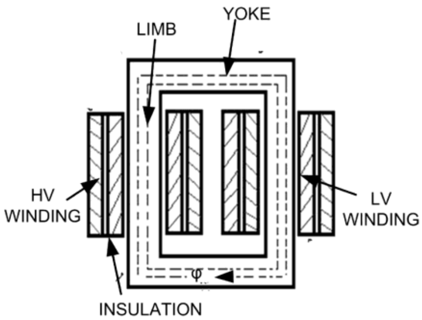


Fig. 6.2: Sectional view of a single phase core type transformer

b. Transformer Tank: In transformers with ratings more than 50 kVA, the whole transformer assembly i.e. the winding and core is placed in a fabricated sheet metal tank and immersed in the oil which serves both the purposes of providing insulation and cooling. The heat generated in the windings and the core is carried by the oil to the external surface of the tank. Cooling tubes are provided to increase the surface area of the tank for more effective cooling.

c. Terminal Bushings: The terminals of the primary and secondary windings of the transformer are brought out from the tank and are insulated from the tank body with the help of porcelain bushings. These bushings are fitted to the tank.

d. Conservator: In a transformer, provision of some space above the oil level is always essential to take up the expansion and contraction of the oil with changes of temperature in service. When the transformer becomes warm, the oil expands and the air at the top of the oil is expelled. When the transformer cools, oil contracts and outside air is drawn into the transformer. This process is known as breathing of the transformer. Unless proper precautions are taken, the outside air which enters the transformer during this process can have considerable moisture. When the oil in the transformer is exposed to such moist air, it readily absorbs the moisture from the air and looses its insulating value to some extent. This deterioration of oil can be prevented by using a conservator. The conservator is an airtight cylindrical metal drum supported on the transformer tank. This drum is connected by pipe to the transformer tank and is always partly filled with oil. The expansion and contraction of the oil in the main tank with the changes of temperature is now taken up by the conservator. With this arrangement, since the main tank remains always full with oil, the surface of the oil is not directly exposed to air.

e. Breather: The displacement of air above the oil level in the conservator during the breathing process of the transformer takes place through the apparatus known as a breather. It contains a drying agent, such as calcium chloride or silica gel, which extracts the moisture from the air. The breather also cleans the air by removing the dust particles present in it. Thus, only dry and clean air is allowed to come in contact with the oil in the transformer.

f. Buchholz Relay: It is a type of protective device mounted in the pipeline connecting the main tank to the conservator. During fault conditions excessive heat is developed due to losses in the winding, the oil in the tank in the vicinity of the winding gets decomposed and different types of the gases are liberated. These gases operate the Buchholz relay which in the initial condition gives alarm to the operator. If the fault developed is converted into a serious type of fault, then this relay trips off the main circuit breaker.

g. Explosion Vent: The bent up pipe fitted on the upper surface of the tank is known as explosion vent or relief valve. It is provided with a diaphragm made out of glass sheet or aluminium foil sheet. In the event of the fault condition, if excessive pressure is developed inside the tank due to vaporization of cooling oil, the diaphragm in the explosion vent bursts and releases the pressure, thus avoiding damage to the transformer.

Types of Transformers

Depending upon the arrangement of the core and the windings, there are two main types of the transformer: Core type and Shell type. Fig. 6.3 shows the two types of transformers.

a. Core Type Transformers: The distinguishing features of a core type transformer are as follows:

- The core type transformer is built of laminations to form a rectangular structure as shown in Fig. 6.3 (a) and provides a single magnetic circuit.
- The winding coils are normally cylindrical in form and concentric to reduce wastage, the low voltage winding being placed near the core. These windings surround considerable portion of the core
- The primary/ secondary or the low voltage/ high voltage windings are uniformly distributed over two limbs of the core.
- The windings being distributed on two limbs; the natural cooling becomes much more effective.
- The coils can be withdrawn for repair just by dismantling the top yoke.

b. Shell Type Transformers: The salient features of a shell type transformer are listed below:

- The core of this type of transformer provides double magnetic circuit.
- The windings are normally sandwich type, always placed on the central limb of the core.
- The H.V. and L.V. coils are wound in the form of pancakes and are interleaved. The top and bottom coils which are near the yoke of the core are of L.V. winding only.
- The core nearly surrounds the windings placed on the central limb of the core. The feature helps in providing mechan ical protection to the windings.
- The coils being placed on the central limb only and are surrounded by the outer core limbs, the natural cooling is therefore poor.
- The repair of coils is not as simple as it is for core type transformers.

