Lecturer Notes

on

Control System Engineering

6^{th} Semester

Submitted By: -

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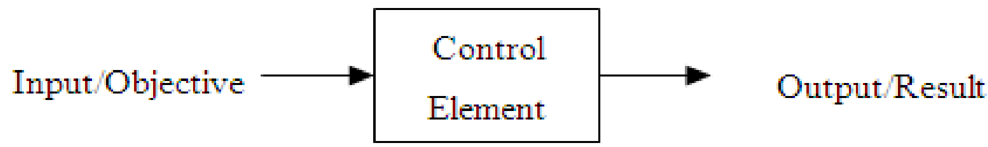
UNIT-I

CONTROL SYSTEMMODELING

Basicelementsofcontrolsystem

In recent years, control systems have gained an increasingly importance in the development and advancement of the modern civilization and technology. Figure shows thebasic components of a control system. Disregard the complexity of the system; it consists of an input (objective), the control system and its output (result). Practically our day-to-day activities are affected by some type of control systems. There are two main branches of control systems:

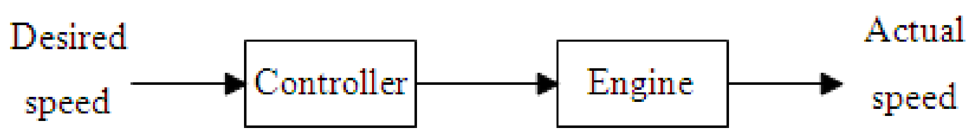
1) Open-loopsystemsand
2) Closed-loopsystems.



BasicComponentsofControlSystem

Open-loopsystems:

The open-loop system is also called the non-feedback system. This is the simpler of the two systems. A simple example is illustrated by the speed control of an automobile as shown in Figure 1-2. In this open-loop system, there is no way to ensure the actual speed is close to the desired speed automatically. The actual speed might be way off the desired speed because of the wind speed and/or road conditions, such as uphill or downhill etc.



BasicOpenLoopSystem

Closed-loopsystems:

The closed-loop system is also called the feedback system. A simple closed-system is shown in Figure 1-3. It has a mechanism to ensure the actual speed is close to the desired speed automatically.

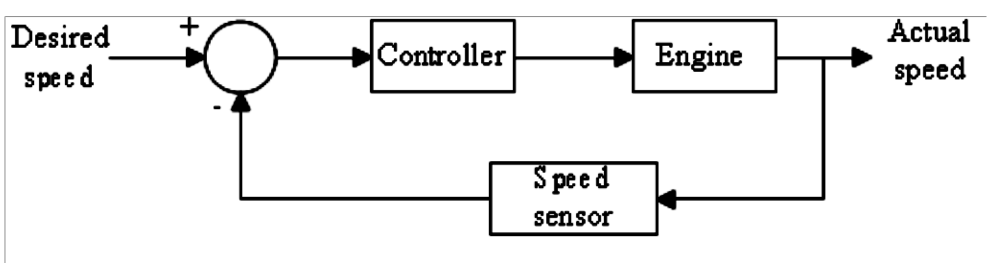


Fig. 1-3. Basic closed-loop system.

Transfer Function

- A simpler system or element maybe governed by first order or second order differential equation. When several elements are connected in sequence, say "n" elements, each one withfirst order, the total order of the system will be nth order
- In general, a collection of components or system shall be represented by nth order differential equation.

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \cdots + b_0 u(t)$$

- In control systems, transfer function characterizes the input output relationship of components or systems that can be described by Liner Time Invariant Differential Equation
- In the earlier period, the input output relationship of a device was representedgraphically.
- In a system having two or more components in sequence, it is very difficult to find graphical relation between the input of the first element and the output of the last element. This problem is solved by transfer function

DefinitionofTransferFunction:

Transfer function of a LTIV system is defined as the ratio of the Laplace Transform ofthe output variable to the Laplace Transform of the input variable assuming all the initial condition as zero.

PropertiesofTransferFunction:

- Thetransferfunctionofasystemisthemathematicalmodelexpressingthedifferential equation that relates the output to input of the system.
- Thetransferfunctionisthepropertyofasystemindependentofmagnitudeandthenature of the input.
- Thetransferfunctionincludesthetransferfunctionsoftheindividualelements.Butatthe same time, it does not provide any information regarding physical structure of the system. Thetransfer functions ofmanyphysicallydifferentsystems shall beidentical.
- Ifthetransferfunctionofthesystemisknown,theoutputresponsecanbestudiedfor various
- types of inputs to understand the nature of the system.
- Ifthetransferfunctionisunknown,itmaybefound outexperimentallybyapplying known inputs to the device and studying the output of the system.

Howyoucanobtainthetransferfunction(T. F.):

- Writethedifferential equationofthe system.
- TaketheL.T.ofthedifferentialequation,assumingallinitialconditiontobezero. Take the
- ratio of the output to the input. This ratio is the T. F.

MathematicalModelofcontrol systems

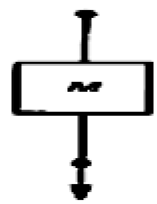
A control system is a collection of physical object connected together to serve an objective. The mathematical model of a control system constitutes a set of differential equation.

1. MechanicalTranslationalsystems

The model of mechanical translational systems can obtain by using three basic elements mass, spring and dashpot. When a force is applied to a translational mechanical system, it is opposed by opposing forces due to mass, friction and elasticity of the system. The force actingon a mechanical body is governed by Newton‘s second law of motion. For translational systems it states that the sum of forces acting on a body is zero.

Forcebalanceequationsof idealizedelements:

Consider an ideal mass element shown in fig. which has negligible friction and elasticity. Let a force be applied on it. The mass will offer an opposing force which is proportional to acceleration of a body.

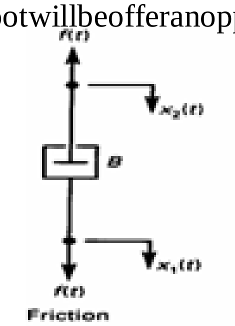


f(t)
Mass

Letf=appliedforce
fm=opposingforceduetomass Here
fm α M $d^2 x / dt^2$

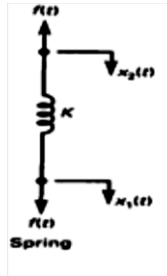
ByNewton‘ssecond law, f=f m= M d^2x/ dt^2

Consider an ideal frictional element dash-pot shown in fig. which has negligible mass and elasticity.Letaforcebeappliedonit.Thedashpotwillbeofferanopposingforcewhichis proportional to velocity of the body.



Letf=appliedforce
fb=opposingforceduetofriction Here, f
b α B dx / dt

ByNewton‘ssecond law, f=f_b=M d x/ dt
Consideranidealelastic elementspringisshowninfig.Thishasnegligible massand friction.



Letf=appliedforce
fk=opposingforceduetoelasticity Here,
f k α x
ByNewton‘ssecond law, f=fk =*x*

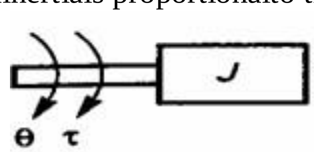
MechanicalRotational Systems:

The model of rotational mechanical systems can be obtained by using three elements, moment of inertia [J] of mass, dash pot with rotational frictional coefficient [B] and torsional spring with stiffness[k].

When a torque is applied to a rotational mechanical system, it is opposed by opposing torques due to moment of inertia, friction and elasticity of the system. The torque acting on rotational mechanical bodies is governed by Newton‘s second law of motion for rotational systems.

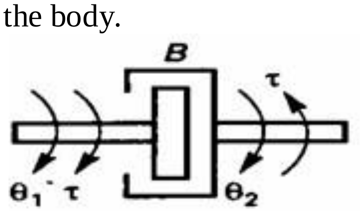
Torquebalanceequationsofidealized elements

Consideranidealmasselementshowninfig.whichhasnegligible frictionandelasticity. Theopposingtorquedue to momentofinertiais proportionalto theangularacceleration.



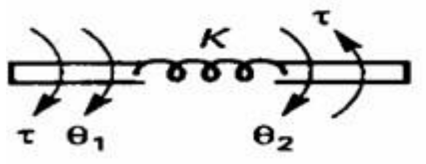
LetT =appliedtorque
Tj=opposingtorqueduetomomentofinertiaofthebody Here
$Tj= \alpha J d^2 \theta / dt^2$
By Newton‘s law
$T=Tj=Jd^2\theta/dt^2$

Consider an ideal frictional element dash pot shown in fig. which has negligible moment of inertia and elasticity. Let a torque be applied on it. The dash pot will offer an opposing torque is proportional to angular velocity of the body.



LetT =appliedtorque
Tb=opposingtorqueduetofriction Here
$Tb = \alpha B d / dt (\theta_1- \theta_2)$
ByNewton‘s law
$T=Tb = Bd / dt (\theta_1-\theta_2)$

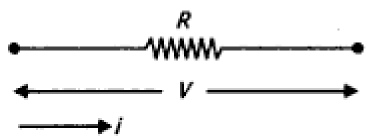
.Consideranidealelasticelement,torsionalspringasshowninfig.whichhasnegligible moment of inertia and friction. Let a torque be applied on it. The torsional spring will offer an opposing torque which is proportional to angular displacement of the body



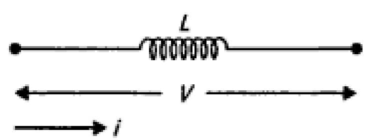
LetT =appliedtorque
Tk=opposingtorqueduetofriction Here
Tk α K (θ_1- θ_2)
By Newton's
lawT=Tk=K(θ_1-θ_2)

Modelingofelectrical system

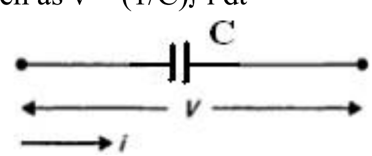
- Electricalcircuitsinvolvingresistors,capacitorsandinductorsareconsidered.The behaviour of such systems is governed by Ohm's law and Kirchhoff's laws
- Resistor: Consider a resistance of'R' Ω carrying current'i' Amps as shown in Fig (a),then the voltage drop across it is v = R I



- **Inductor:** Consider an inductor ―L' H carrying current ‗i' Amps as shown in Fig (a),then the voltage drop across it can be written as v = L di/dt



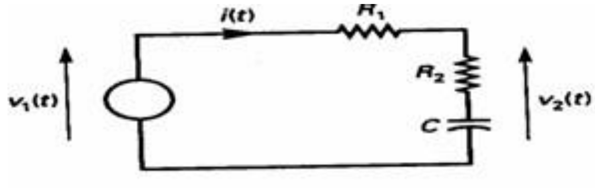
- **Capacitor:**Consideracapacitor'C'Fcarryingcurrent'i'AmpsasshowninFig(a), then the voltage drop across it can be written as v = (1/C)∫ i dt



Stepsformodelingofelectricalsystem

- ApplyKirchhoff'svoltagelaworKirchhoff'scurrentlawtoformthedifferential equations describing electrical circuits comprising of resistors, capacitors, and inductors.
- FormTransferFunctionsfromthedescribingdifferentialequations. Then
- simulate the model.

Example

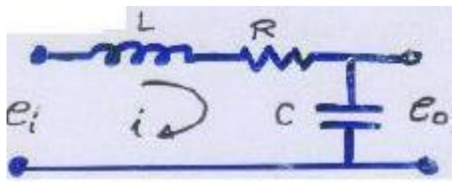


$R_1 i(t)+R_2 i(t)+1/C\int i(t)dt=V_1(t)$ $R_2\, i(t)$
$+ 1/ C \int i(t)\, dt = V_2(t)$

Electricalsystems

LRCcircuit.ApplyingKirchhoff'svoltagelawtothesystemshown.Weobtainthe following equation;

Resistancecircuit



$L(di/dt) +Ri+ 1/ C\int i(t)dt =e_i$..(1)

$1/ C \int i(t)\, dt =e_0$...(2)

Equation(1)&(2)giveamathematicalmodelofthecircuit.TakingtheL.T.ofequations (1)&(2), assuming zero initial conditions, we obtain

$$LsI(s)+RI(s)+\frac{1}{C}\frac{1}{s}I(s)=E_i(s)$$

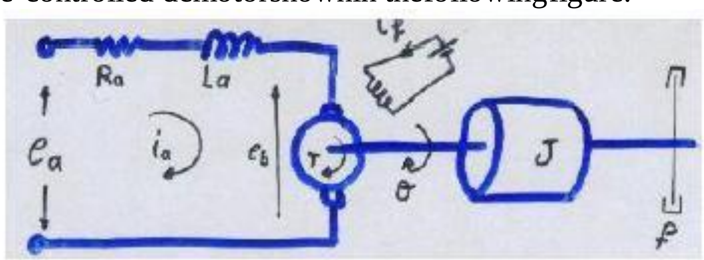
$$\frac{1}{C}\frac{1}{s}I(s)=E_o(s)$$

$$\textit{the transfer function}\quad \frac{E_0(s)}{E_i(s)}=\frac{1}{LCs^2+RCs+1}$$

Armature-Controlleddc motors

The dc motors have separately excited fields. They are either armature-controlled with fixed field or field-controlled with fixed armature current. For example, dc motors used in instruments employ a fixed permanent-magnet field, and the controlled signal is applied to the armature terminals.

Considerthearmature-controlled dcmotorshownin thefollowingfigure.



Ra = armature-winding resistance, ohms
La=armature-windinginductance,henrys ia
= armature-winding current, amperes
if=fieldcurrent, a-pares
ea=appliedarmaturevoltage,volt eb
= back emf, volts
θ=angulardisplacementofthemotorshaft,radians T =
torque delivered by the motor, Newton*meter
J=equivalent momentofinertiaof themotorand loadreferred tothemotor shaft kg.m2
f=equivalentviscous-frictioncoefficientofthemotorandloadreferredtothemotorshaft. Newton*m/rad/s
T=k1iaψwhereψistheairgap flux,ψ=kfif,k1isconstant For the
constant flux

$$e_b = k_b \frac{d\vartheta}{dt}$$

WhereKb is aback emfconstant ---------------- (1)
Thedifferentialequationforthearmaturecircuit

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \text{........}(2)$$

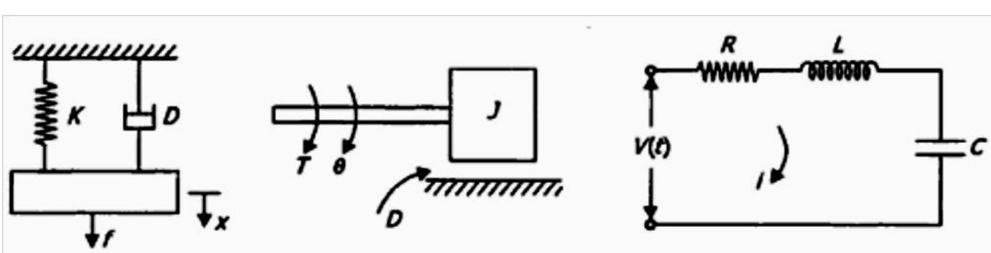
Thearmaturecurrentproducesthetorquewhichisappliedtotheinertiaandfriction; hence

$$\frac{Jd^2\vartheta}{dt^2} + f\frac{d\vartheta}{dt} = T = Ki_a \text{..........}(3)$$

Assumingthatallinitialconditionsareconditionarezero/andtakingtheL.T.ofequations(1),
(2)& (3), weobtain
$K_p s\ \theta\ (s) = E_b(s)$
$(L_a s + Ra) I_a(s) + E_b(s) = E_a(s) (Js^2 + fs)\ \theta\ (s) =$
$T(s) = K\ I_a(s)$
TheT.F can beobtainedis

$$\frac{\theta(s)}{E_a(s)} = \frac{K}{s(L_a J s^2 + (L_a f + R_a J)s + R_a f + KK_b)}$$

Analogous Systems
Letusconsideramechanical(bothtranslationalandrotational)and electricalsystemasshownin the fig.



(a) Translational (b) Rotational (c) Electrical system

Fromthefig(a)
Weget $Md^2x/dt^2+D\,dx/dt+Kx=f$

Fromthefig(b)
Weget$Md^2\theta/dt^2+Dd\theta/dt+K\theta=T$ From the fig (c)

Weget$Ld^2q/dt^2+Rd\,q/dt+(1/C)\,q=V(t)$

Whereq $=\int i\,dt$

They are two methods to get analogous system. These are (i) force- voltage (f-v) analogy and (ii) force-current (f-c) analogy

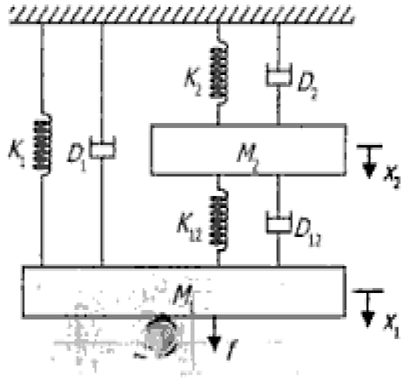
Translational	Electrical	Rotational
Force (f)	Voltage (v)	Torque (T)
Mass (M)	Inductance (L)	Inertia (J)
Damper (D)	Resistance (R)	Damper (D)
Spring (K)	Elastance $\left(\frac{1}{C}\right)$	Spring (K)
Displacement (x)	Charge (q)	Displacement (θ)
Velocity (u)	Current (i)	Velocity (ω)

Force–VoltageAnalogy
Force–CurrentAnalog

Translational	Electrical	Rotational
Force (f)	Current (i)	Torque (T)
Mass (M)	Capacitance (C)	Inertia (J)
Spring (K)	Reciprocal of Inductance $\left(\frac{1}{L}\right)$	Damper (D)
Damper (D)	Conductance $\left(\frac{1}{K}\right)$	Spring (K)
Displacement (x)	Flux Linkage (ψ)	Displacement (θ)
Velocity $\left(u=\frac{dx}{dt}\right)$	Voltage (v) $=\frac{d\psi}{dt}$	Velocity $\left(\omega=\frac{d\theta}{dt}\right)$

Problem

1.Findthesystemequationforsystemshowninthefig.Andalsodeterminef-vandf-i analogies



Forfreebodydiagram M1

$$f = M_1 \frac{d^2x_1}{dt^2} + D_1 \frac{dx_1}{dt} + K_1 x_1 + D_{12} \frac{d}{dt}(x_1 - x_2) + K_{12}(x_1 - x_2) \quad (1)$$

Forfreebodydiagram M2

$$K_{12}(x_1 - x_2) + D_{12} \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2x_2}{dt^2} + D_2 \frac{dx_2}{dt} + K_2 x_2 \quad (2)$$

Force–voltageanalogy From

$$f \to v,\ M \to L,\ D \to R,\ K \to \frac{1}{C},\ x \to q$$

eq (1) we get

$$v = L_1 \frac{d^2q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{1}{C_1} q_1 + R_{12} \frac{d}{dt}(q_1 - q_2) + \frac{1}{C_{12}}(q_1 - q_2)$$

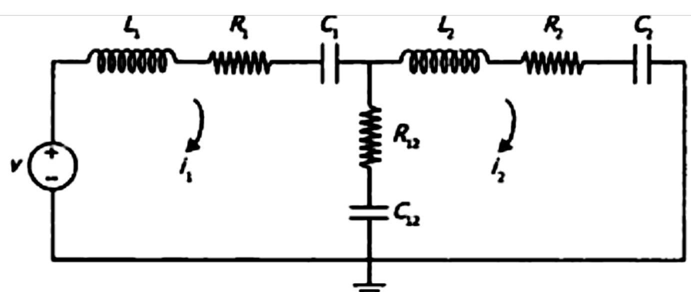
$$v = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt + R_{12}(i_1 - i_2) + \frac{1}{C_{12}}\int (i_1 - i_2) dt \quad (3)$$

Fromeq(2)we get

$$\frac{1}{C_{12}}(q_1 - q_2) + R_{12} \frac{d}{dt}(q_1 - q_2) = L_2 \frac{d^2q_2}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{1}{C_2} q_2$$

$$\frac{1}{C_{12}}\int (i_1 - i_2) dt + R_{12}(i_1 - i_2) = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2 dt \quad \ldots..(4)$$

Fromeq(3)and(4)we candraw f-v analogy



Force–currentanalogy

$$f \rightarrow i,\ M \rightarrow C,\ D \rightarrow \frac{1}{R},\ K \rightarrow \frac{1}{L},\ x \rightarrow \psi$$

From eq (1) we get

$$i = C_1 \frac{d^2\psi_1}{dt^2} + \frac{1}{R_1}\frac{d\psi_1}{dt} + \frac{1}{L_1}\psi_1 + \frac{1}{R_{12}}\frac{d}{dt}(\psi_1 - \psi_2) + \frac{1}{L_{12}}(\psi_1 - \psi_2)$$

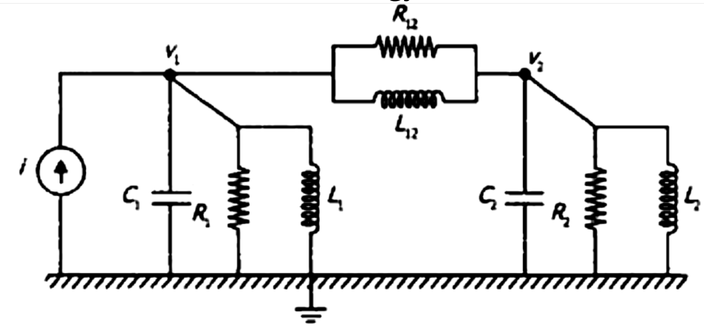
$$i = C_1 \frac{dv_1}{dt} + \frac{1}{R_1}v_1 + \frac{1}{L_1}\int i_1 dt + \frac{v_1 - v_2}{R_{12}} + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt \quad \text{........(5)}$$

Fromeq(2)we get

$$\frac{1}{L_{12}}(\psi_1 - \psi_2) + \frac{1}{R_{12}}\frac{d}{dt}(\psi_1 - \psi_2) = C_2\frac{d^2\psi_2}{dt^2} + \frac{1}{R_2}\frac{d\psi_2}{dt} + \frac{1}{L_{12}}\psi_2$$

$$\frac{1}{L_{12}}\int (v_1 - v_2)dt + \frac{1}{R_{12}}(v_1 - v_2) = C_2\frac{dv_2}{dt^2} + \frac{v_2}{R_2} + \frac{1}{L_{12}}\int v_2 dt \quad \text{............(6)}$$

Fromeq(5)and(6)we candraw force-currentanalogy



Thesystemcanberepresentedintwoforms: Block

- diagram representation
- Signalflow graph

Block diagram

A pictorial representation of the functions performed by each component and of the flow of signals.