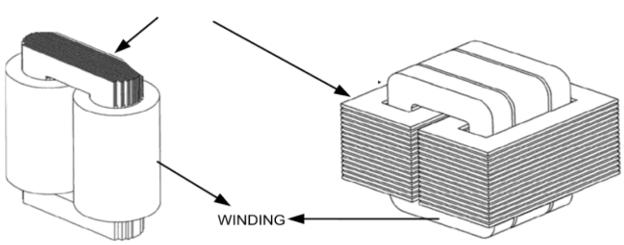


Fig. 6.3: (a) Core type transformer. (b) Shell type transformer

Principle of Working

The operation of the transformer is based on the principle of mutual induction between two circuits linked by a common magnetic field. Consider the transformer in its elementary form shown in Fig. 6.4.

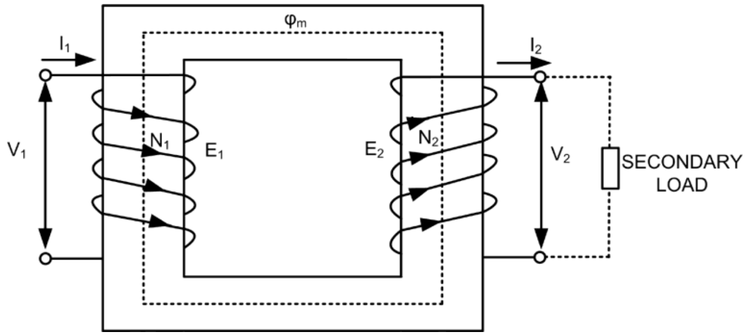


Fig. 6.4: Elementary Transformer

It essentially consists of two windings, primary and secondary winding, electrically separate but wound on a common laminated steel core. The vertical portions of the core on which these windings are placed are called as the limbs and the top and bottom portions are the yokes. The winding which is connected to the existing supply system and which receives energy from it is called as the primary winding. The other winding delivering energy to the load at the desired voltage is called the secondary winding.

When the primary winding is connected to an AC supply, an alternating current circulates through it. This current flowing through the primary winding produces an alternating flux. Most of this varying flux links with the secondary winding through the iron core and induces an emf in it in accordance with Faraday's law of electromagnetic induction. The phenomenon, due to which an alternating current in the primary winding produces an emf in the secondary winding, is known as mutual induction and the emf induced in the secondary winding is known as mutually induced emf. The frequency of this emf is the same as that of the supply voltage.

EMF Equation of Transformer

Suppose a transformer having N_1 and N_2 number of turns in the primary and secondary winding as shown in Fig. 6.4. When an AC voltage V_1 of frequency f is applied across the primary winding, a current I_m will flow through the primary winding and this will produce an alternating flux which complete its path through the core linking both the primary and secondary winding. The equation of the alternating flux is

$$\varphi = \varphi_m \cos\omega t \qquad ...(6.1)$$

As per Faraday law the induced emf equation in primary winding due to the alternating flux is given

$$e_1 = -N_1 \frac{d\varphi}{dt} \qquad ...(6.2)$$

Substituting the value of flux of equation 6.1.1 in 6.1.2 the equation becomes

$$e_1 = -N_1 \frac{d\varphi_m \cos\omega t}{dt}$$

$$e_1 = N_1 \omega \varphi_m \sin \omega t$$

$$e_1 = N_1 \omega \varphi_m \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$\text{or } e_1 = E_m \cos\left(\omega t - \frac{\pi}{2}\right) \qquad ...(6.3)$$

where $E_m = 2\pi f N_1 \varphi_m$, the maximum value of the induced emf

The root mean square value of the induced emf in the primary winding is given by

$$E_1 = \frac{2\pi f N_1 \varphi_m}{\sqrt{2}} = 4.44 f N_1 \varphi_m \qquad ...(6.4)$$

Similarly, the emf in the secondary winding is given by

$$E_2 = 4.44 f N_2 \varphi_m \qquad ...(6.5)$$

Voltage Transformation ratio

Voltage Transformation ratio is defined as the ratio of the secondary voltage to the primary voltage. It is denoted by K. If K < 1, then the secondary voltage will be less than the primary voltage and the transformer will be called as step down transformer. If K > 1, then the transformer is a step up transformer.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{K}$$

In an ideal transformer, the following assumptions are made:

- Winding resistance are negligible
- All the flux produced in confined to the core of the transformer and links fully both the windings.
- The permeability of the core is high so that the magnetizing current required to produce the flux and establish it in the core is negligible.
- Hysteresis and Eddy current losses are negligible.

With the above assumption, the input volt ampere and output volt ampere of a transformer can be approximated as equal i.e. $V_1 I_1 = V_2 I_2$. The above equation becomes

$$\frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} = \frac{1}{K} \qquad ...(6.6)$$

Transformer Under No Load Condition

When a transformer is under no load condition, the current I_2 in the secondary winding as shown in Fig. 6.4 is zero while the primary winding carries a small current I_0 known as no load current . The current I_0 consists of following two components.

a. A reactive or magnetising component I_m and
b. An active or power component I_μ.