Basicelementsofablockdiagram

- Blocks
- Transferfunctionsofelementsinsidetheblocks
- Summing points
- Takeoffpoints
- Arrow

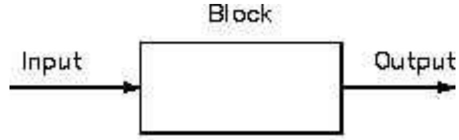
Block diagram

A control system may consist of a number of components. A block diagram of a systemis a pictorial representation of the functions performed by each component and of the flow of signals.

Theelements ofablock diagram areblock, branchpoint andsummingpoint.

Block

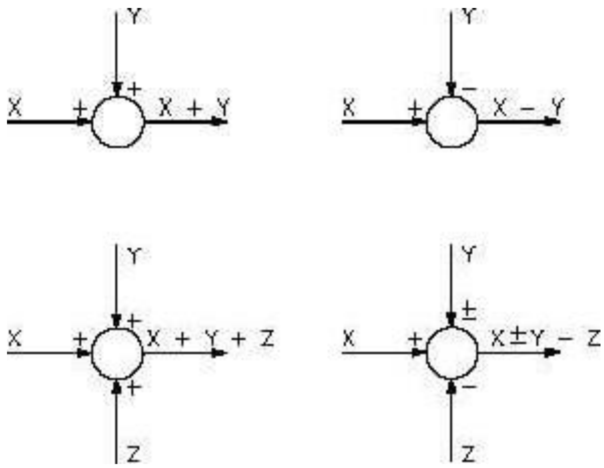
Inablockdiagramallsystemvariablesarelinkedtoeachotherthroughfunctional blocks. The functional block or simply block is a symbol for the mathematical operation on the input signal to the block that produces the output.



Summing point

Although blocks are used to identify many types of mathematical operations, operations of addition and subtraction are represented by a circle, called a summing point. As shown in Figure a summing point may have one or several inputs. Each input has its own appropriate plus or minus sign.

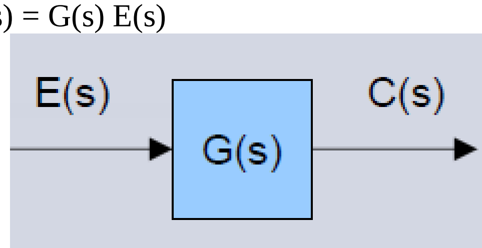
Asummingpoint has onlyoneoutput and isequalto thealgebraicsum of theinputs.



Atakeoffpointisusedto allowasignaltobeusedbymorethanoneblockorsummingpoint. The transfer function is given inside the block

• TheinputinthiscaseisE(s)
• TheoutputinthiscaseisC(s) •

C(s) = G(s) E(s)



Functionalblock –eachelementofthepracticalsystemrepresentedbyblockwithits T.F.
Branches–linesshowingthe connectionbetweentheblocks
Arrow –associatedwitheachbranchtoindicatethedirectionofflowof signal
Closedloopsystem
Summingpoint–comparingthedifferentsignals
Takeoffpoint–pointfromwhichsignalistakenforfeed back

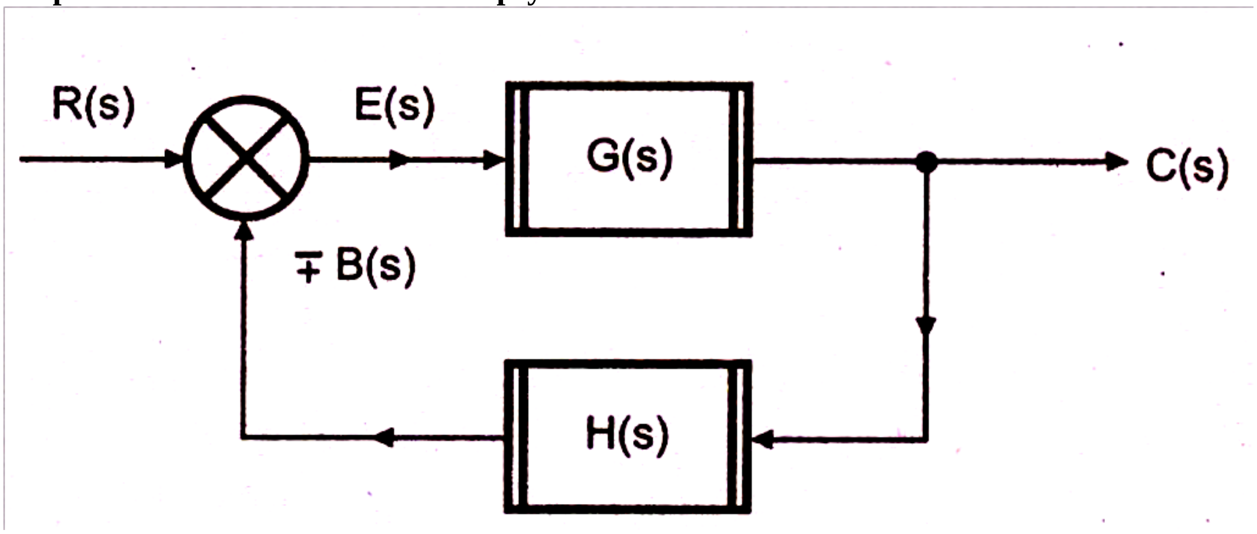
Advantagesof Block Diagram Representation

- Verysimpletoconstructblockdiagramforacomplicatedsystem Function
- of individual element can be visualized
- Individual & Overall performance can be studied
- Overalltransferfunctioncanbecalculatedeasily.

Disadvantages of Block Diagram Representation

- Noinformationaboutthephysicalconstruction
- Source of energy is not shown

SimpleorCanonical formofclosed loopsystem



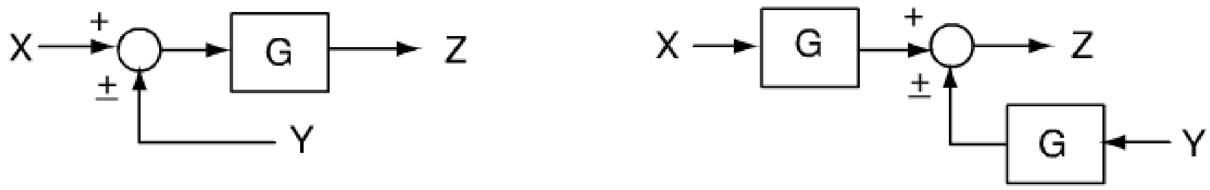
R(s) – Laplace of reference input r(t)
C(s)–Laplaceofcontrolledoutputc(t)
E(s) – Laplace of error signal e(t)
B(s) – Laplace of feed back signal b(t)
G(s) – Forward path transfer function
H(s)–Feedbackpathtransferfunction

Blockdiagramreduction technique

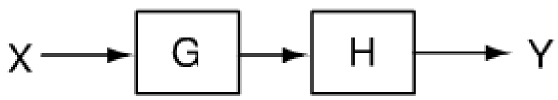
Because of their simplicity and versatility, block diagrams are often used by control engineers to describe all types of systems. A block diagram can be used simply to represent the composition and interconnection of a system. Also, it can be used, together with transfer functions, to represent the cause-and-effect relationships throughout the system. Transfer Function is defined as the relationship between an input signal and an output signal to a device.

Block diagramrules

Cascadedblocks

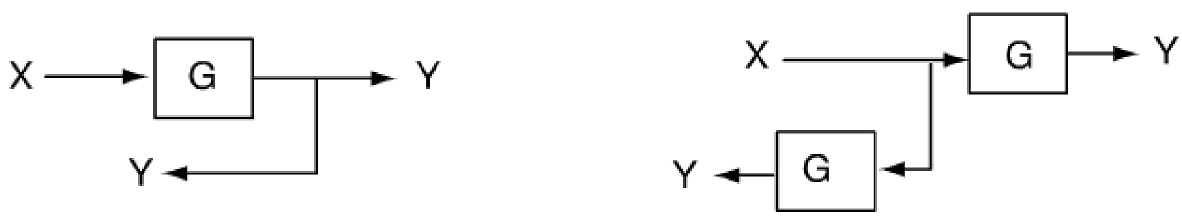


Movingasummerbeyond theblock



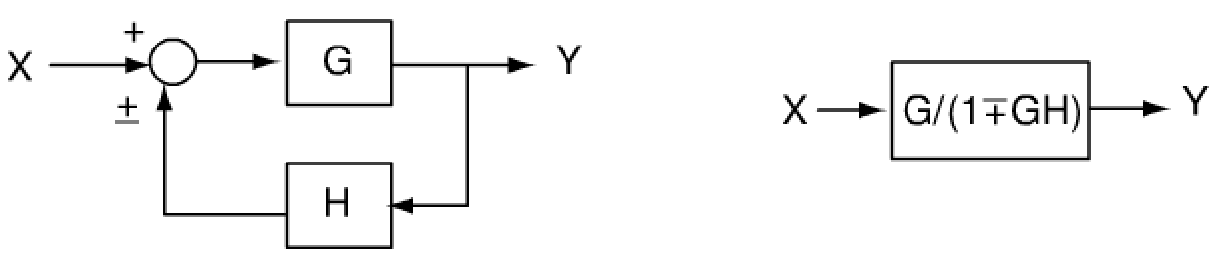
Movingasummer aheadofblock

Movingapick-offaheadof block

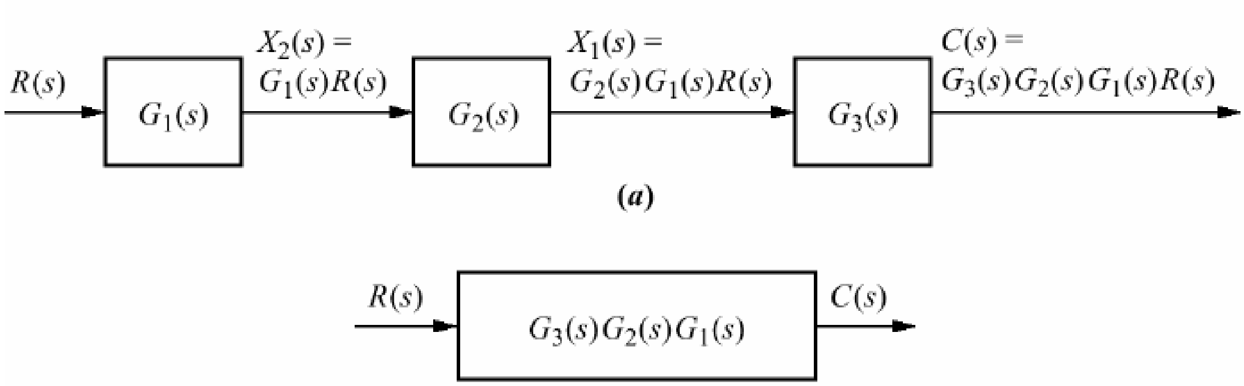


Movingapick-off behind a block

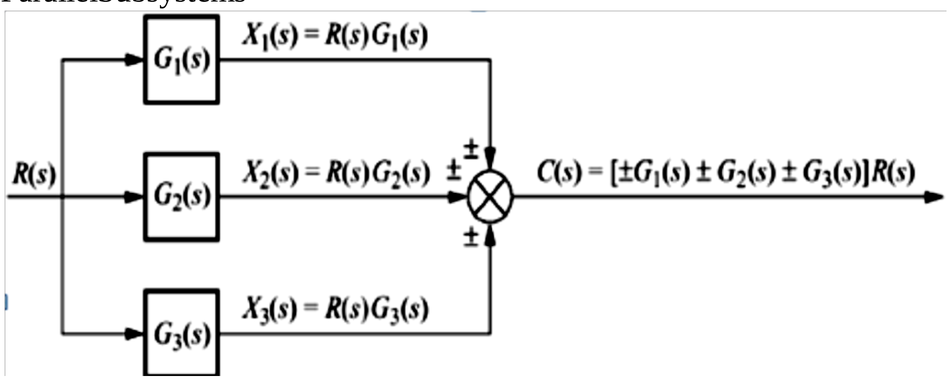
Eliminatingafeedbackloop



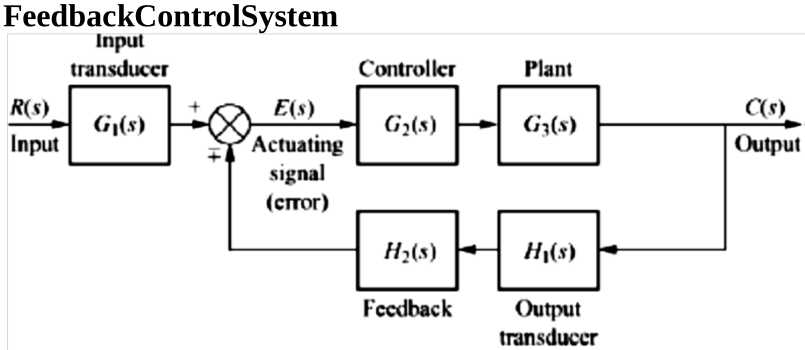
CascadedSubsystems



ParallelSubsystems



FeedbackControlSystem

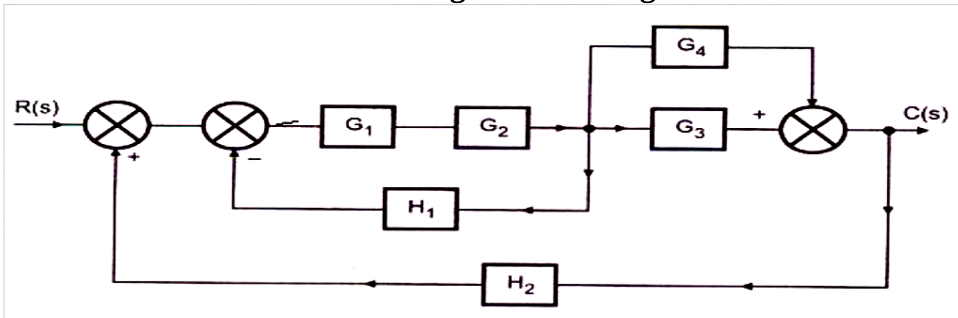


ProceduretosolveBlockDiagramReductionProblems

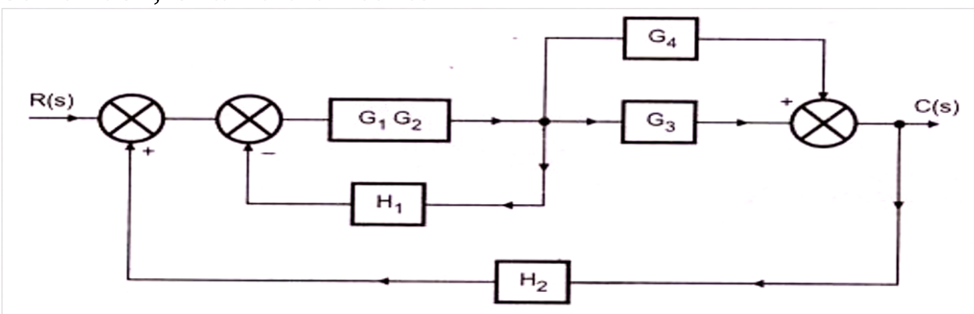
Step 1: Reduce the blocks connected in series
Step2:Reducetheblocksconnectedinparallel
Step 3: Reduce the minor feedback loops
Step4:Trytoshifttakeoffpointstowardsrightand Summingpointtowardsleft Step 5: Repeat steps 1 to 4 till simple form is obtained
Step6:ObtaintheTransferFunctionofOverall System

Problem1

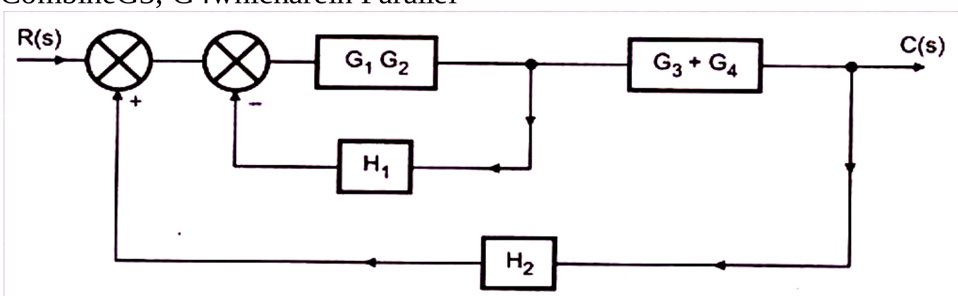
ObtaintheTransferfunctionof thegivenblockdiagram