The magnetising component produce the magnetising flux, so it is in phase with the flux. The active component produces the power to supply the hysteresis and eddy current losses in the iron core, the active component is in phase with the applied voltage V_1. The induced emf E_1 in the primary winding lags the magnetizing flux by 90^° as shown in equation 6.1.3. Normally the active component is very small compared to the magnetising component of no load current. Fig. 6.5 shows the phasor diagram at no load condition of a transformer

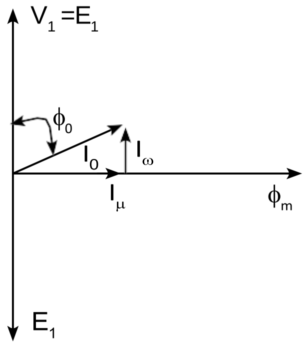


Fig. 6.5: Phasor diagram at No load

From the phasor diagram the magnetizing current $I_\mu = I_0 \sin Ø_0$ and the core loss component $I_\omega = I_0 \cos Ø_0$. The power input to the transformer at no load condition is given by

$$P_o = V_1 I_0 \cos Ø_0 \qquad ...(6.7)$$

where $\cos Ø_o$ is the no load power factor and the no load current I_0 is given as

$$I_0 = \sqrt{I_\mu^2 + I_\omega^2} \qquad ...(6.8)$$

Autotransformer

A transformer in which a part of the winding is common to both the primary and secondary circuit is known as an auto transformer. The primary is electrically connected to the secondary as well as magnetically coupled to it as shown in Fig. 6. 6. Unlike a two winding transformer, an autotransformer is not electrically isolated.

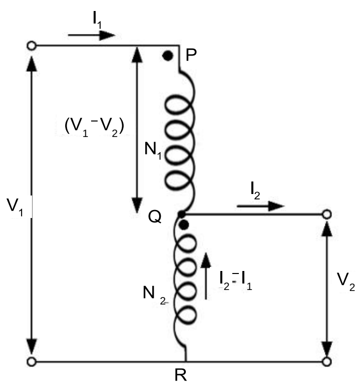


Fig. 6.6: Autotransformer

In Fig. 6.6, PR is the primary winding having N_1 turns and QR is the secondary winding having N_2 number of turns. The input voltage and current are V_1 and I_1 and the output voltage are V_2 and I_2 respectively. If the internal impedance drop &losses are neglected, then $V_1 I_1 = V_2 I_2$ or

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = \frac{N_1}{N_2} = \frac{1}{K} \quad ...(6.9)$$

The current in the section QR is $(I_2 - I_1)$ where $I_2 > I_1$

In an auto transformer only a part of the power input is transferred from the primary to the secondary side by transformer action. The remaining power is transferred directly from the primary to the secondary side. The relative amount of power inductively transferred and power conductively transferred depends upon the ratio of transformation.

Let the volt ampere power delivered to load by the auto transformer = $V_2 I_2$

The power transformed which is equal to the power in winding QR. The transformed power or the inductive power is given as

$$V_2(I_2 - I_1) = V_2 I_2\left(1 - \frac{I_1}{I_2}\right)$$

Substituting equation 6.9, the equation = $V_2 I_2(1 - K)$ ...(6.10)

The power that is conducted directly equals the power delivered to load minus the power transformed and is equal to

$$V_2 I_2 - V_2 I_2(1 - K) = K V_2 I_2 \quad ...(6.11)$$

Following are the advantages of an autotransformer when compared with a two winding transformer

- For the same capacity and voltage ratio the weight of copper required for an autotransformer is less.
- The size of an autotransformer is less for the same rating.

Applications of Autotransformer

1. To compensate for voltage drops on long feeder circuits.
2. To provide variable voltage control.
3. To adjust the transformer output voltage in order to keep the system voltage constant with varying load

Activities

1. Visit the college main power supply substation. Note down the name plate details of the distribution transformer installed. Prepare a report on the specification details given in the name plate.
2. Measure the primary and secondary winding resistance of a given 1-phase two winding transformer. Note down the resistance value and infer which winding is a HV winding.

Example 6.1.1: A single phase transformer has 400 primary and 1000 secondary winding turns. The cross sectional area of the core is 60 cm^2. Determine the peak value of the flux density in the core if the primary supply voltage is 500V, with frequency = 50 Hz.

Solution: The rms value of the induced emf in the primary winding equals $E1 = 4.44\ fN1\ \phi_m$.