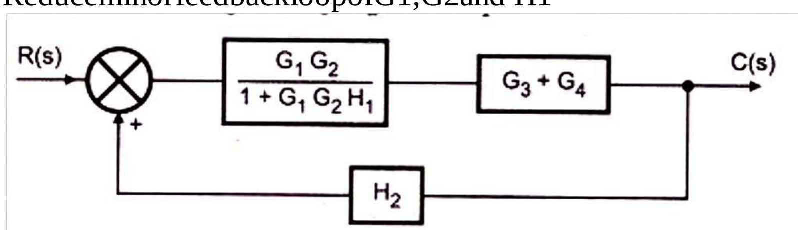
CombineG1, G2whicharein series

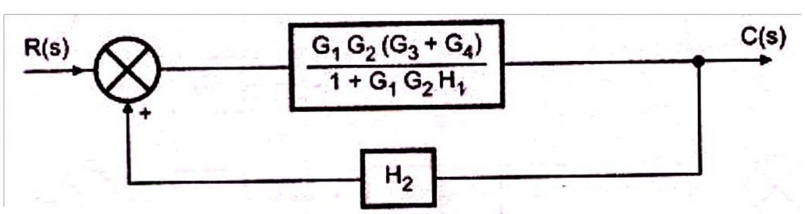


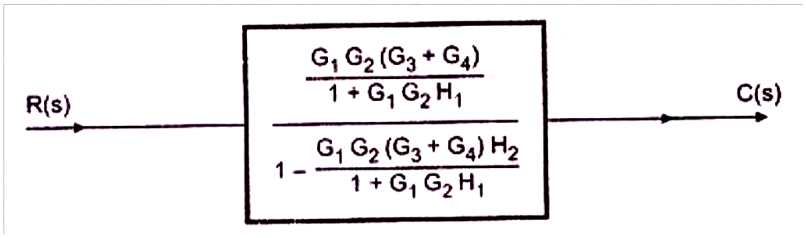
CombineG3, G4whicharein Parallel



ReduceminorfeedbackloopofG1,G2and H1



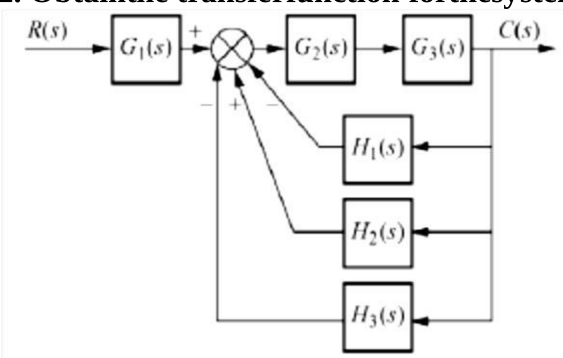




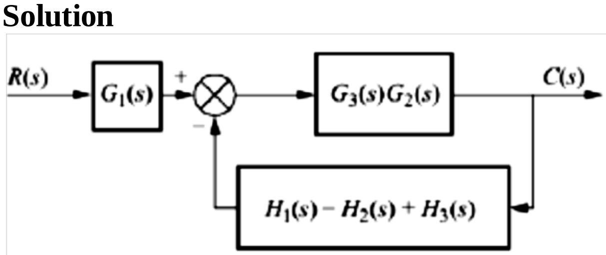
Transfer function

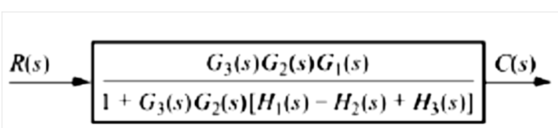
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 (G_3 + G_4)}{1 + G_1 G_2 H_1 - G_1 G_2 (G_3 + G_4) H_2}$$

2. Obtainthe transferfunction forthesystemshownin thefig

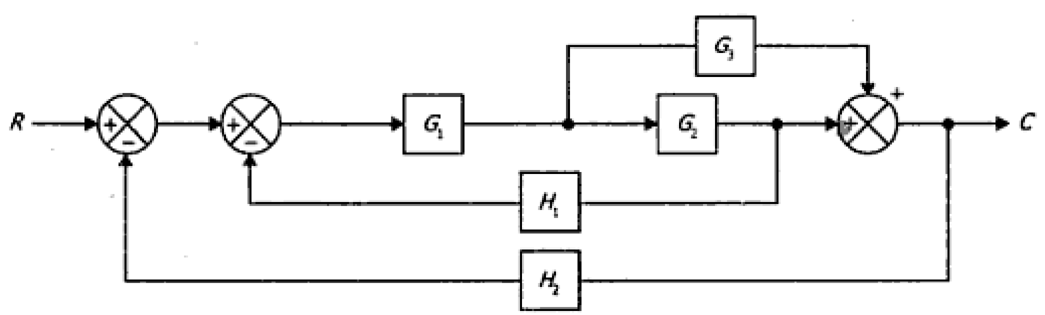


Solution



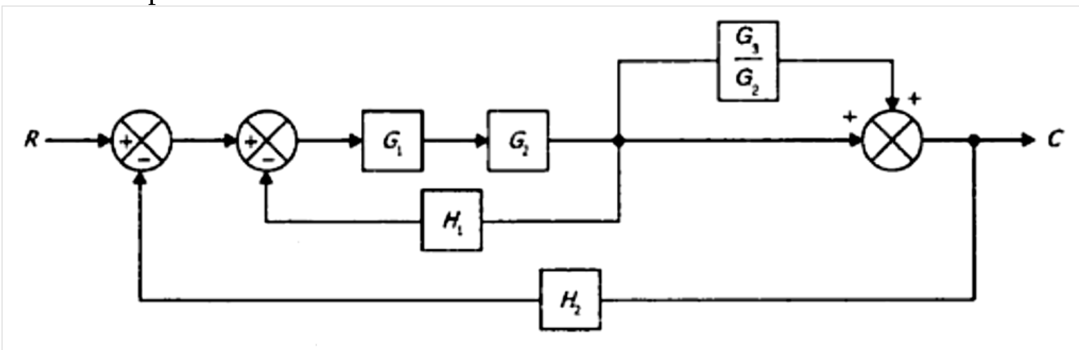


3. Obtainthetransferfunction C/Rfortheblockdiagramshownin thefig

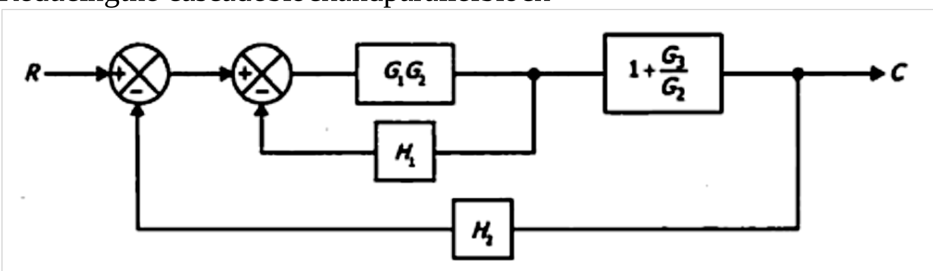


Solution

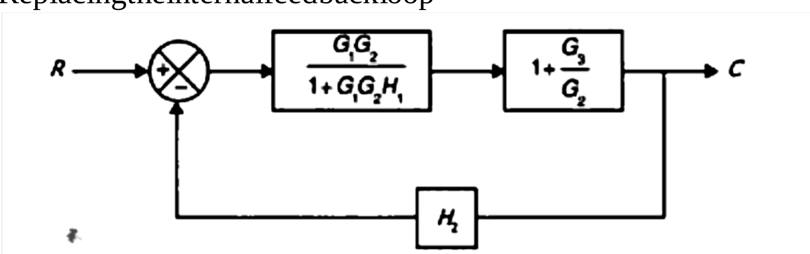
Thetake-offpointis shifted aftertheblock G2



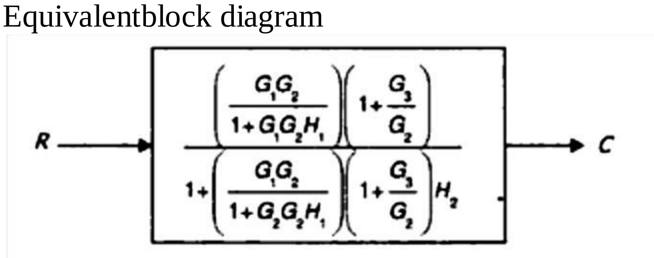
Reducingthe cascadeblockandparallelblock



Replacingtheinternalfeedbackloop



Equivalentblock diagram



Transfer function

$$\frac{C}{R}=\frac{\dfrac{G_1\,(G_2+G_3)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2+G_3)\,H_2}{1+G_1G_2H_1}}$$

$$=\frac{G_1(G_2+G_3)}{1+G_1G_2(H_1+H_2)+G_1G_3H_2}$$

SignalFlow GraphRepresentation

Signal Flow Graph Representation ofasystem obtained from the equations, which shows the flow of the signal

Signalflowgraph

A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. Bytaking Laplacetransfer,thetimedomain differential equations governing a control systemcanbetransferredtoasetofalgebraicequationins-domain.Asignal-flowgraphconsists of a network in which nodes are connected by directed branches. It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

BasicElementsofaSignalflowgraph

Node-apoint representingasignal or variable.
Branch–unidirectionallinesegmentjoiningtwonodes.
Path–abranchoracontinuoussequenceofbranchesthatcanbetraversedfromonenodeto another node.
Loop – aclosedpath that originates and terminates on thesame node and alongthe path no node is met twice.
Nontouchingloops –two loops aresaid to be nontouchingif theydo not have acommonnode.

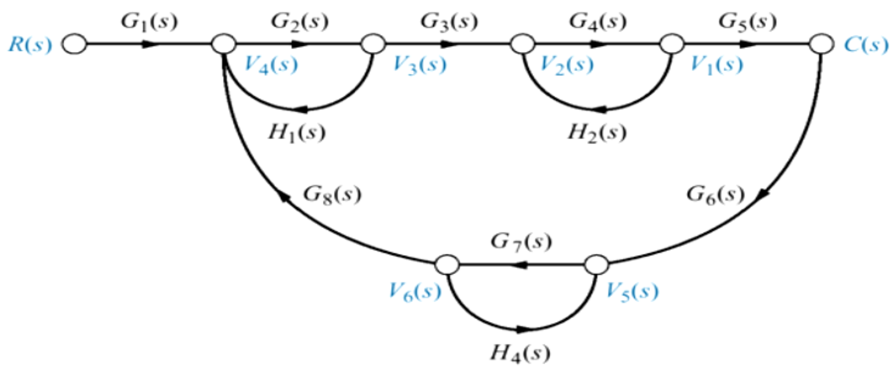
Mason'sgainformula

The relationship between an input variable and an output variable of signal flow graph is given by the net gain between the input and the output nodes is known as overall gain of the system. Mason's gain rule for the determination of the overall system gain is given below.

$$M = \frac{1}{\Delta}\sum_{k=1}^{N} P_k \Delta_k = \frac{X_{out}}{X_{in}}$$

WhereM=gainbetweenXinandXout
Xout =output node variable
Xin=input nodevariable
N = total number of forward paths
Pk=pathgainofthekthforwardpath
Δ=1-(sum of loop gains of all individual loop) + (sum of gain product of all possible combinations of two nontouching loops) – (sum of gain products of all possible combination of three nontouching loops)

Problem



- Forward path gain: $T_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$
- Closed loop gain

(1) $G_2(s)H_1(s)$ (2) $G_4(s)H_2(s)$

(3) $G_7(s)H_4(s)$ (4) $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$

- Nontouching loops taken two at a time
  - (5) loop (1) and loop (2): $G_2(s)H_1(s)G_4(s)H_2(s)$
  - (6) loop (1) and loop (3): $G_2(s)H_1(s)G_7(s)H_4(s)$
  - (7) loop (2) and loop (3): $G_4(s)H_2(s)G_7(s)H_4(s)$
- Nontouching loops taken three at a time
  - (8) loops (1), (2), (3): $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$
- Now, $\Delta = 1 - \{(1) + (2) + (3) + (4)\} + \{(5) + (6) + (7)\} - (8)$
- Portion of Δ not touching the forward path

$$\Delta_1 = 1 - G_7(s)H_4(s)$$

- Hence,

$$G(s) = \frac{C(s)}{R(s)} = \frac{T_1\Delta_1}{\Delta}$$

$$= \frac{G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)[1 - G_7(s)H_4(s)]}{\Delta}$$

TIMERESPONSE ANALYSIS

Introduction

- After deriving a mathematical model of a system, the system performance analysis can be done in various methods.
- In analyzing and designing control systems, a basis of comparison of performance of various control systems should be made. This basis may be set up by specifying particular test input signals and by comparing the responses of various systems to these signals.
- The system stability, system accuracy and complete evaluation are always based on the time response analysis and the corresponding results.
- Nextimportantstepafteramathematicalmodelofasystemisobtained. To
- analyze the system's performance.
- Normallyusethestandardinputsignalstoidentifythecharacteristicsofsystem'sresponse Step
  - function
  - Ramp function
  - Impulse function
  - Parabolic function
  - Sinusoidal function

Timeresponseanalysis

It is an equation or a plot that describes the behavior of a system and contains much information about it with respect to time response specification as overshooting, settling time, peaktime,risetimeand steadystateerror.Timeresponseisformed bythetransientresponseand the steady state response.

Timeresponse=Transientresponse+Steadystate response

Transient time response (Natural response) describes the behavior of the system in its firstshort time until arrives the steady state value and this response will be our study focus. If the input is step function then the output or the response is called step time response and if the input is ramp, the response is called ramp time response ... etc.

ClassificationofTimeResponse

- Transient response
- Steadystateresponse

y(t)= yt(t)+yss(t)

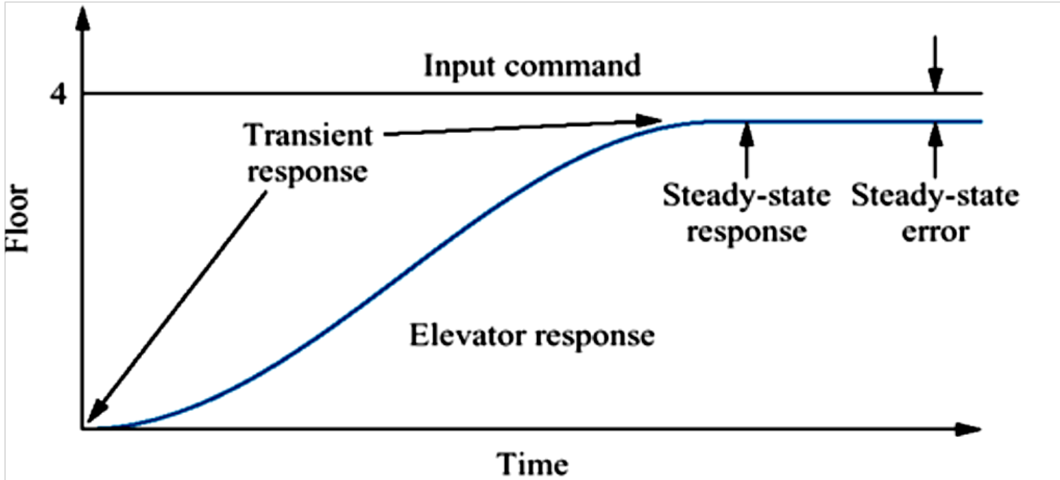
TransientResponse

Thetransient responseis defined as thepartofthe timeresponsethat goes to zeroas time becomes very large. Thus yt(t) has the property

Limyt(t)=0 t

-->∞

The time required to achieve the final value is called transient period. The transient response may be exponential or oscillatory in nature. Output response consists of the sum of forced response (form the input) and natural response (from the nature of the system).The transientresponseisthechangeinoutputresponsefromthebeginningoftheresponsetothe finalstateoftheresponseandthesteadystateresponseistheoutputresponseastimeisapproaching infinity (or no more changes at the output).

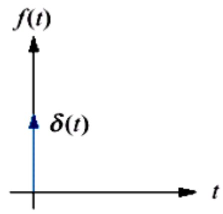
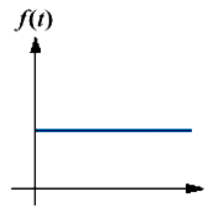
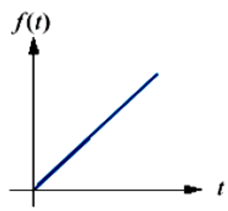
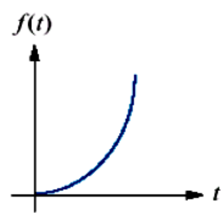
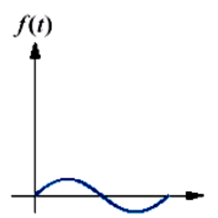


SteadyState Response

The steady state response is the part of the total response that remains after the transient has died out. For a position control system, the steady state response when compared to with the desired reference position gives an indication of the final accuracy of the system. If the steady state response of the output does not agree with the desired reference exactly, the system is said to have steady state error.

TypicalInputSignals

- Impulse Signal
- Step Signal
- Ramp Signal
- ParabolicSignal

Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)\,dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error

TimeResponseAnalysis&Design

Twotypesofinputscanbeappliedtoacontrolsystem.

CommandInputorReferenceInputyr(t).

DisturbanceInputw(t)(Externaldisturbancesw(t)aretypically uncontrolledvariationsinthe load on a control system).

Insystems controllingmechanical motions,load disturbancesmayrepresent forces.

Involtageregulatingsystems,variationsinelectricalloadareamajorsourceof disturbances.

TestSignals

Input	r(t)	R(s)
StepInput	A	A/s
RampInput	At	A/s^2
ParabolicInput	$At^2 / 2$	A/s^3
ImpulseInput	$\delta(t)$	1

Transfer Function

- Oneof thetypes of Modelinga system
- Usingfirstprinciple, differentialequationis obtained
- Laplace Transformis appliedtotheequationassumingzeroinitial conditions
- RatioofLT(output)toLT(input)isexpressedasaratioofpolynomialin *s*inthetransfer function.

Orderofa system

- TheOrderofasystemisgivenbytheorderofthedifferentialequationgoverningthe system Alternatively,ordercanbeobtainedfromthetransferfunction
- In the transfer function, the maximum power of s in the denominator polynomialgivesthe
- order of the system.

DynamicOrderof Systems

- Orderofthesystemistheorderofthedifferentialequationthatgovernsthedynamic behaviour
- Workinginterpretation:Numberofthedynamicelements/capacitancesorholdup elements between a
- manipulatedvariableand acontrolledvariable
- Higherordersystemresponsesareusuallyverydifficulttoresolvefromoneanother The
- response generally becomes sluggish as the order increases.

SystemResponse

First-ordersystemtime response

- Transient
- Steady-state

Second-ordersystemtime response

- Transient
- Steady-state

FirstOrder System

Ys/ R(s)=K /(1+K+sT)=K /(1+sT)

StepResponseofFirst OrderSystem

Evolution of the transient response is determined by the pole of the transfer functionats=-1/t where t is the time constant

Also,thestep response can befound:

Impulseresponse	K / (1+sT)	**Exponential**
Step response	(K/S)– (K/ (S+(1/T)))	**Step, exponential**
Rampresponse	(K/S^2)-(KT/S)-(KT/ (S+1/T))	**Ramp,step, exponential**

Second-ordersystems

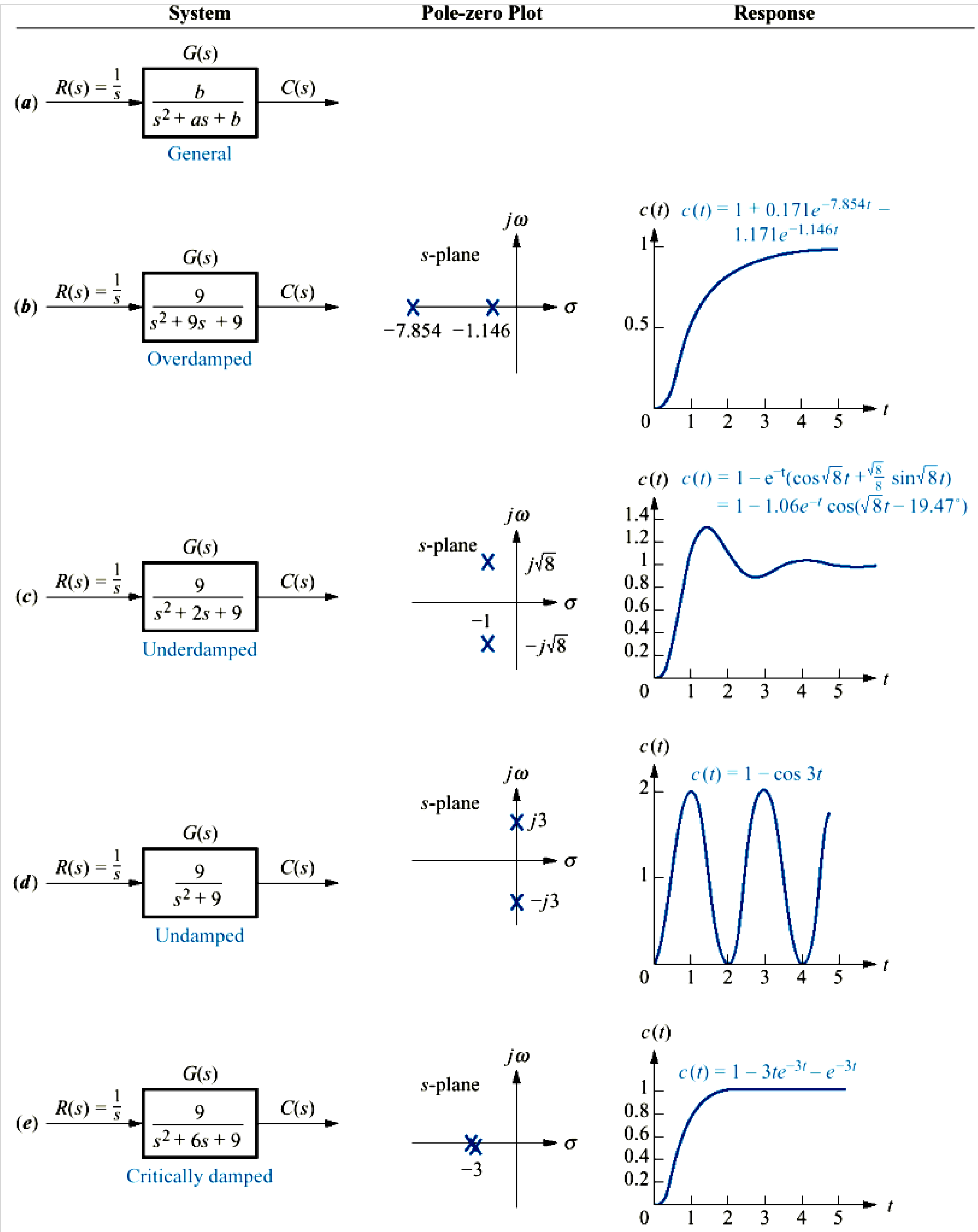
LTIsecond-ordersystem

$$G(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)\, C(s) = \omega_n^2\, R(s)$$

$$c(t) + 2\zeta\omega_n c(t) + \omega_n^2\, c(t) = \omega_n^2\, r(t)$$

Second-OrderSystems



System
Pole-zero Plot
Response
(a)
R(s) = 1/s
G(s)
b / (s² + as + b)
C(s)
General
(b)
R(s) = 1/s
G(s)
9 / (s² + 9s + 9)
C(s)
Overdamped
s-plane
jω
σ
−7.854
−1.146
c(t)
c(t) = 1 + 0.171e^(−7.854t) − 1.171e^(−1.146t)
1
0.5
0
1
2
3
4
5
t
(c)
R(s) = 1/s
G(s)
9 / (s² + 2s + 9)
C(s)
Underdamped
s-plane
jω
σ
j√8
−1
−j√8
c(t)
c(t) = 1 − e^(−t)(cos√8t + (√8/8) sin√8t)
= 1 − 1.06e^(−t) cos(√8t − 19.47°)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
t
(d)
R(s) = 1/s
G(s)
9 / (s² + 9)
C(s)
Undamped
s-plane
jω
σ
j3
−j3
c(t)
c(t) = 1 − cos 3t
2
1
0
1
2
3
4
5
t
(e)
R(s) = 1/s
G(s)
9 / (s² + 6s + 9)
C(s)
Critically damped
s-plane
jω
σ
−3
c(t)
c(t) = 1 − 3te^(−3t) − e^(−3t)
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
t

Secondordersystem responses

Overdampedresponse:

Poles:Tworealat

$-\sigma_1$ $--\sigma_2$

Naturalresponse:Twoexponentialswithtimeconstantsequaltothereciprocalofthepole location

$C(t)=k_1e^{-\sigma 1}+k_2e^{-\sigma 2}$

Poles:Twocomplexat

Underdampedresponse:

$$-\sigma_1 \pm jW_d$$

Natural response: Damped sinusoid with an exponential envelope whose time constant is equal to the reciprocal of the pole's radian frequency of the sinusoid, the damped frequency of oscillation, is equal to the imaginary part of the poles

UndampedResponse:

Poles:Twoimaginaryat

$$\pm jW1$$

Natural response: Undamped sinusoid with radian frequency equal to the imaginary part of the poles

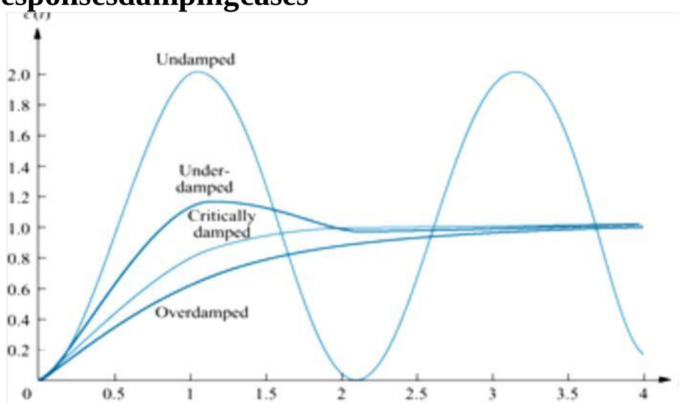
$$C(t)=A\cos(w_1t-\varphi)$$

Criticallydampedresponses:

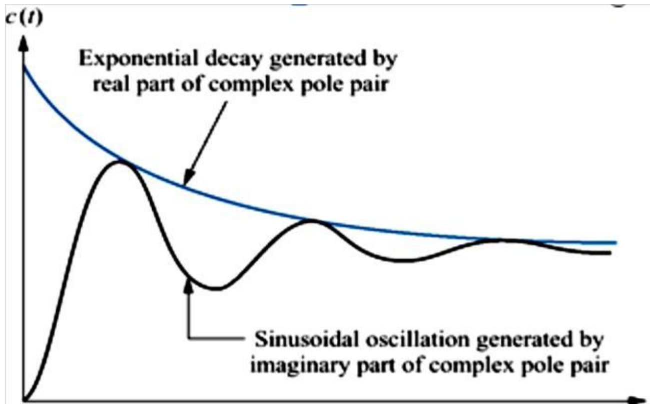
Poles:Tworealat

Natural response: One term is an exponential whose time constant is equal to the reciprocal of the pole location. Another term product of time and an exponential with time constant equal to the reciprocal of the pole location.

Secondordersystemresponsesdampingcases



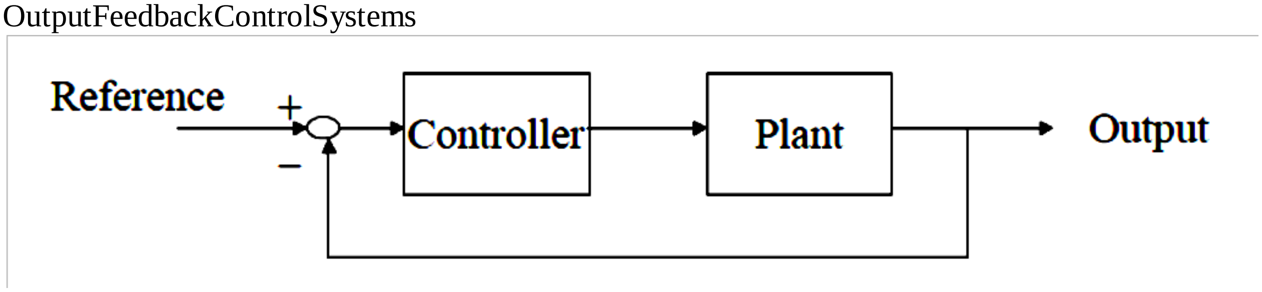
Second-orderstepresponse
Complexpoles



SteadyState Error
Consider a unity feedback system
Transferfunctionbetweene(t)andr(t)

Type of system	Error constants			Steady state error e_{ss}		
	K_p	K_v	K_a	Unit step input	Unit ramp input	Unit parabolic input
0	K	0	0	1/(1+K)	∞	∞
1	∞	K	0	0	1/K	∞
2	∞	∞	K	0	0	1/K
3	∞	∞	∞	0	0	0

OutputFeedbackControlSystems



Feedback onlythe outputsignal
– Easyaccess
– Obtainablein practice

PIDControllers

Proportionalcontrollers
– puregainor attenuation

Integralcontrollers
– integrate error

Derivativecontrollers
– differentiate error

ProportionalController

U = K_p e

- Controllerinputiserror(referenceoutput) Controller
- output is control signal
- Pcontrollerinvolvesonlyaproportionalgain(orattenuation)

IntegralController

- Integralo ferrorwithaconstantgain
- Increase system type by 1
- Infinitysteady-stategain
- Eliminatesteady-stateerrorforaunit step input

IntegralController

$$\frac{Y(s)}{R(s)} = \frac{G_p(s)}{1+G_p(s)}$$

$$Y(s) = E(s)G_p(s)$$

$$E(s) = \frac{R(s)}{1+G_p(s)}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G_p(s)} = \lim_{s\to 0} \frac{1}{1+G_p(s)} = \frac{1}{1+\infty} = 0$$

DerivativeControl

$$u = K_d \frac{de}{dt}$$

- Differentiationo ferrorwithaconstantgain
- Reduce overshoot and oscillation
- Donotaffectsteady-stateresponse
- Sensitive to noise

ControllerStructure

- Single controller
- Pcontroller,Icontroller,Dcontroller
- Combination of controllers
- PIcontroller,PDcontroller
- PID controller

Controller Performance

- P controller
- PI controller
- PDController
- PIDController

DesignofPIDControllers

- Basedontheknowledge ofP,Iand D
- – trialanderror
- – manual tuning
- – simulation

DesignofPIDControllers

- Timeresponse measurements areparticularlysimple.
- A step input to a system is simplya suddenly applied input - often just a constant voltage applied through a switch.
- The system output is usually a voltage, or a voltage output from a transducer measuring the output.
- A voltage output can usually be captured in a file using a C program or a Visual Basic program.
- Youcanuseresponsesinthetimedomaintohelpyoudeterminethetransferfunctionofa system.
- First we will examine a simple situation. Here is the step response of a system. This is an exampleofreally"clean" data,betterthan you might havefrommeasurements. Theinput to the system is a step of height 0.4. The goal is to determine the transfer function of the system.

ImpulseResponseofAFirstOrder System

- The impulse response of a system is an important response. The impulse response is the response to a unit impulse.
- The unit impulse has a Laplace transform of unity (1).That gives the unit impulse aunique stature. If a system has a unit impulse input, the output transform is G(s), where G(s) is the transfer function of the system. The unit impulse response is therefore the inversetransformofG(s),i.e.g(t),thetimefunctionyougetbyinversetransforming G(s). If you haven't begun to study Laplace transforms yet, you can just file these last statements away until you begin to learn about Laplace transforms. Still there is an important fact buried in all of this.

- Knowing that the impulse response is the inverse transform of the transfer function of a system can be useful in identifying systems (getting system parameters from measured responses).

In this section we will examine the shapes/forms of several impulse responses. We will start with simple first order systems, and give you links to modules that discuss other, higher order responses.

Ageneralfirstordersystemsatisfiesadifferentialequationwiththisgeneral form

If the input, u(t), is a unit impulse, then for a short instant around t = 0 the input isinfinite. Let us assume that the state, x(t), is initially zero, i.e. x(0) = 0. We will integrate both sides of the differential equation from a small time,, before t = 0, to a small time, after t = 0. We are just taking advantage of one of the properties of the unit impulse.

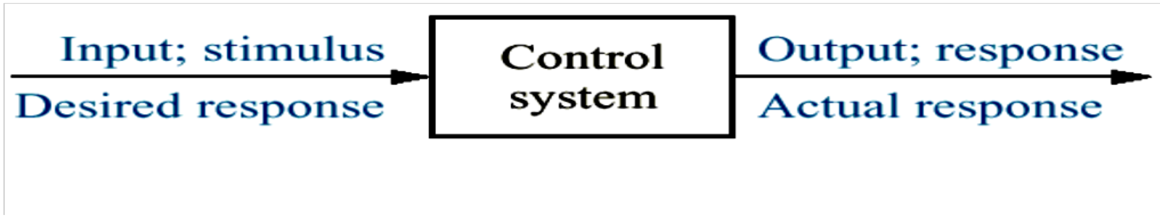
The right hand side of the equation is just G**dc** since the impulse is assumed to be a unit impulse - one with unit area. Thus, we have:

We can also note that x(0) = 0, so the second integral on the right hand side is zero. In other words, what the impulse does is it produces a calculable change in the state, x(t), and this change occurs in a negligibly short time (the duration of theimpulse) after t = 0 That leads us toa simple strategy for getting the impulse response. Calculate the new initial condition after the impulse passes. Solve the differential equation - with zero input - starting from the newly calculated initial condition.

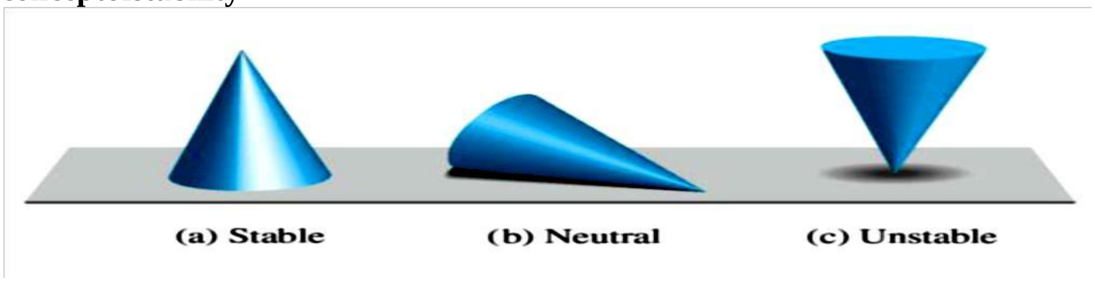
STABILITYANALYSIS

Stability

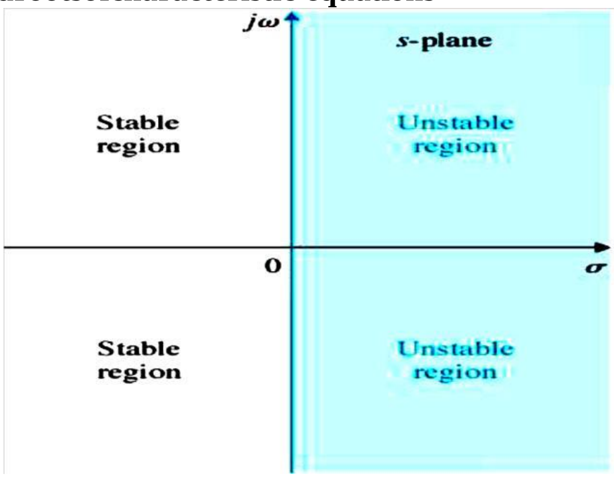
Asystemisstableif anyboundedinputproduces aboundedoutput for all boundedinitial conditions.



Basicconceptofstability



Stabilityofthesystemandrootsofcharacteristic equations



CharacteristicEquation

Consider an nth-order system whose the characteristic equation (which is also the denominatorof the transfer function) is

$$a(S) = S^n + a_1 S^{n-1} + a_2 S^{n-2} + \ldots\ldots + a_{n-1} S^1 + a_0 S^0$$

Routh Hurwitz Criterion

Goal: Determining whetherthe system is stableorunstable from a characteristicequation in polynomial form without actually solving for the roots Routh's stability criterion is useful for determining the ranges of coefficients of polynomials for stability, especially when the coefficients are in symbolic (non numerical) form.

Tofind Kmar &ω

Anecessarycondition forRouth's Stability

- Anecessaryconditionforstabilityofthesystemisthatallofthe rootsofitscharacteristic equation have negative real parts, which in turn requires that all the coefficients be positive.
- A necessary (but not sufficient) condition for stability is that all the coefficients of the polynomial characteristic equation are positive & none of the co-efficient vanishes.
- Routh's formulation requires the computation of a triangular array that is a function ofthe coefficients of the polynomial characteristic equation.
- A system is stable if and onlyif all the elements ofthe first column of the Routh arrayare positive

MethodfordeterminingtheRoutharray

Considerthecharacteristic equation

$$a(S) = 1X\ S^n + a_1 S^{n-1} + a_2 S^{n-2} + \ldots\ldots + a_{n-1} S^1 + a_0 S^0$$

Routharraymethod

Thenadd subsequentrows tocomplete theRouth array

Computeelementsforthe3rd row:

$$b_1 = -\frac{1 \times a_3 - a_2 a_1}{a_1},$$

$$b_2 = -\frac{1 \times a_5 - a_4 a_1}{a_1},$$

$$b_3 = -\frac{1 \times a_7 - a_6 a_1}{a_1}$$

$$\ldots$$

Giventhecharacteristic equation,

$$a(s) = s^6 + 4s^5 + 3s^4 + 2s^3 + s^2 + 4s + 4$$

Isthesystemdescribedbythischaracteristic equation stable?

Answer:

- Allthecoefficients arepositiveand nonzero
- Therefore,thesystem satisfiesthe necessaryconditionfor stability
- WeshoulddeterminewhetheranyofthecoefficientsofthefirstcolumnoftheRouth array are negative.

$$\begin{array}{lcccc}
s^6: & 1 & 3 & 1 & 4 \\
s^5: & 4 & 2 & 4 & 0 \\
s^4: & 5/2 & 0 & 4 & \\
s^3: & 2 & -12/5 & 0 & \\
s^2: & ? & ? & & \\
s^1: & ? & ? & & \\
s^0: & ? & & &
\end{array}$$

S^6:	1	3	1	4
S^5:	4	2	4	0
S^4:	5/2	0	4	
S^3:	2	-12/5	0	
S^2:	3	4		
S^1:	-76/15	0		
S^0:	4			

Theelements ofthe 1stcolumn arenotall positive.Then thesystem is unstable

SpecialcasesofRouth'scriteria:

Case1: Allthe elementsofarow inaRA arezero

- Form Auxiliary equation by using the co-efficient of the row which is just above the row of zeros.
- FindderivativeoftheA.E.
- Replacetherowofzerosbytheco-efficientofdA(s)/ds
- Complete the array in terms of these coefficients.
- analyze for any sign change, if so, unstable
- no sign change, find the nature of roots of AE
- non-repeatedimaginaryroots-marginallystable
- repeated imaginary roots – unstable

Case2:

- Firstelement ofanyof therows ofRA is
- Zero and the same remaining row contains atleast one non-zero element
- Substituteasmallpositiveno._ε'inplaceofzeroandcompletethearray. Examine the
- sign change by taking Lt ε = 0

RootLocus Technique

- Introduced byW. R. Evans in1948
- Graphical method, in which movement of poles in the s-plane is sketched when some parameter is varied The path taken by the roots of the characteristic equation when open loop gain K is varied from 0 to ∞ are called root loci
- Direct Root Locus = $0 < k < \infty$
- InverseRootLocus=$-\infty < k < 0$

RootLocus Analysis:

- Therootsoftheclosed-loopcharacteristicequationdefinethesystemcharacteristicresponses
- Their location in the complex s-plane lead to prediction of the characteristics of the time domain responses in terms of:
- damping ratio ζ,
- naturalfrequency,wn
- dampingconstantσ, first-ordermodes
- Considerhowtheserootschangeastheloopgainisvariedfrom0to∞

Basics ofRoot Locus:

- Symmetricalaboutrealaxis
- RLbranchstartsfromOLpolesandterminatesatOLzeroes No. of
- RL branches = No. of poles of OLTF
- Centroid is common intersection point of all the asymptotes on the real axis
- Asymptotesarestraightlineswhichareparallelto RLgoingto∞andmeettheRLat∞ No. of
- asymptotes = No. of branches going to ∞
- AtBreakAwaypoint,theRLbreaksfromrealaxistoenterintothecomplex plane At BI
- point, the RL enters the real axis from the complex plane

ConstructingRootLocus:

- LocatetheOLpoles&zerosintheplot Find
- the branches on the real axis
- Findangleofasymptotes&centroid Φa=
- ±180º(2q+1) / (n-m)
- ζa=(Σpoles-Σzeroes)/(n-m) Find
- BA and BI points
- FindAngleOfdeparture(AOD)andAngleOfArrival(AOA)
- AOD=180º-(sumofanglesofvectorstothecomplexpolefromallotherpoles)+(Sum of angles of vectors to the complex pole from all zero)
- AOA=180º-(sumofanglesofvectorstothecomplexzerofromallotherzeros)+(sum of angles of vectors to the complex zero from poles)
- Find the point of intersection of RL with the imaginary axis.