Considering an ideal transformer $E_1 = V_1$,

Given, the primary supply voltage = 500 V.

Therefore, $500 = 4.44 \times 50 \times 400 \times B_m \times 60 \times 10^{(-4)}$, $B_m = 0.938\ ^{Wb}/m^2$

Example 6.1.2: A 200/100 V, 50 Hz transformer to be excited at 40 Hz from the 100 V side. Find the voltage to be applied at the low voltage side if the exciting current to remain same.

Solution: Let the induced emf equation at 100 V side at 50 Hz equals $100 = 4.44 \times 50 \times \phi_m \times N_2$ .... (1)

Given the exciting current I_μ has to remain same at 40 Hz also, i.e. ϕ_m to remain same.

The emf equation at 40 Hz = $E2 = 4.44 \times 40 \times \phi_m \times N2$, ... (2)

Equating equation (1) and (2) E2 = 80 Volts.

ELECTRIC MOTORS

Introduction

A rotating electrical machine consists mainly of two parts, the stator the stationary part and rotor, the rotating part. The stator is generally a cylindrical shaped magnetic core and the rotor again made of magnetic core rotates inside the stator. The stator and rotor core are separated by means of an air gap. The stator and rotor magnetic carries winding to establish a magnetic flux. The rotor is mounted on a bearing supported shaft and the shaft is connected to the mechanical loads by means of belt and pulley arrangement or through gear boxes.

DC Motor

An electric motor is a machine which coverts electrical energy into mechanical energy. If the electric energy is supplied in form of DC supply, the motor is called DC motor.

Construction of DC Motor

The field poles of a DC machine are located on the stator. The iron poles are projected inwards from the inside surface of the cylindrical shaped magnetic core called the stator yoke. The yoke serves as a return path for the magnetic flux. The iron pole consists of a narrow portion on which the field winding coils are placed. A pole shoe usually laminated distribute the pole flux over the rotor surface. The rotor or armature made of cylindrical silicon steel core consists of a stack of slotted laminations. The slots are cut on the surface of the laminated core along the axial length of the core, in which the coil sides of the armature winding are placed. The coils in the form of conductor wire or bars are made of copper or aluminium and the conductor size depends on the current and voltage requirement of the machine. The armature coils are held in place by wood wedges driven into the slot along the slot length. The coil terminal ends are connected to the commutator. The commutator consists of segments made of copper, the segments separated from each other by insulating material usually mica.

The current is conducted to the armature coils by carbon brushes. The brushes are held in brush holder and is fitted in such a way that they should slide freely over the commutator surface. To maintain proper contact between the brush contact and commutator, adjustable springs are placed in the brush holder assembly to ensure the contact force. The brushes must be inspected regularly and replaced if wear and tear of the brush occurs. Fig. 6.7 shows the sectional view of a DC machine.

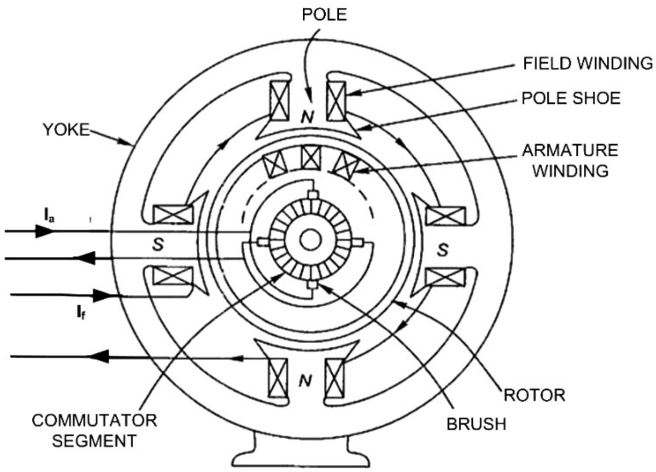


Fig. 6.7: Sectional view of a DC machine

Working Principle of DC Motor

The principle on which the DC motors work is based on Fleming's left hand rule. When a current carrying conductor is placed in a steady magnetic field, such that the conductor makes right angle with the field, it experiences a mechanical force, whose direction is given by the Flemings left hand rule. The movement of the conductor is in the direction of force. In short, when electric fields and magnetic fields interact, a mechanical force arises. The magnitude of the mechanical force in Newton experienced by the conductor is given by eq. 6.12.

$$F = BIL \qquad ...(6.12)$$

where, B is the field strength in $^{Wb}/m^2$, I is the current flowing through the conductor in amperes and L is the length of conductor in metres.