Applicationof theRootLocusProcedure

Step1:Writethecharacteristicequation as

$$1+F(s)=0$$

Step2:Rewriteprecedingequationintotheformofpolesandzerosas follows

$$1+K\frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-p_i)}=0$$

Step 3:

- Locate the poles and zeros with specific symbols, the root locus begins at the open-loop poles and ends at the open loop zeros as K increases from 0 to infinity
- Ifopen-loopsystemhas n-mzerosatinfinity,therewillben-mbranchesoftherootlocus approaching the n-m zeros at infinity

Step 4:

- Therootlocusontherealaxisliesinasectionoftherealaxistotheleftofanodd number of real poles and zeros

Step 5:

- Thenumberof separate loci is equal to the number ofopen-loop poles

Step 6:

- Therootlocimustbecontinuousandsymmetricalwithrespecttothehorizontalrealaxis

Step 7:

- Thelociproceedtozerosatinfinityalongasymptotescenteredatcentroidandwith angles

$$\sigma_a = \frac{\sum_{i=1}^{n} p_i - \sum_{j=1}^{m} z_j}{}$$

$$\phi_a = \frac{(2k+1)\pi}{n-m} \qquad (k = 0, 1, 2, \cdots n-m-1)$$

Step 8: Theactualpointatwhichtherootlocuscrossestheimaginaryaxisisreadilyevaluatedby using

- Routh‘s criterion

Step 9: Determinethebreakawaypointd (usuallyon the real axis)

-

Step 10:

- Plot the root locus that satisfy the phase criterion

$$\angle P(s) = (2k+1)\pi \qquad k = 1, 2, \cdots$$

Step 11:

Determinetheparameter value *K1*ata specificrootusingthemagnitudecriterion

$$K_1 = \left. \frac{\prod_{i=1}^{n} |(s - p_i)|}{\prod_{j=1}^{m} |(s - z_j)|} \right|_{s=s_1}$$

NyquistStability Criteria:

The Routh-Hurwitz criterion is a method for determining whether a linear system isstable or not by examining the locations of the roots of the characteristic equation of the system. In fact, the method determines onlyif there are roots that lie outside oftheleft half plane; it does not actually compute the roots. Consider the characteristic equation.

Todeterminewhetherthissystemis stableornot,checkthe following conditions

$$1 + GH(s) = D(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 = 0$$

1. Twonecessarybut notsufficient conditionsthat alltheroots havenegativerealparts are

a) Allthepolynomialcoefficientsmusthavethesamesign.

b) Allthepolynomialcoefficientsmustbenonzero.

2. Ifcondition(1)issatisfied,then computetheRouth-Hurwitz arrayas follows

$$
\begin{array}{c|ccccc}
s^{n} & a_{n} & a_{n-2} & a_{n-4} & a_{n-6} & \cdots \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \cdots \\
s^{n-2} & b_{1} & b_{2} & b_{3} & & \cdots \\
s^{n-3} & c_{1} & c_{2} & c_{3} & & \cdots \\
s^{n-4} & & & \vdots & & \\
\vdots & & & \vdots & & \\
s^{1} & & & \vdots & & \\
s^{0} & & & \vdots & &
\end{array}
$$

Wherethea$_i$'Sarethepolynomialcoefficients,andthecoefficientsin therestofthetableare computed using the following pattern

$$b_1 = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix} = \frac{-1}{a_{n-1}} (a_n a_{n-3} - a_{n-2} a_{n-1})$$

$$b_2 = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}$$

$$b_3 = \frac{-1}{a_{n-1}} \begin{vmatrix} a_n & a_{n-6} \\ a_{n-1} & a_{n-7} \end{vmatrix} \quad \cdots$$

$$c_1 = \frac{-1}{b_1} \begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}$$

$$c_2 = \frac{-1}{b_1} \begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix} \quad \cdots$$

3. Thenecessaryconditionthatallrootshavenegativerealpartsisthatalltheelementsofthe first column of the array have the same sign. The number of changes of sign equals the number of roots with positive real parts.
4. Special Case 1: The first element of a row is zero, but some other elements in that row are nonzero.Inthiscase,simplyreplacethezeroelementsby"",completethetabledevelopment, and then interpret the results assuming that "" is a small number of the same sign as the

elementaboveit. The resultsmust beinterpretedin thelimitasεto 0.

5. SpecialCase2:Alltheelementsofaparticularrowarezero.Inthiscase,someoftherootsof thepolynomialarelocatedsymmetricallyabout theoriginofthe *s*-plane, e.g., apairofpurely imaginaryroots. Thezero rows will always occurin a row associated with an odd power of *s*. Therowjustabovethezerorowsholdsthecoefficientsoftheauxiliarypolynomial. Theroots of the auxiliary polynomial are the symmetrically placed roots. Be careful to remember that the coefficients in the array skip powers of *s* from one coefficient to the next.

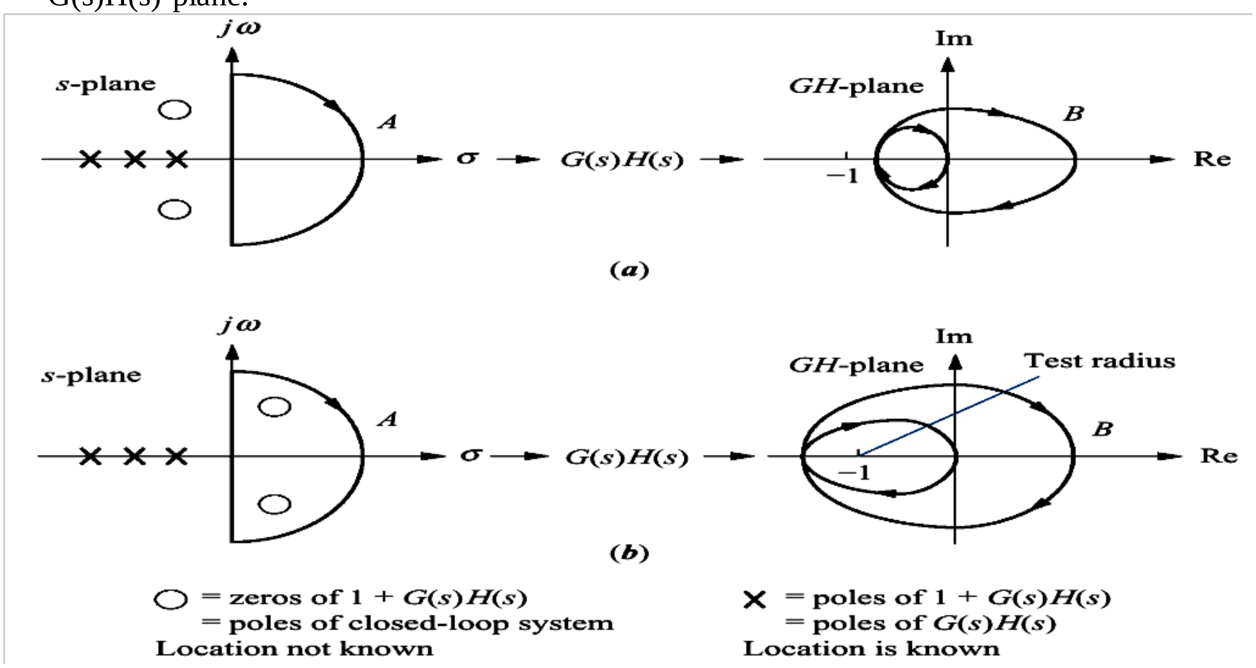
LetP= no.of polesofq(s)-planelyingonRightHalfofs-planeandencircledbys-plane contour.
LetZ=no.ofzerosofq(s)-planelyingonRightHalfofs-planeandencircledbys-plane contour.
Forthe CLsystemtobe stable,theno.ofzerosof q(s)whicharethe CLpolesthatlieinthe right half of s-plane should be zero. That is Z = 0, which gives N = -P.
Therefore,for a stable systemthe no.of ACWencirclementsof the origininthe q(s)-plane by the contour Cq must be equal to P.

Nyquistmodifiedstability criteria

- Weknowthatq(s)=1+G(s)H(s)

Therefore G(s)H(s) = [1+G(s)H(s)] – 1

- The contour Cq, which has obtained due to mapping of Nyquist contour from s-plane to q(s)-plane (ie)[1+G(s)H(s)] -plane, will encircle about the origin.
- The contour CGH, which has obtained due to mapping of Nyquist contour from s-planeto G(s)H(s) -plane, will encircle about the point (-1+j0).
- Thereforeencirclingtheorigininthеq(s)-planeis equivalenttoencirclingthepoint -1+j0 in the G(s)H(s)-plane.



Problem

Sketch the Nyquist stabilityplot for a feedback system with the following open-loop transfer function

$$G(s)H(s) = \frac{1}{s(s^2+s+1)}$$

Solution

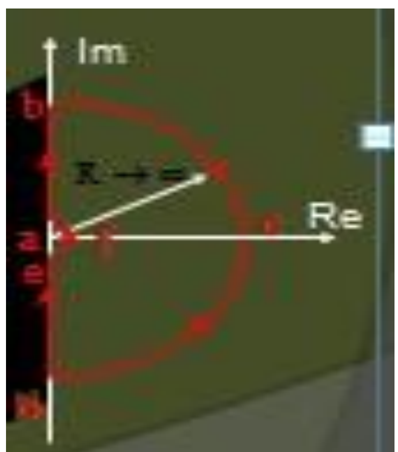
For section ab, s = jω, ω : 0 →∞

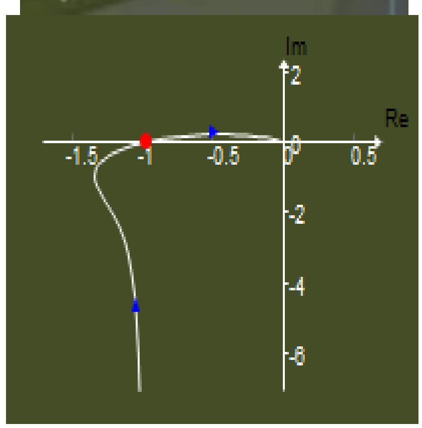
$$G(j\omega)H(j\omega) = \frac{1}{j\omega(1-\omega^2+j\omega)}$$

(i) $\omega \to 0$: $G(j\omega)H(j\omega) \to -1 - j\infty$

(ii) $\omega = 1$: $G(j\omega)H(j\omega) \to -1 + j0$

(iii) $\omega \to \infty$: $G(j\omega)H(j\omega) \to 0\angle -270^\circ$

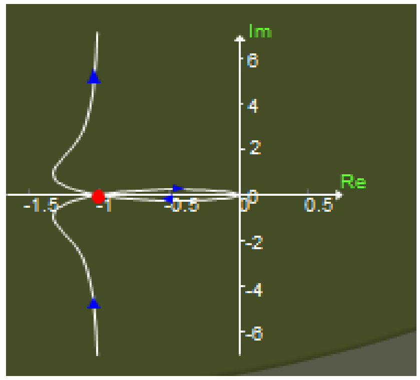




On section bcd, $s = Re^{j\theta}|_{R\to\infty}$; therefore i.e. section bcd maps onto the origin of the G(s)H(s)-plane

$$|G(s)H(s)| \to \frac{1}{R^3} \to 0$$

Sectiondemaps asthe compleximageof thepolar plotas before



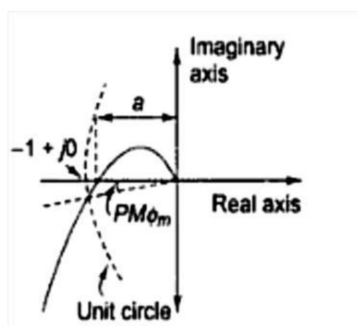
Relativestability

The main disadvantage of a Bode plot is that we have to draw and consider two different curvesatatime,namely,magnitudeplotandphaseplot.Informationcontainedinthesetwo plots can be combined into one named polar plot. The polar plot is for a frequency range of 0<w<α . while the Nyquist plot is in the frequency range of - α<w<α. The information on the negative frequency is redundant because the magnitude and real part of *G*(*jw*) *an* are even functions. In this section. We consider how to evaluate the system performance in terms of relative stability using a Nyquist plot. The open-loop system represented by this plot will become unstablebeyond a certain value. As shown in the Nyquist plot of Fig. the intercept of magnitude 'a on the negativereal axis corresponds lost phaseshift of - 180° and -1 represents theamount ofincrease ingainthatcanbetoleratedbeforeclosed-loopsystemtendstowardinstability.As*'a'*approaches (-1+ *j*0) point the relative stability is reduced; the gain and phase margins are represented as follows in the Nyquist plot.

Gainmargin

As system gain is increased by a factor 1/a, the open loopmagnitude of *G* (*jw*)*H*(*jw*)will increase by a factor a(1/a) = 1 and the system would be driven to instability. Thus, the gain margin is the reciprocal of the gain at the frequency at which the phase angle of the Nyquist plot is - 180^0. The gain rnargin, usually measured in dB, is a positive quantity given by

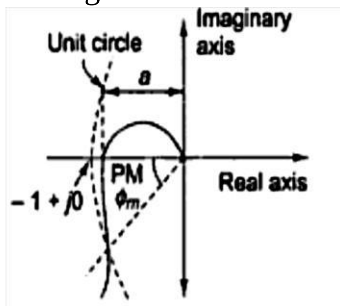
GM =-20log*adB*



Gain margin representation in Nyquist plot with small PM

PhaseMarginϕ$_m$

Importance of the phase margin has already in the content of Bode. Phase margin is defined as the change in open-loop phase shift required al unity gain to make a closed loop system unstable. A closed-loop system will be unstable if the Nyquist plot encircles -1 +*j*0 point. Therefore, the angle required to make this system marginally stable in a closed loop is the phase margin .In order to measure this angle, we draw a circle with a radius of 1, and find the point of intersection of the Nyquist plot with this circle, and measure the phase shift needed for this point to be at an angle of 1800. If may be appreciated that the system having plot of Fig with largerPM is more stable than the one with plot of Fig.



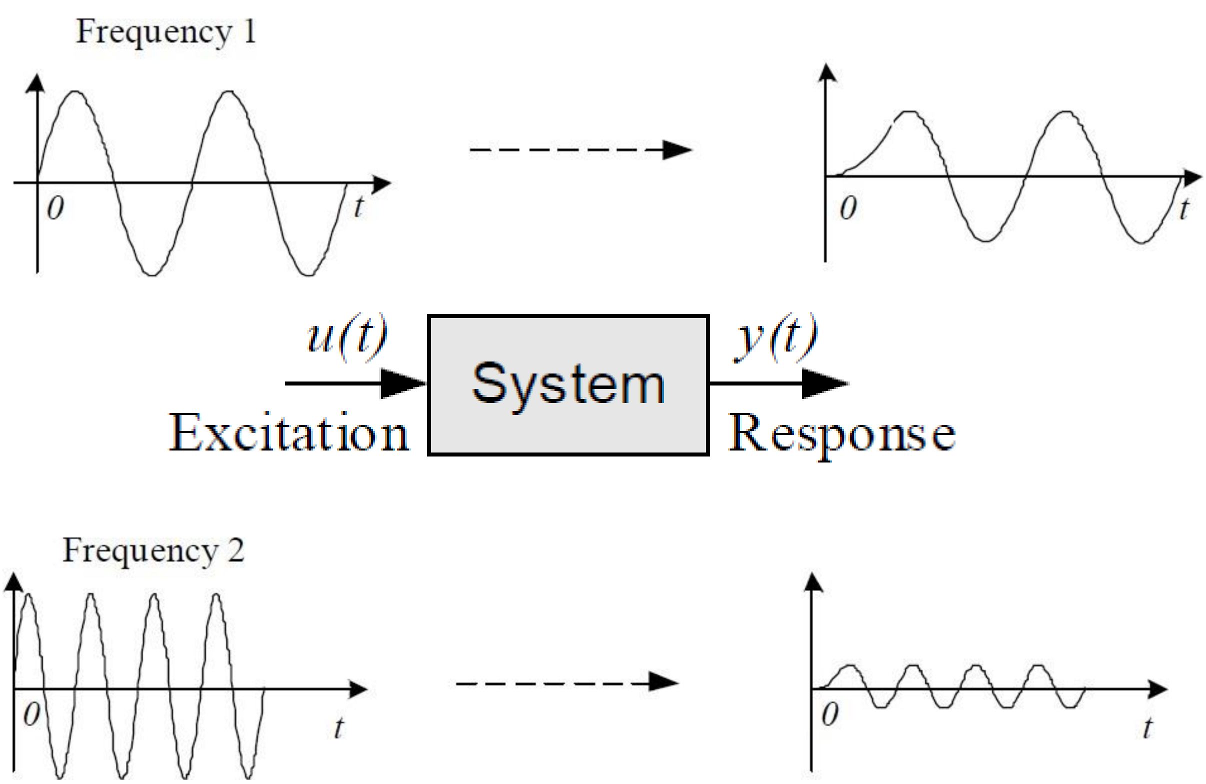
Phase margin representation

FREQUENCYRESPONSE ANALYSIS

FrequencyResponse

The frequency response of a system is a frequency dependent function which expresses how a sinusoidal signal of a given frequency on the system input is transferred through the system. Time-varying signals at least periodical signals —which excite systems, as the reference (set point) signal or a disturbance in a control system or measurement signals which are inputs signals to signal filters, can be regarded as consisting of a sum of frequency components. Each frequency component is a sinusoidal signal having certain amplitude and a certain frequency. (The Fourier series expansion or the Fourier transform can be used to express these frequency components quantitatively.) The frequency response expresses how each of these frequency components is transferred through the system. Some components may be amplified, others may be attenuated, and there will be some phase lag through the system.

The frequency response is an important tool for analysis and design of signal filters (as low pass filters and high pass filters), and for analysis, and to some extent, design, of control systems.Both signalfilteringand controlsystemsapplicationsaredescribed(briefly)laterinthis chapter. The definition of the frequency response — which will be given in the next section — applies onlyto linearmodels, but thislinearmodel mayverywell bethe local linearmodel about someoperatingpoint ofanon-linearmodel. Thefrequencyresponsecan found experimentallyor from a transfer function model. It can be presented graphically or as a mathematical function.



Bodeplot

• Plotsofthemagnitudeandphasecharacteristicsareusedtofullydescribethefrequency response

• A**Bodeplot**isa(semilog)plotofthetransferfunctionmagnitudeandphaseangleasa function of frequency.

Thegain magnitudeis manytimesexpressed in terms ofdecibels (dB)

$$db = 20 \log 10 A$$

BODEPLOTPROCEDURE:

Thereare4basic formsin anopen-loop transfer function $G(j\omega)H(j\omega)$

- GainFactorK
- $(j\omega)\pm p$factor:poleandzeroatorigin $(1+j\omega T)\pm q$
- factor
- Quadraticfactor
  $1+j2\zeta(W / W_n)-(W^2 / W_n^2)$

GainmarginandPhasemargin

Gain margin:

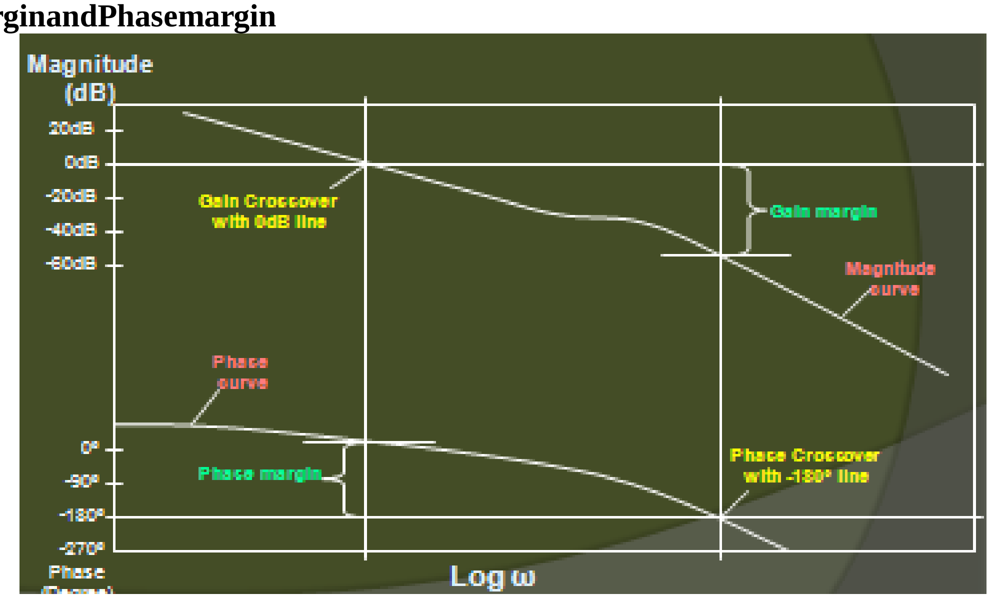
Thegainmarginisthenumberofdbthatisbelow0dBatthephasecrossoverfrequency (ø=-180º). It can also be increased before the closed loop system becomes unstable

Term	Corner Frequency	Slopedb /dec	Changein slope
20/jW	-----	-20	
1/ (1+4jW)	$WC_1=1/4= 0.25$	-20	-20-20=-40
1/(1+j3w)	wc2=1/3=0.33	-20	-40-20=-60

Phase margin:

Thephasemarginisthenumberofdegreesthephaseofthatisabove-180ºatthegain crossover frequency

GainmarginandPhasemargin



BodePlot–Example

ForthefollowingT.FdrawtheBodeplotandobtainGaincrossoverfrequency(wgc), Phase cross over frequency , Gain Margin and Phase Margin.