Working of a DC Motor

When direct current is passed through armature and field-winding of a DC motor, magnetic flux is established by the field current (Ampere turns). Since the armature conductors are perpendicular to the magnetic field and they are carrying current, they experience mechanical force. The resultant of these forces is a torque. Under the influence of this torque rotor starts rotating. Any mechanical device(load) coupled to it does useful work. If the mechanical load is increased more torque will be produced by drawing more current from DC supply. Thus motor converts electrical energy into mechanical energy.

Back EMF: When armature of a motor rotates, an emf is induced in the conductors as they cut the lines of magnetic force. The induced emf is in opposition to the applied voltage (V) and is called back or counter emf (E_b). Its magnitude is given by

$$E_b \propto \phi N \qquad ...(6.13)$$

where Ø is the field flux and N the armature speed.

Types of DC Motors

Depending on the nature of connection of armature winding and field windings, DC motors can be classified into two types:

i. DC series motor
ii. DC shunt motor

Another type of DC motor is the DC Compound motor, in which field winding is connected in series as well as in parallel that is not being discussed in this book.

i. **DC series motor:** A series motor is one in which field winding is connected in series with the armature as shown in Fig. 6.8 that the current drawn by the motor passes through the field winding as well as armature Field winding has a few turns of thick conductors. Magnetic flux varies with current till saturation.
ii. **DC shunt motor:** A shunt motor is one in which the field winding consisting of large number of turns of comparatively thin wire is connected in parallel with armature as shown in Fig. 6.9. In the case of shunt motor, the field current is costant because of the DC supply is constant. Therefore, flux remains practically constant.

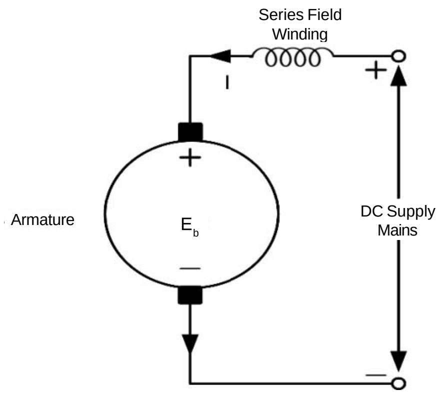


Fig. 6.8: DC series motor

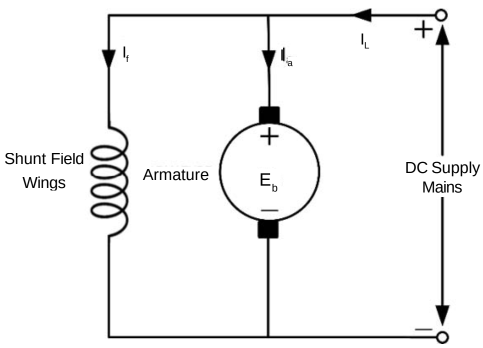


Fig. 6.9: DC shunt motor

Characteristics of DC Motors

The two most important characteristics of a DC motor are the torque characteristics and the speed characteristics.

a. Torque Characteristic (T vs. I_a): The torque characteristic represents the variation of torque with armature current. The torque developed in a motor is the result of the interaction between the magnetic flux produced by field current and the current flowing through armature conductor. If the magnetic flux increases due to increase in field current, torque produced for the same armature current will increase i.e. $T \alpha\ Ø$ for I_a constant. Similarly, if the armature current increases because of increase in shaft load, then also torque will increase for the same value of magnetic flux i.e. $T \alpha I_a$ for Ø constant. Now if both Ø and I_a are changing then in general, it can be written as

$$T \alpha\ ØI_a \quad ...(6.14)$$

i. DC series motor: The torque equation is given as $T \alpha\ ØI_a$. For a series motor as shown in Fig. 6.8, the same current flows in the field winding as well as in the armature winding. So, up to magnetic saturation, the field flux $Ø \alpha I_a$ and therefore the torque developed is

$$T \alpha I_a2 \quad ...(6.15)$$

This means that the torque is proportional to square of the current up to magnetic situation.

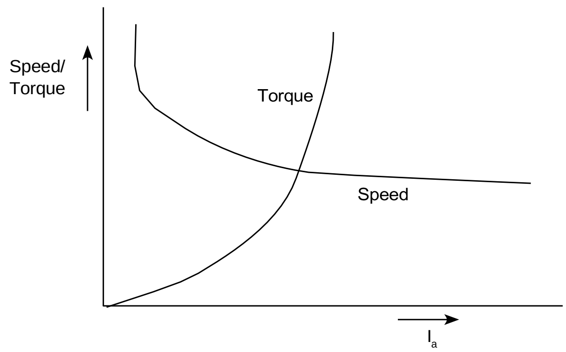


Fig. 6.10: Speed-Torque characteristics of a DC series motor

This part of the characteristic is a parabola. However, after magnetic saturation, T vs. I_a curve becomes straight line because flux Ø becomes independent of armature current and hence torque increases with armature current only. The characteristic curve is shown in Fig. 6.10. Since the torque is proportional to the square of current, the starting torque is extremely high. The high starting torque is advantageous for certain applications. Hence DC series motors are used where large starting torque is required.

ii. DC Shunt motor: In case of a DC shunt motor, the flux Ø is constant Hence the torque $T \alpha\ ØI_a$ is directly proportional to the armature current, whatever the speed may be. As armature current (I_a) increases, torque (T) increases and vice-versa. Fig. 6.11 shows the torque characteristic of a DC shunt motor.

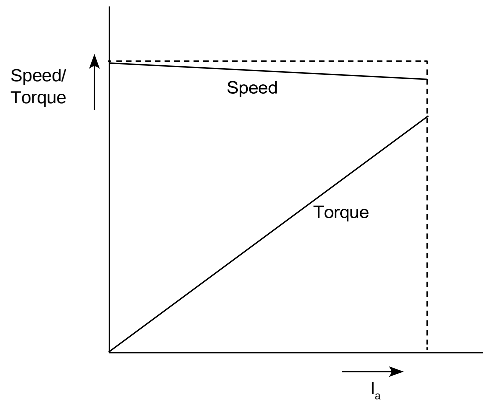


Fig. 6.11: Speed - Torque characteristics of a DC shunt motor

b. Speed characteristic (N vs. Ia):

The running or speed characteristic of a motor normally represents the variation of speed with input current.

i. DC Series motor: The speed equation of a DC motor is $N \propto \frac{E}{\emptyset}$

where, E_b is the back emf, Ø is the flux and N is the motor speed in rpm. For very low armature resistance the change in back emf, for different load currents is very small and so can be neglected.

Therefore, the rotor speed is inversely proportional to the field flux or $N \propto \frac{1}{\phi}$ ...(6.16)

In a DC series motor, the flux (Ø) increases, with increase in armature current, that is, $\emptyset \,\alpha\, I_a$, the equation 6.16 modifies to

$$N \propto \frac{1}{I_a} \qquad \text{...(6.17)}$$

This means that as load current i.e. the armature current (I_a) increases, the speed decreases and vice-versa. The characteristic is shown in Fig. 6.10. From the characteristic curve it is seen that, when load is large the speed is low. When the load is light, the speed is very high. Therefore, series motor should never be run without load otherwise it may get damaged due to very high centrifugal force.

ii. DC shunt motor: In DC shunt motor, the flux remains constant. As flux is constant, speed is also constant. Theoretically it is true but practically it is not possible. Actually, as the load is increased, the back emf (E_b) decreases and due to this fact, speed N = Eb/Ø decreases slightly. This decrease in speed is not significant and therefore DC shunt motor for all practical purposes is considered as a constant speed motor.

Applications of DC Motors

DC motors are used for many industrial applications, particularly those requiring constant torque across the motor's entire speed range. In portable applications using battery power, DC motors are a natural choice. The main applications of DC Series Motors and DC shunt motors are as follows:

i. DC series motors

The DC series motors are used where high starting torque is required, where constancy of speed is not required. and variations in speed are possible. Some of the applications of series motors are:

- Cranes
- Air compressor
- Lifts and Elevators
- Vacuum cleaner
- Electric traction
- Hair drier
- Sewing machine
- Power tools
- Electric footing, etc.

ii. DC shunt motors

The shunt motors are used where constant speed is required from no load to full load. and starting conditions are not severe. The various applications of DC shunt motor are :

- Lathe Machines
- Centrifugal pumps
- Conveyors
- Fans
- Boring machines
- Weighing machine
- Spinning machine
- Blowers
- Line shaft, etc.

AC Motors

Motor Construction

From the previous topic, the performance of a DC motor in terms of its characteristics and its applications has been studied. For operation of a DC motor, DC power supply is required. For this the AC supply is rectified to make it a DC supply by using semiconductor devices.

It would be more convenient if the single or three phase AC power supply can be used to directly drive a AC motor. Like a DC motor, the AC motor has also stator and rotor. A large number of identical slots are cut on the stator on which coil sides are placed. The coil ends are connected and the leads are brought out, depending on whether the type of AC supply is single phase or three phase. Accordingly, the motor is classified as 3-phase or 1-phase AC motor.

The rotor construction depends on the type of AC motor. Fig. 6.12 shows the rotor construction of a 3-phase induction motor.