G(s)= 20 /[s(1+3s) (1+4s)]

Solution:

ThesinusoidalT.FofG(s)isobtainedbyreplacingsbyjwinthegivenT.F G(jw) = 20 / [jw (1+j3w) (1+j4w)]

Cornerfrequencies:

wc1=1/4=0.25rad/sec;

wc2 = 1/3 = 0.33 rad /sec

ChoosealowercornerfrequencyandahigherCornerfrequency wl= 0.025 rad/sec ;

wh=3.3rad / sec

CalculationofGain(A)(MAGNITUDEPLOT) A

@ wl ; A= 20 log [20 / 0.025] = 58 .06 dB

A@wc1; A=[Slopefrom wlto wc1xlog(wc1/wl] +Gain (A)@wl

=-20 log[0.25 / 0.025] +58.06

=38.06 dB

A@wc2; A =[Slope from wc1to wc2 xlog(wc2 /wc1]+Gain (A)@wc1

=-40 log[0.33 / 0.25]+ 38

=33 dB

A@wh ; A=[Slope from wc2to wh xlog(wh /wc2] +Gain (A) @wc2

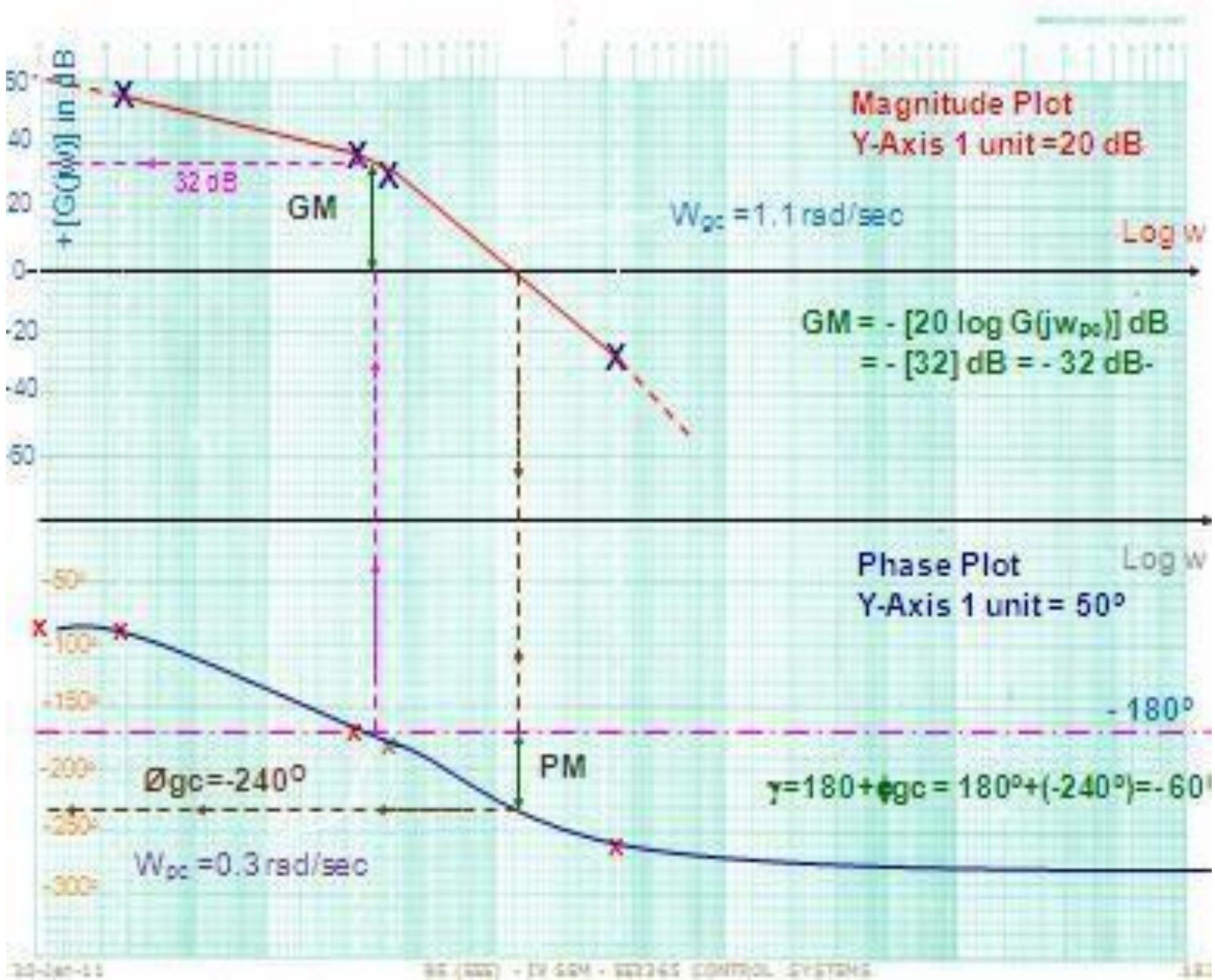
=-60 log[3.3 / 0.33]+33

=-27dB

CalculationofPhaseanglefordifferentvaluesoffrequencies[PHASEPLOT] Ø = - 90^{O}- $\tan^{-1} 3w - \tan^{-1} 4w$

When

Frequencyinrad/sec	**PhaseanglesinDegree**
w=0	Ø=-90 ^{0}
w=0.025	Ø=-99^{0}
w=0.25	Ø=-172^{0}
w=0.33	Ø=-188^{0}
w=3.3	Ø=-259^{0}
w=∞	Ø=-270^{0}



- **CalculationsofGain crossover frequency**

ThefrequencyatwhichthedBmagnitudeisZero wgc =
1.1 rad / sec

- **Calculations of Phase cross over**

frequencyThefrequencyatwhichthePhaseofthesystemis -
180o wpc = 0.3 rad / sec

- Gain Margin

  The gain margin in dB is given by the negative of dB magnitude of G(jw) at phase cross over frequency

  GM=-{ 20 log[G(jwpc)]= - { 32 } =-32dB

- **Phase Margin**

  Ґ$=180^0+$Øgc$=180^0+(-2400^o)=-60^0$

- **Conclusion**

  For this system GM and PM are negative in values. Therefore the system is unstable innature.

Polarplot

Tosketchthepolarplotof*G*(*j*ω)fortheentirerangeoffrequencyω,i.e.,from0to infinity, there are four key points that usually need to be known:
(1) thestart of plotwhere ω= 0,
(2) theendofplotwhere ω= ∞,
(3) wherethe plotcrosses thereal axis,i.e.,Im(*G*(*j*ω))=0, and
(4) wherethe plot crosses the imaginaryaxis, i.e., Re(*G*(*j*ω))= 0.

BASICSOFPOLAR PLOT:

- The polar plot of a sinusoidal transfer function G(jω) is a plot of the magnitude of G(jω) Vs the phase of G(jω) on polar co-ordinates as ω is varied from 0 to ∞.
  (ie)|G(jω)|Vs angleG(jω)asω→0to∞.
- Polargraphsheethasconcentriccirclesandradiallines.
- Concentric circles represents the magnitude.
- Radiallinesrepresentsthephaseangles. In
- polar sheet
  +vephase angleismeasuredinACW from 0^0
  -vephase angleismeasuredinCW from 0^0

PROCEDURE

- ExpressthegivenexpressionofOLTFin(1+sT)form.
- Substitutes=jωintheexpressionforG(s)H(s)andgetG(jω)H(jω). Get the
- expressions for | G(jω)H(jω)| & angle G(jω)H(jω).
- Tabulate various values of magnitude and phase angles for different values of ω ranging from 0 to ∞.
- Usuallythechoiceoffrequencieswillbethecornerfrequencyandaroundcorner frequencies. Chooseproperscaleforthemagnitudecircles.
- Fixallthepointsinthepolargraphsheetandjointhepointsbyasmoothcurve. Write the
- frequency corresponding to each of the point of the plot.
-

MINIMUMPHASESYSTEMS:

- Systemswithallpoles&zerosintheLefthalfofthes-plane–MinimumPhase Systems. ForMinimum PhaseSystems withonlypoles
- TypeNo.determinesatwhatquadrantthepolarplotstarts. Order
- determines at what quadrant the polar plot ends.
- TypeNo.→No.ofpoles lyingatthe origin
- Order→Maxpowerof‘s’inthedenominatorpolynomialofthetransfer function.

GAINMARGIN

- Gain Margin is defined as "the factor by which the system gain can be increased to drive the system to the verge of instability".
- Forstablesystems,

ωgc<ωpc

MagnitudeofG(j)H(j)at ω=ωpc<1

GM =in positive dB
Morepositivethe GM, morestable is thesystem.

- Formarginallystable systems,

$\omega gc=\omega pc$

magnitudeofG(j)H(j)atω=ωpc=1 GM
= 0 dB
ForUnstablesystems,

$\omega gc>\omega pc$

magnitudeofG(j)H(j)atω=ωpc>1 GM
= in negative dB
Gainisto bereduced tomakethesystem stable

Note:

- If the gain is high, the GM is low and the system's step response shows high overshoots and long settling time.
- On the contrary, verylow gains give high GM and PM, butalso causes higher ess, higher values of rise time and settling time and in general give sluggish response.
- Thusweshouldkeepthegainashighaspossibletoreduceessandobtainacceptable response speed and yet maintain adequate GM & PM.
- An adequate GM of 2 i.e. (6 dB) and a PM of 30 is generally considered good enough asa thumb rule.

Atw=wpc,angle ofG(jw)H(jw) = -180^0

- LetmagnitudeofG(jw)H(jw) at $w=w_{pc}$ betakenaB
- Ifthe gainofthesystemisincreasedbyfactor1/B,thenthemagnitudeofG(jw)H(jw)at w = wpc becomes B(1/B) = 1 and hence the G(jw)H(jw) locus pass through -1+j0 point driving the system to the verge of instability.
- GM is defined as the reciprocal of the magnitude of the OLTF evaluated at the phasecross over frequency.
  GMindB=20log(1/B)=-20logB

PHASE MARGIN

PhaseMarginisdefinedas" theadditional phaselag thatcanbe introducedbefore the system becomes unstable".

'A'bethepoint ofintersectionofG(j)H(j)plot andaunitcircle centeredattheorigin.

Drawalineconnecting thepoints'O'& 'A'andmeasurethephaseanglebetweenthe line OA and

+verealaxis.

Thisangleisthephaseangleofthesystemat the gaincrossoverfrequency.

AngleofG(jwgc)H(jwgc)=φ gc

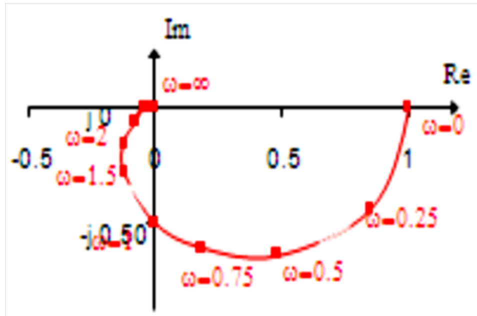
If an additional phase lag of φ PM is introduced at this frequency, then the phase angle G(jwgc)H(jw gc) will become 180 and the point 'A' coincides with (-1+j0) drivingthe system to the verge of instability.

ThisadditionalphaselagisknownasthePhaseMargin. γ= 180^0 + angle of G(jwgc)H(jw gc)

$\gamma=180^0+\varphi gc$

[SinceφgcismeasuredinCWdirection,itistakenasnegative] For a stable system, the phase margin is positive.

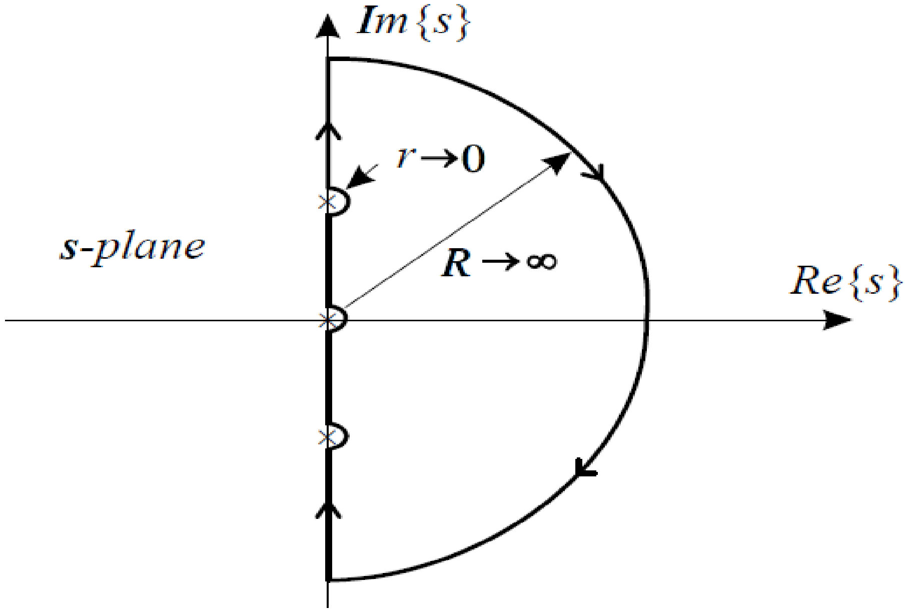
- APhasemarginclosetozero corresponds tohighlyoscillatorysystem.



- Apolarplotmaybeconstructedfromexperimentaldataorfromasystem transfer function
- Ifthevaluesofwaremarkedalongthecontour,apolarplothasthesameinformationas a bode plot. Usually,the shapeofapolarplot is of most interest.
-

NyquistPlot:

TheNyquist plot is a polarplot of the function



The Nyquist stability criterion relates the location of the roots of the characteristic equation to the open-loop frequency response of the system. In this, the computation of closed-loop poles is not necessary to determine the stability of the system and the stability study can be carried out graphically from the open-loop frequency response. Therefore experimentally determined open-loop frequency response can be used directly for the study of stability. When the feedback path is closed. The Nyquist criterion has the following features that make it an alternative method that is attractive for the analysis and design of control systems. 1. In addition to providing information on absolute and relative.

NyquistPlotExample

Considerthefollowingtransfer function

$$G(s) = \frac{k(s+1)}{s^2(s+4)(s+5)}$$

Changeitfrom"s"domainto"jw"domain:

$$G(jw) = \frac{k(j\omega+1)}{(j\omega)^2(j\omega+4)(j\omega+5)}$$

Findthemagnitudeandphaseangle equations:

$$\frac{k(\sqrt{\omega^2+1})}{\omega^2(\sqrt{\omega^2+16})(\sqrt{\omega^2+25})}\angle -180+\tan^{-1}\omega-\tan^{-1}(\frac{\omega}{4})-\tan^{-1}(\frac{\omega}{5})$$

Evaluatemagnitudeandphaseangle atω=0+andω= +∞

At $\omega = 0+$

$$|G(jw)|\angle G(jw) \Rightarrow \infty\angle -180+\varepsilon$$

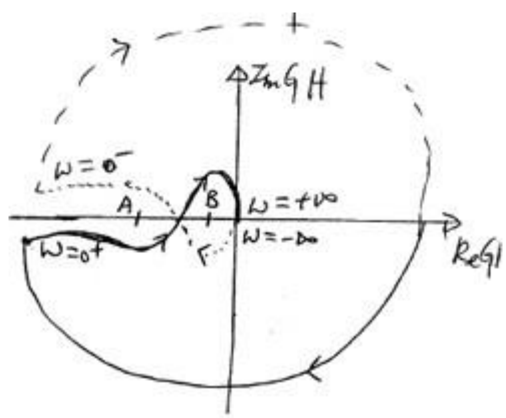
At $\omega = \infty$

$$|G(jw)|\angle G(jw) \Rightarrow \infty\angle -180+\varepsilon$$

At $\omega = \infty$

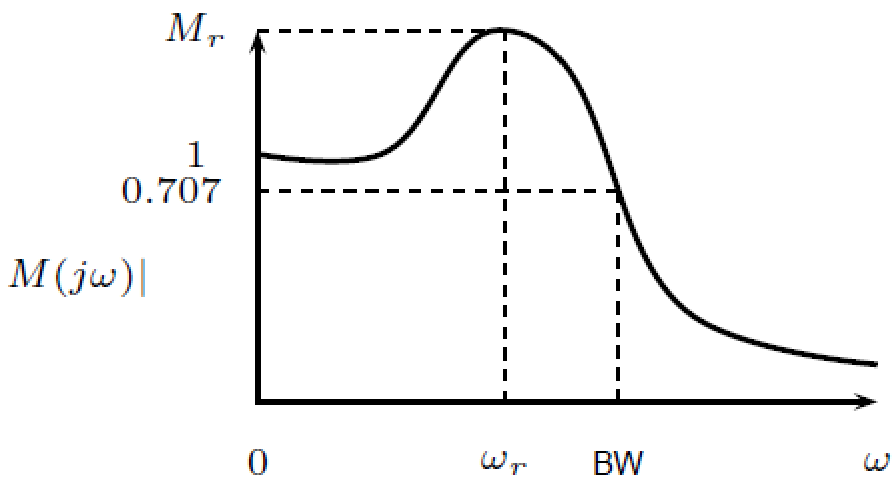
$$|G(jw)|\angle G(jw) \Rightarrow 0\angle -270$$

Drawthenyquistplot:



Frequencydomain specifications

- Theresonant peak Mr isthe maximum valueof jM(jw)j.
- Theresonantfrequency!r is the frequencyat which the peak resonanceMroccurs.
- ThebandwidthBWisthefrequencyatwhich(jw)dropsto70:7%(3dB)ofitszero- frequency value.

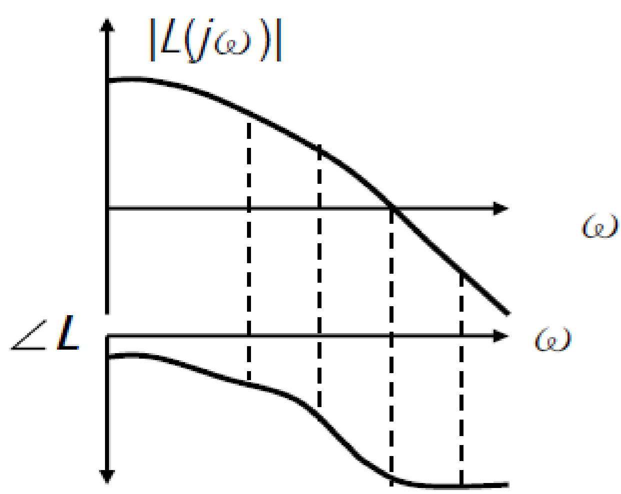


- Mr indicates therelativestabilityof astableclosed loopsystem.
- AlargeMrcorrespondstolargermaximumovershootofthestepresponse. Desirable
- value: 1.1 to 1.5
- BW gives anindicationofthetransientresponsepropertiesofacontrol system.
- Alargebandwidthcorrespondstoafasterrisetime.BWandrisetimetrare inversely proportional.
- BWalsoindicatesthenoise-filteringcharacteristicsandrobustnessofthesystem.
- Increasing wn increases BW.
- BWandMrareproportionaltoeachother.