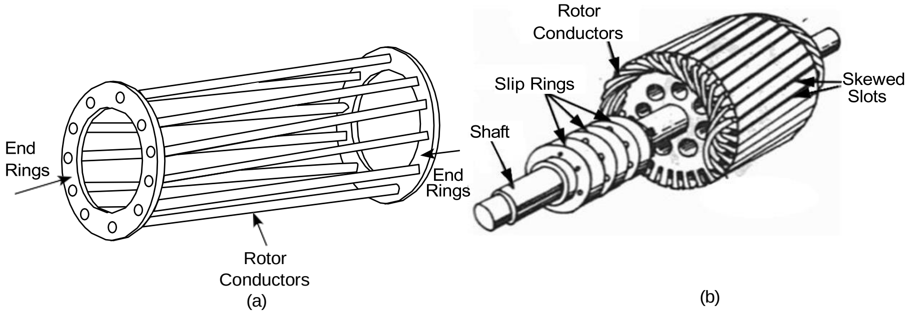


Fig.6.12: 3- Phase induction motor (a) Squirrel cage (b) Wound rotor

Table 6.1 shows the rotor construction details of two main types of 3 phase AC motors i.e. induction motor and synchronous motor. further, there are two types of 3 phase induction motor on the basis of construction : squirrel cage and wound rotor. similarly, synchronous motors are of two types on the basis of construction of rotors: salient pole rotors and non-salient pole rotors.

Table 6.1: Rotor construction details of 3-phase AC Motor

Sr. No.	Type of 3-phase AC motor	Type of rotor and their construction details	
1.	**Three phase Induction Motor**	**Squirrel Cage**	**Wound Rotor**
		i. Rotor core is cylindrical with slotted periphery. ii. The rotor conductor is made of uninsulated bars or rods made of copper or aluminum. iii. The bars are permanently shorted at each end with the help of conducting rings made of copper and are known as end rings.	i. Rotor core is cylindrical with slotted periphery. ii. The rotor winding is similar to the stator winding. iii. The three ends of the three phase rotor winding are permanently connected to the slip rings iv. The slip rings are mounted on the rotor shaft v. The rotor terminals are brought out for external connections through brushes mounted on brush holders placed on the slip rings

2.	Three phase Synchronous Motor	Salient pole rotors	Non salient pole rotors
		i. The term salient means projecting. A salient pole consists of poles that are projected out from the surface of rotor core. ii. Used for rotors with more than four poles. iii. The rotor winding coils known as field coils are placed on the pole body. iv. The two ends of the field windings are connected to the slip rings and connected externally to DC supply through brushes.	i. Non salient pole rotors are also known as cylindrical rotor. ii. The rotor is cylindrical in shape with no physical pole as in salient pole construction. iii. Slots are cut on the rotor periphery to place the rotor or field winding. iv. External connection of the field winding is same as that of salient pole rotor.

Three phase AC Motor

The fundamental principle of operation of AC machines is the generation of a rotating magnetic field. When a three phase balanced supply is given to the three phase coils placed on the stator slots which are space displaced by 120° a rotating magnetic field is created. The rotating magnetic field causes the rotor to turn at a speed that depends on the speed of the rotating magnetic field. The speed of the rotating magnetic field known as synchronous speed and is given as

$$N_s = \frac{120f}{P} \qquad ...(6.18)$$

where f is the frequency of the AC supply and P the number of poles present in the stator. The electromagnetic torque developed in a motor is the interaction of the two magnetic fields in the air gap, F_s created by the stator currents and F_r created by the rotor currents. The torque equation is given by

$$T = F_s\, F_r \sin\lambda \qquad ...(6.19)$$

For creation of a steady torque the following two conditions must be fulfilled i.e.

c. The two fields must be stationary with respect to each other and

d. The two fields must have the same number of poles.

Fig. 6.13 shows the torque interaction of an AC machine.

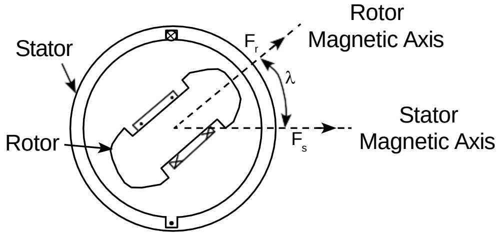


Fig. 6.13: Torque in round rotor machine

The three phase induction motor is a singly fed machine, the stator is excited from AC mains. The current flowing in the three phase stator winding give rise to a constant rotating magnetic field. As per Faraday's law the rotating magnetic field induces voltage in the stationary rotor conductors which is short circuited. The induced voltage circulates current in the rotor conductors which produce rotor magnetic field and the interaction of the rotor and stator magnetic field give rise to torque and the rotor starts rotating in the direction of magnetic field produced by stator winding according to Lenz's law. The rotor frequency automatically adjusts in accordance with the rotor speed, thus fulfilling the first condition required for a steady torque. In a synchronous machine, the stator carries the alternating current, while the rotor is DC excited. The two fields would be relatively stationary, causing torque production if and only if the rotor runs at synchronous speed i.e. the speed of the rotating magnetic field produced by stator. Fig. 6.14 shows the torque versus speed characteristics of a 3-phase induction motor and synchronous motor.