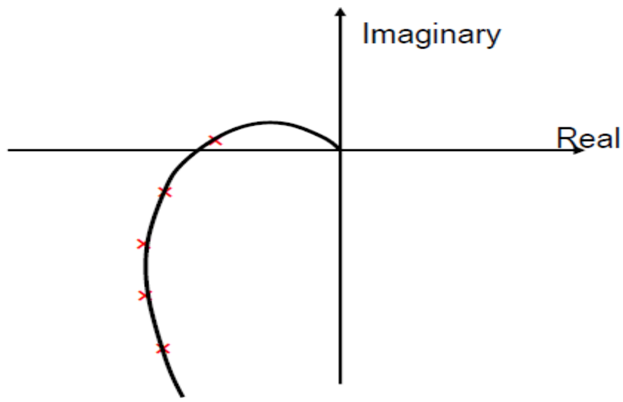
Constant Mand N circles

Consideracandidatedesign ofaloop transfer function$L(j\omega)$ shown on the RHS.

$$T(j\omega)=\frac{L(j\omega)}{1+L(j\omega)}$$



Evaluate$T(j\omega)$ from $L(j\omega)$ inthemanner offrequencypoint byfrequency point.
Alternatively,theBodeplotof$L(j\omega)$canalsobeshowonthecomplexplanetoformitsNyquist plot.



Mcircles(constantmagnitudeof T)

Inordertopreciselyevaluate$|T(j\omega)|$fromtheNyquistplotof$L(j\omega)$,atoolcalledMcircleis developed as followed.

Let$L(j\omega)=X+jY$, where$X$is thereal and Ytheimaginarypart . Then

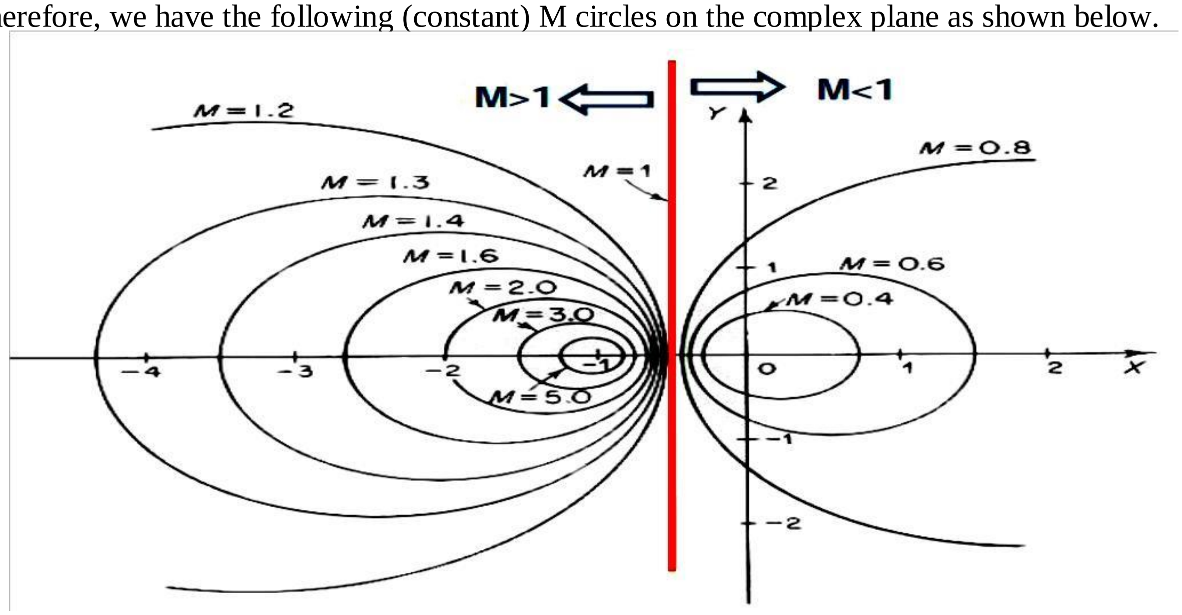
$$|T(j\omega)| = M = \frac{|X(j\omega)+jY(j\omega)|}{|1+X(j\omega)+jY(j\omega)|},$$

$$M(j\omega)^2 = \frac{X(j\omega)^2+Y(j\omega)^2}{\left(1+X(j\omega)\right)^2+Y(j\omega)^2}$$

Rearranging the above equations, it gives

X2(1-M2)-2M2X-M2+(1-M2)Y2=0

That is, all (X, Y) pair corresponding to a constant value of M for a circle on the complex plane. Therefore, we have the following (constant) M circles on the complex plane as shown below.

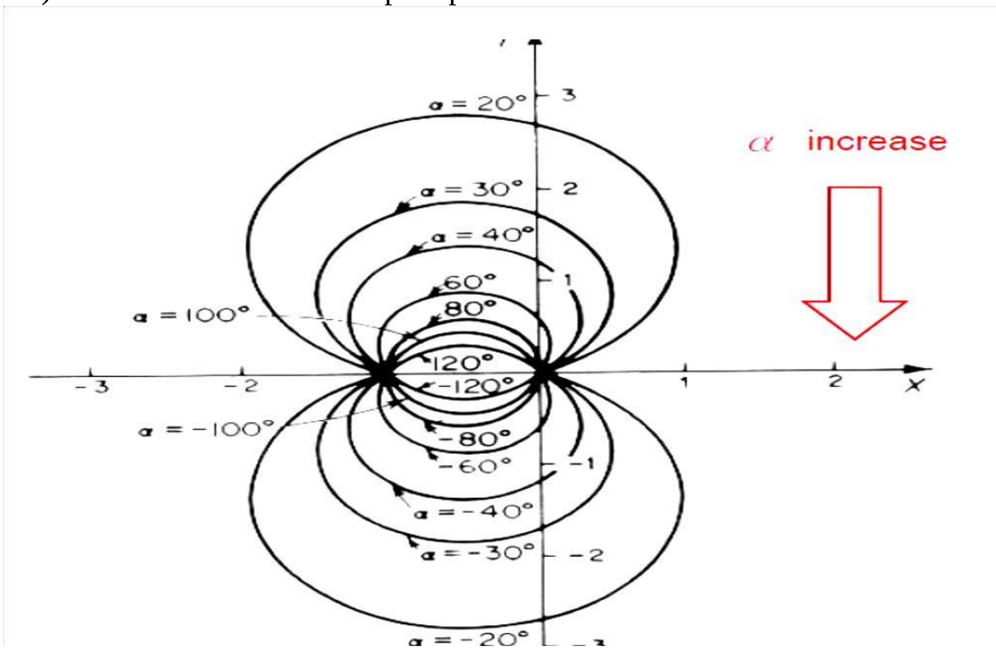


Ncircles(constantphase ofT)

Similarly,itcanbeshownthatthephaseof$T(j\omega)$be

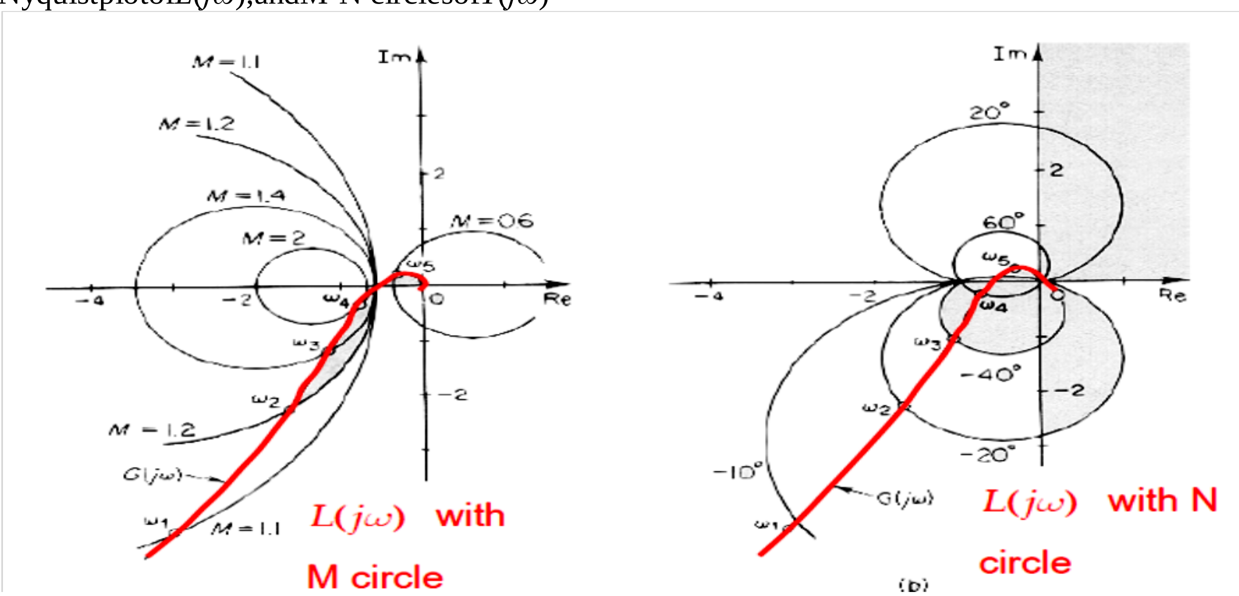
$$\alpha \triangleq \angle T(j\omega) = \tan^{-1}\left[\frac{Y}{X}\right] - \tan^{-1}\left[\frac{Y}{1+X}\right]$$

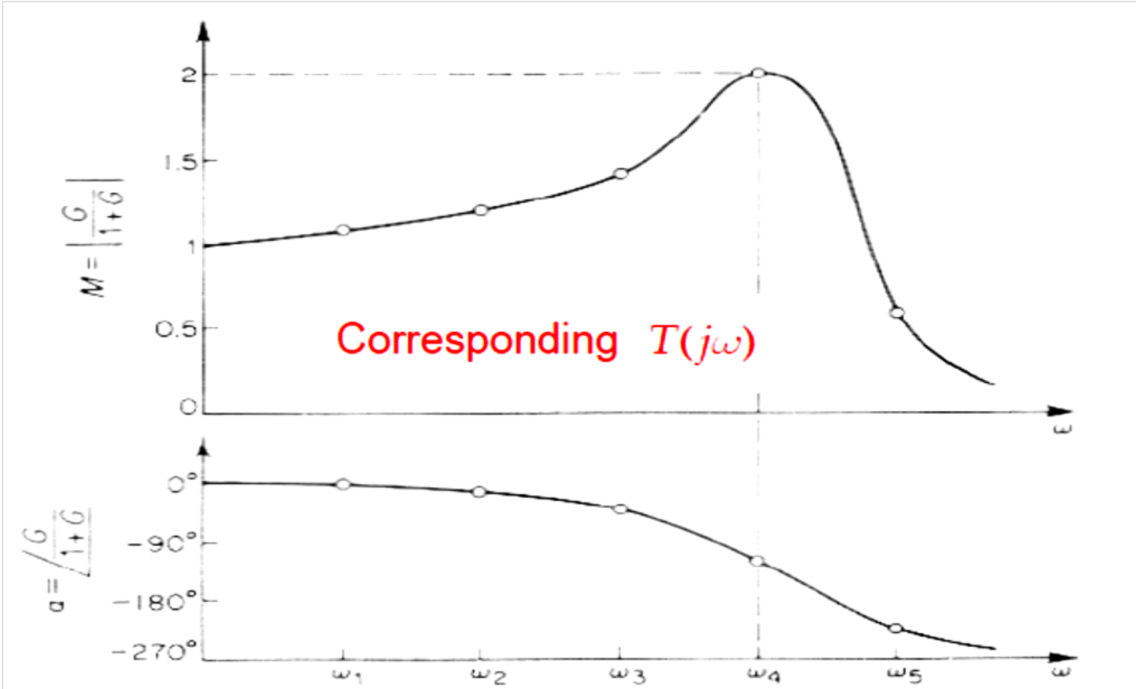
Itcan be shown that all(X, Y) pair which corresponds to the sameconstant phase of T(i.e., constant N) forms a circle on the complex plane as shown below.



Example

NyquistplotofL($j\omega$),andM-N circlesof$T(j\omega)$

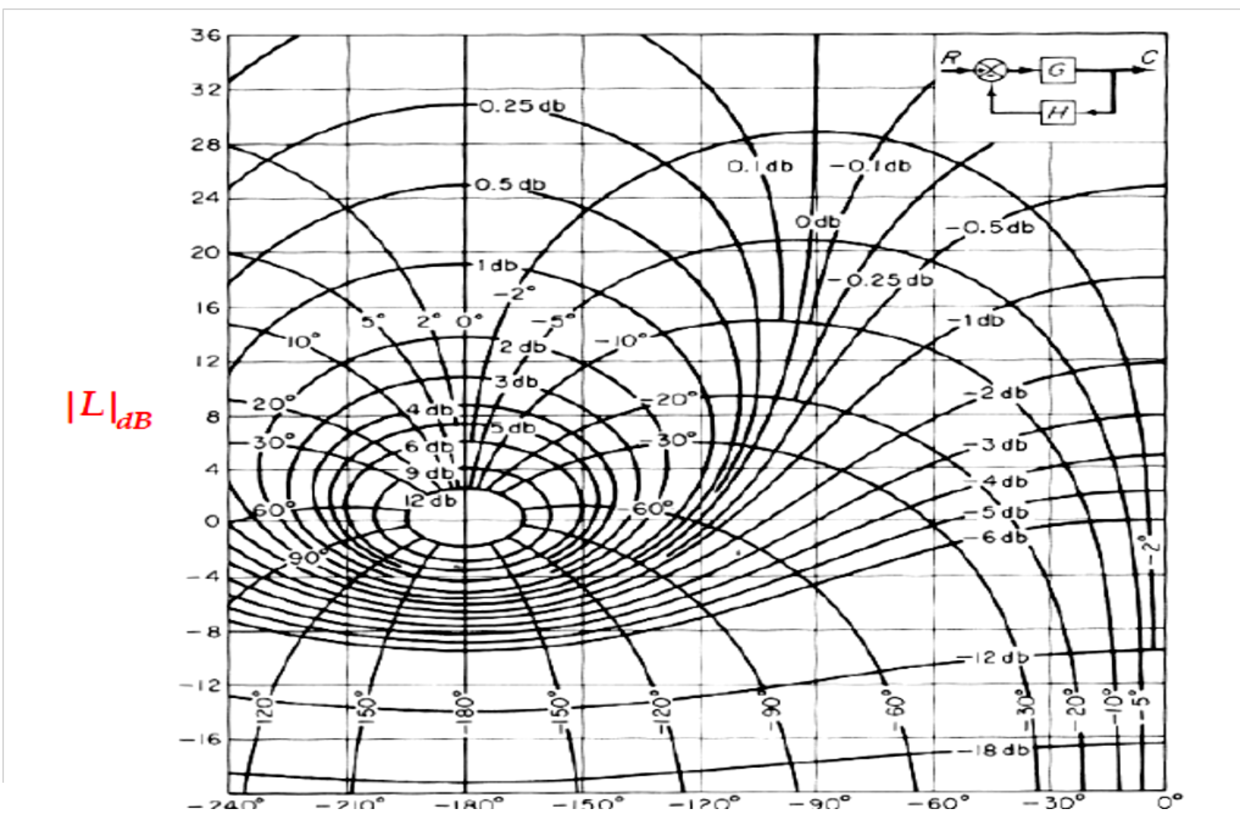




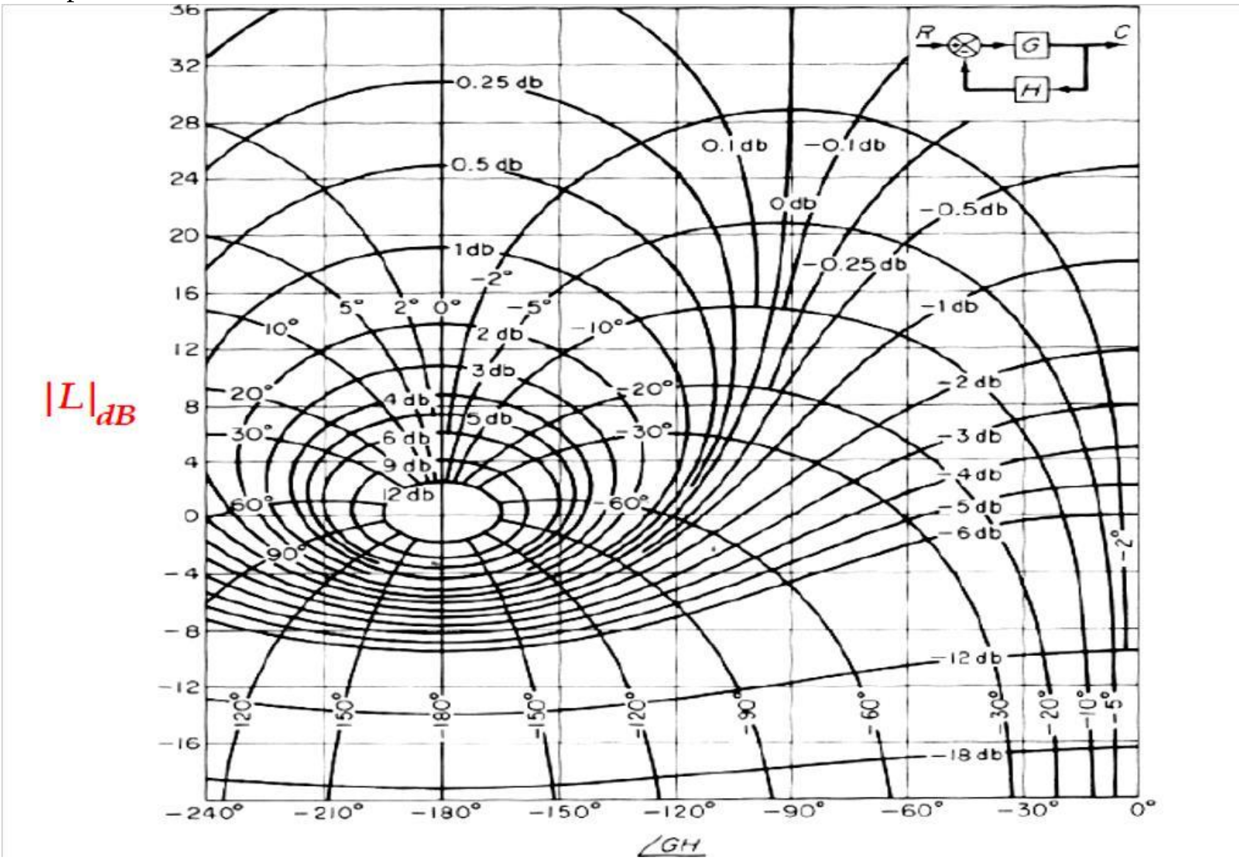
NicholsChart

TheNyquistplotof$L(j\omega)$canalsoberepresentedbyitspolarformusingdBas magnitude and degree as phase.