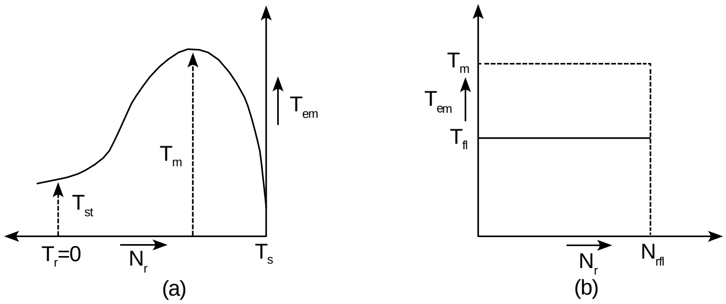


Fig. 6.14: Torque-Speed Characteristics of (a) 3-Phase Induction motor (b) 3-Phase Synchronous motor

Applications of 3-phase Motors

i. 3-phase Induction Motor

- Squirrel cage induction motor has not a very good conventional speed control method, so it used for constant speed applications.
- The three phase squirrel cage induction motor is used for centrifugal pump, milling machines, lathe machines, drilling machines, and large blowers and fans.
- The Slip ring induction motor has a high starting torque and good speed control method, so it can operate a high load with less speed.
- The slip ring motor is used for high load applications like elevators, cranes, hoists and equipment's of process industries.

ii. 3-phase Synchronous Motor

- Synchronous motor having no load connected to its shaft is used for power factor improvement. It is used in power system in situations where static capacitors are expensive.
- Synchronous motor finds application where operating speed is less than 500 rpm and high power ranging from 100 kW to 2500 KW is required. Ex- Reciprocating pump, compressor, crusher motors for rotary kiln of cement plants, motors in steel rolling mills etc.

Single phase AC Motors

The three phase AC motors are used for high power rating applications. Generally, the single phase AC supply is available to most homes and offices. This has led to the availability of a wide variety of small size motors or fractional horse power motors for domestic applications like fan, refrigerator, room air conditioners, kitchen and office equipment's etc. A single phase induction motor comprises a single phase winding on the stator and a squirrel cage rotor. Due to the pulsating magnetic field produced the single phase induction motors are not self-starting. To overcome this problem, 2-winding single phase motors are developed, in which the two windings, named as main and auxiliary winding are placed at 90° electrical in space, but are fed from single phase supply. The time difference in winding currents so as to develop a rotating magnetic field is obtained by placing suitable impedance in series with the auxiliary winding. Depending on the method of phase splitting the 2 –winding single phase motors are classified as Resistance split phase motor and Capacitor split phase motor.

Capacitor Split Phase AC Motors

The most widely used single phase AC motors for household applications. The capacitor split phase motors are classified as Capacitor start induction motor, Permanent split capacitor motor and capacitor start capacitor run motor. The connection diagram of a permanent split capacitor induction motor is shown in Fig. 6.15.

In order to make the single phase induction motor self-starting a capacitor is connected in series with the auxiliary winding. The auxiliary winding is generally made of thin copper wire as compared to the main winding which is made of thick copper wire. The two windings are connected across the single phase supply. The current I_a flowing through auxiliary winding leads the main winding current I_m due to the capacitor as shown in the phasor diagram. Thus the motor becomes a 2-phase motor with the main and auxiliary winding displaced electrically in space by 90° electrical. A starting torque is created and the rotor starts rotating. The typical rating of the capacitor is 40 - 100 µF.

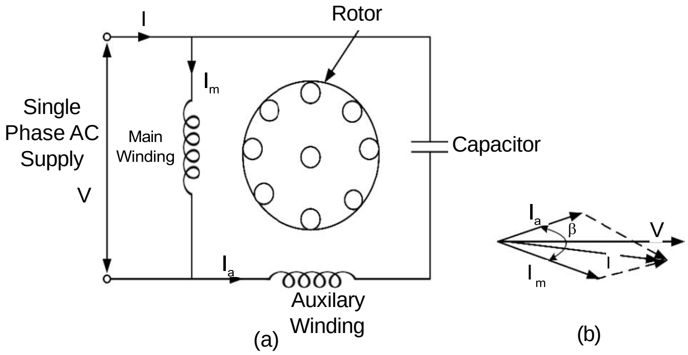


Fig. 6.15: 1-Phase capacitor split phase motor (a) Connection diagram (b) Phasor diagra