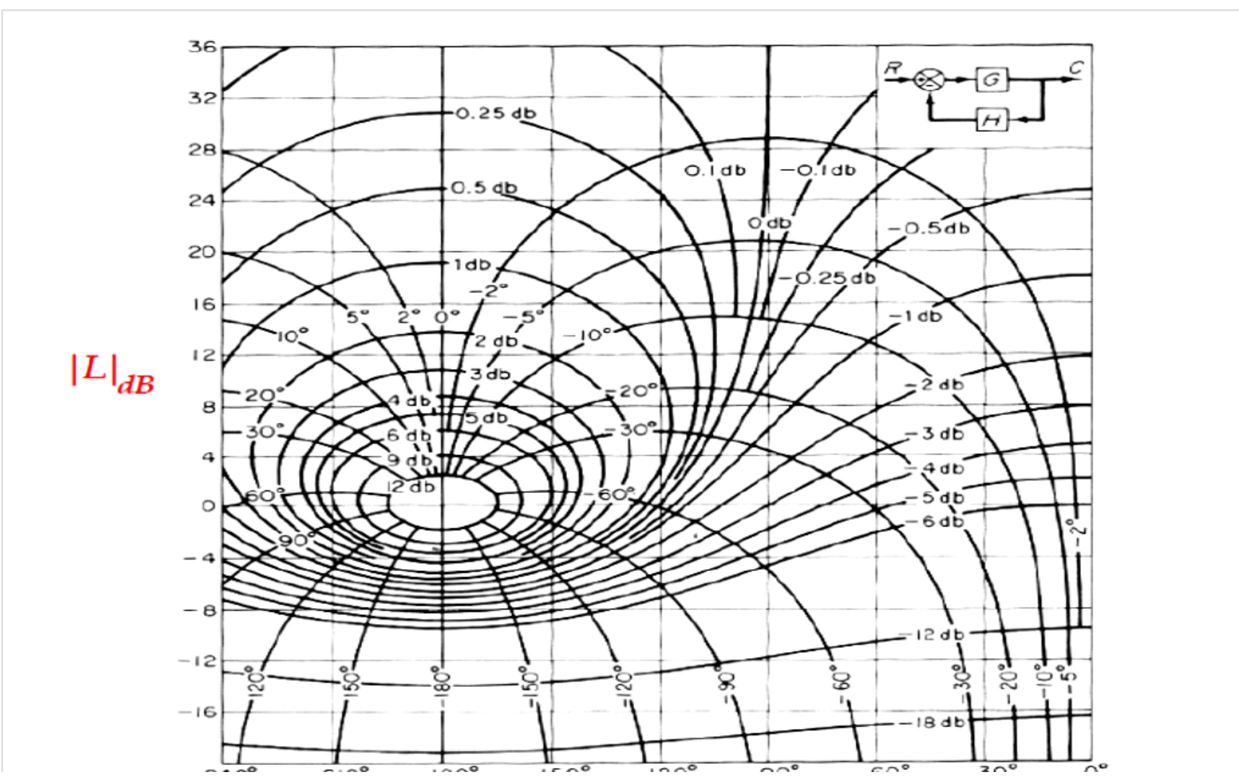
$$L(j\omega) = |L|_{dB}\, e^{j\alpha}$$



And ll $L(j\omega)$ which corresponds to a constant $\alpha(j\omega)$ can be draw as a locus of M circleon this plane as shown below.



CombiningtheabovetwographsofMcirclesandNcircles,wehavetheNicholaschart below.

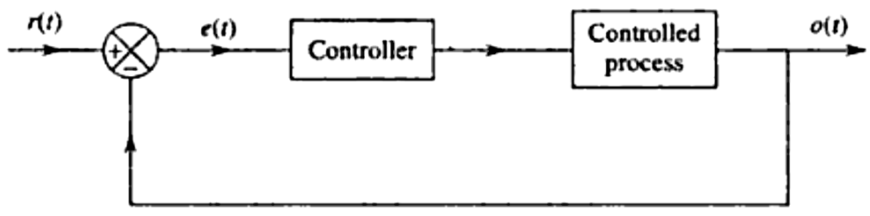


TYPESOFCOMPENSATION

- Series Compensation or Cascade Compensation

This is the most commonly used system where the controller is placed in series with the controlled process.

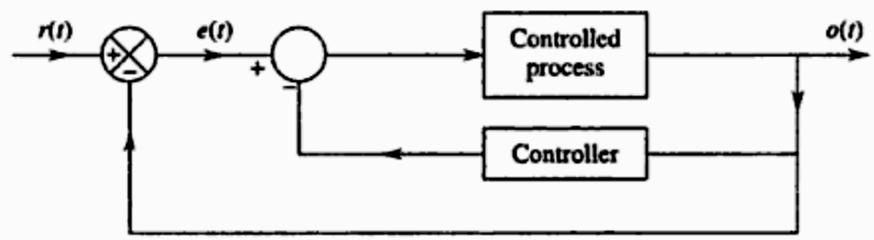
Figureshowstheseriescompensation



Series compensation.

FeedbackcompensationorParallel compensation

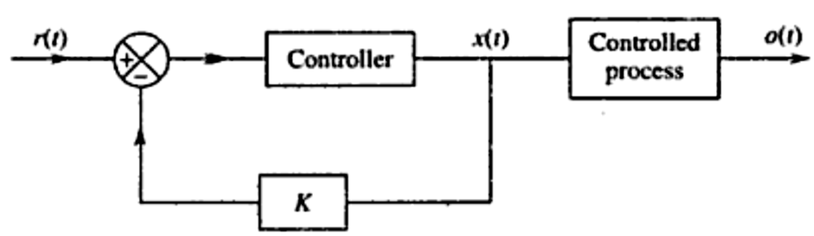
Thisisthesystemwherethecontrollerisplacedinthesensorfeedbackpathasshownin fig.



Feedback compensation or parallel compensation.

StateFeedbackCompensation

This is a system which generates the control signal by feeding back the state variables through constant real gains. The scheme is termed state feedback. It is shown in Fig.



State feedback compensation.

The compensation schemes shown in Figs above have one degree of freedom, since there is only one controller in each system. The demerit with one degree of freedom controllers is that the performance criteria that can be realized are limited.

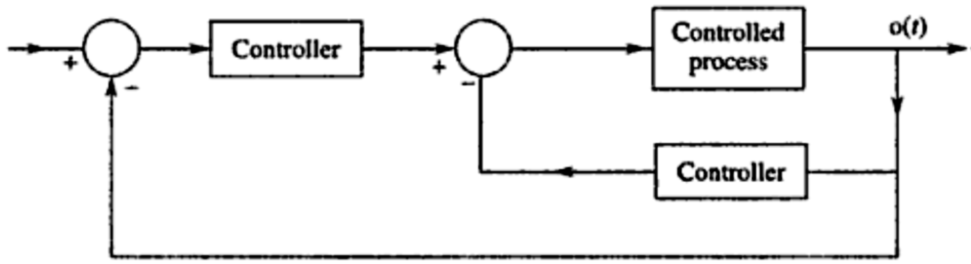
Thatis whytherearecompensation schemes which havetwo degreefreedoms, suchas:

(a) Series-feedback compensation

(b) Feedforwardcompensation

Series-FeedbackCompensation

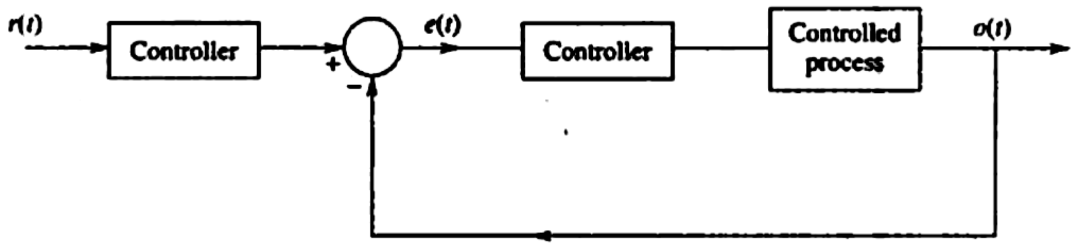
Series-feedback compensation is the scheme for which a series controller and a feedback controller are used. Figure 9.6 shows the series-feedback compensation scheme.



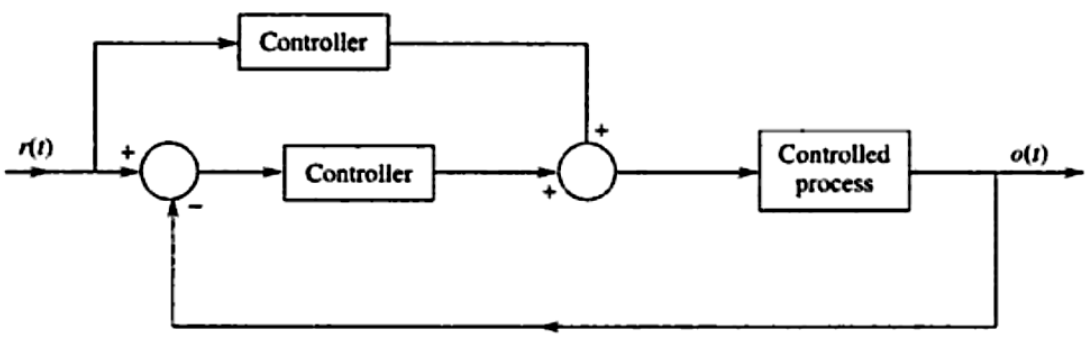
Series-feedback compensation.

FeedforwardCompensation

The feed forward controller is placed in series with the closed-loop system which has a controller in the forward path Orig. 9.71. In Fig. 9.8, Feed forward the is placed in parallel with the controller in the forward path. The commonly used controllers in the above-mentioned compensation schemes are now described in the section below.



Feedforward controller in series with the closed-loop system.



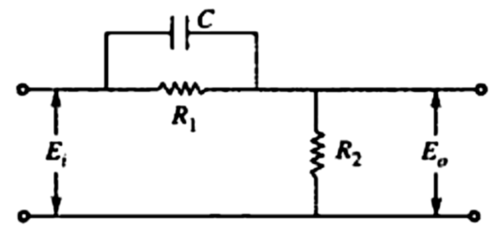
Feedforward controller in parallel with the controller in the forward path.

LeadCompensator

Ithasazeroandapolewithzeroclosertotheorigin.Thegeneralformofthetransfer function of the load compensator is

$$G(s) = \frac{s + \frac{1}{\tau}}{s + \frac{1}{\beta\tau}}$$

$$G(j\omega) = \beta \frac{(\tau j\omega + 1)}{\beta\tau j\omega + 1}$$



Lead compensator.

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2}{\dfrac{R_1 \times \frac{1}{Cs} + R_2\left(R_1 + \frac{1}{Cs}\right)}{R_1 + \frac{1}{Cs}}} = \frac{R_2R_1 + \frac{R_2}{Cs}}{R_1R_2 + \frac{1}{Cs}(R_1 + R_2)}$$

$$= \frac{Cs\,R_1R_2 + R_2}{Cs\,R_1R_2 + R_1 + R_2}$$

$$= \frac{R_2(Cs\,R_1 + 1)}{(R_1 + R_2)\left(\frac{CsR_1R_2}{R_1 + R_2} + 1\right)}$$

$$= \left(\frac{R_2}{R_1 + R_2}\right)\frac{CR_1s + 1}{\left(\frac{CR_1R_2s}{R_1 + R_2} + 1\right)}$$

Subsisting

$$\tau = CR_1; \qquad \beta\tau = \frac{CR_1R_2}{R_1 + R_2} \quad (\because\ \tau = CR_1)$$

Transfer function

$$G(s) = \beta\,\frac{\tau s + 1}{\beta\tau s + 1}$$

Lag Compensator

It has a zero and a pole with the zero situated on the left of the pole on the negative real axis. The general form of the transfer function of the lag compensator is

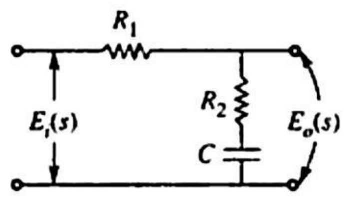
$$G(s) = \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}} = \frac{\alpha(\tau s + 1)}{\alpha\tau s + 1}$$

where $\alpha > 1,\ \tau > 0.$

Therefore,the frequencyresponseofthe above transferfunctionwill be

$$G(j\omega) = \frac{\alpha(\tau j\omega + 1)}{\alpha\tau j\omega + 1}$$

$$E_o(s) = \frac{E_i(s)}{R_1 + R_2 + \frac{1}{Cs}}\left(R_2 + \frac{1}{Cs}\right)$$



Lag compensator.

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}}$$

$$= \frac{R_2Cs + 1}{(R_1 + R_2)Cs + 1}$$

$$= \frac{R_2C\left(s + \frac{1}{R_2C}\right)}{(R_1 + R_2)C\left(S + \frac{1}{(R_1 + R_2)C}\right)}$$

$$= \frac{R_2}{(R_1 + R_2)} \frac{s + \frac{1}{R_2C}}{\left(s + \frac{1}{(R_1 + R_2)C}\right)} = \frac{R_2}{(R_1 + R_2)} \frac{\left(s + \frac{1}{R_2C}\right)}{\left(s + \frac{R_2}{(R_1 + R_2)R_2C}\right)}$$

Nowcomparingwith

$$G(s) = \frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}}$$

$$\frac{1}{\tau} = \frac{1}{R_2 C}; \qquad \frac{1}{\alpha\tau} = \frac{R_2}{(R_1 + R_2)R_2 C}$$

$$\frac{1}{\alpha\tau} = \frac{R_2}{(R_1 + R_2)}\frac{1}{\tau} \qquad \left(\because \frac{1}{\tau} = \frac{1}{R_2 C}\right)$$

$$\alpha = \frac{R_1 + R_2}{R_2}$$

Therefore

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{\alpha}\frac{s + \frac{1}{\tau}}{s + \frac{1}{\alpha\tau}}$$

Lag-Lead Compensator

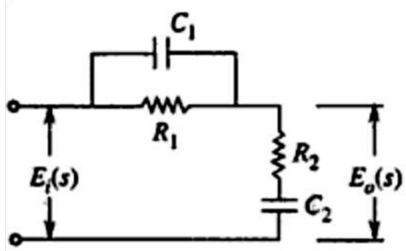
The lag-lead compensator is the combination of a lag compensator and a lead compensator. The lag-section is provided with one real pole and one real zero, the pole being to the right of zero, whereas the lead section has one real pole and one real came with the zerobeing to the right of the pole.

Thetransferfunctionofthelag-leadcompensator willbe

$$G(s) = \left(\frac{s + \frac{1}{\tau_1}}{s + \frac{1}{\alpha\tau_1}}\right)\left(\frac{s + \frac{1}{\tau_2}}{s + \frac{1}{\beta\tau_2}}\right)$$

Thefigureshows laglead compensator

$$E_o(s) = \frac{E_i(s)}{\dfrac{R_1 \times \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} + R_2 + \dfrac{1}{sC_2}}\left(R_2 + \frac{1}{sC_2}\right)$$



Lag-lead compensator.

where $\alpha > 1$, $\beta < 1$.

$$\frac{E_o(s)}{E_i(s)} = \frac{\left(R_1 + \frac{1}{sC_1}\right)\left(R_2 + \frac{1}{sC_2}\right)}{R_1 \frac{1}{sC_1} + \left(R_2 + \frac{1}{sC_2}\right)\left(R_1 + \frac{1}{sC_1}\right)}$$

$$= \frac{\frac{(sC_1R_1 + 1)}{sC_1}\frac{(sC_2R_2 + 1)}{sC_2}}{\frac{R_1}{sC_1} + \frac{(R_2sC_2 + 1)}{sC_2}\frac{(R_1sC_1 + 1)}{sC_1}}$$

$$= \frac{\frac{(1 + sC_1R_1)(1 + sC_2R_2)}{s^2C_1C_2}}{\frac{R_1sC_2 + R_2sC_2 + 1 + R_1R_2s^2C_1C_2 + R_1sC_1}{s^2C_1C_2}}$$

$$= \frac{(1 + sC_1R_1)(1 + sC_2R_2)}{s^2R_1R_2C_1C_2 + s(R_1C_1 + R_2C_2) + 1 + R_1sC_2}$$

$$= \frac{C_1R_1\,C_2R_2\left(s + \frac{1}{C_1R_1}\right)\left(s + \frac{1}{C_2R_2}\right)}{R_1R_2C_1C_2\left[s^2 + \left\{\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right\}s + \frac{1}{R_1R_2C_1C_2}\right]}$$

$$= \frac{\left(s + \frac{1}{C_1R_1}\right)\left(s + \frac{1}{C_2R_2}\right)}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}}$$

Theabovetransfer functions arecomparingwith

$$G(s) = \frac{\left(s + \frac{1}{\tau_1}\right)\left(s + \frac{1}{\tau_2}\right)}{\left(s + \frac{1}{\alpha\tau_1}\right)\left(s + \frac{1}{\beta\tau_2}\right)}$$

Then

$$\frac{1}{\tau_1} = \frac{1}{C_1R_1}, \qquad \frac{1}{\tau_2} = \frac{1}{C_2R_2}$$

$$\frac{1}{\alpha\tau_1} + \frac{1}{\beta\tau_2} = \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2}$$

$$\frac{1}{\alpha\beta\tau_1\tau_2} = \frac{1}{R_1R_2C_1C_2}$$

$$\tau_1 = C_1R_1$$

$$\tau_2 = C_2R_2$$

$$\alpha\beta\tau_1\tau_2 = R_1R_2C_1C_2$$

$$\alpha\beta = 1 \quad \text{or} \quad \beta = \frac{1}{\alpha}$$

Therefore

$$G(s) = \frac{\left(s+\frac{1}{\tau_1}\right)\left(s+\frac{1}{\tau_2}\right)}{\left(s+\frac{1}{\alpha\tau_1}\right)\left(s+\frac{\alpha}{\tau_2}\right)} \quad \text{where } \alpha > 1$$

$$\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} = \frac{1}{\alpha\tau_1} + \frac{\alpha}{\tau_2}$$

STATEVARIABLEANALYSIS

StatespacerepresentationofContinuousTime systems

The state variables may be totally independent of each other, leading to diagonal or normal form or they could be derived as the derivatives of the output. If them is no direct relationship between various states. We could use a suitable transformation to obtain the representation in diagonal form.

PhaseVariableRepresentation

It is often convenient to consider the output of the system as one of the state variable and remaining state variable as derivatives of this state variable. The state variables thus obtained from one of the system variables and its (n-1) derivatives, are known as n-dimensional phase variables.

Inathird-ordermechanicalsystem,theoutputmaybedisplacementx_1, $x_1 = x_2 =$ vandx_2 $= x_3 = a$ in the case of motion of translation or angular displacement θ 1 = x_1, $x_1 = x_2 =$ wand $x_2 = x_3 = \alpha$ if the motion is rotational, Where v *v,w,a*, α respectively, are velocity, angular velocity acceleration, angular acceleration.

ConsideraSISOsystemdescribedbynth-orderdifferential equation.

$$y^{(n)}(t) + a_1 y^{(n-1)}(t) + \dots + a_{n-1} y(t) + a_n y(t) = Ku$$

Where

$$y^{(n)}(t) = d^n y(t)/dt^n,$$

uis,ingeneral,afunctionoftime.

Thenthordertransfer functionofthissystem is

$$G(s) = \frac{y(s)}{u(s)} = \frac{K}{s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n}$$

Withthestates(eachbeingfunctionoftime)bedefinedas

$$x_1 = y(t), \quad x_2 = \dot{y}(t), \; x_3 = \ddot{y}(t), \dots, x_n = y^{(n-1)}(t),$$

Equation becomes

$$\dot{x}_n + a_1 x_n + a_2 x_{n-1} + \dots + a_{n-1} x_2 + a_n x_1 = Ku(t)$$
$$\dot{x}_n = -a_1 x_n - a_2 x_{n-1} - \dots - a_{n-1} x_2 - a_n x_1 + Ku$$

UsingaboveEqsstate equationsin phasesatiable loan canheobtained as

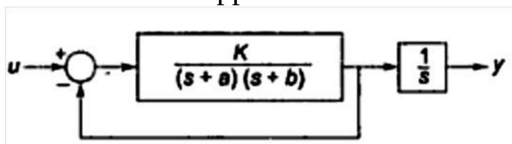
$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & & \cdots & & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ K \end{bmatrix} u$$

Where

$$y = [1 \quad 0 \quad 0 \ldots 0]\mathbf{x}$$
$$\mathbf{x} = [x_1 \quad x_2 \ldots x_n]^T$$

PhysicalVariable Representation

In this representation the state variables are real physical variables, which can be measured and used for manipulation or for control purposes. The approach generally adopted is to break the block diagram of the transfer function into subsystems in such a way that the physical variables can he identified. The governing equations for the subsystems can he used to identify the physical variables. To illustrate the approach consider the block diagram of Fig.

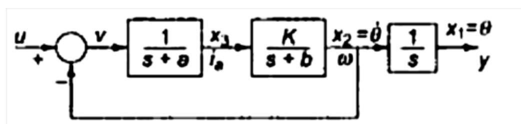


Block diagram

Onemayrepresent thetransfer function ofthis system as

$$T(s) = \frac{y(s)}{u(s)} = \frac{K}{K + (s+a)(s+b)} \cdot \frac{1}{s} = \frac{G(s)}{1 + G(s)H(s)} \cdot \frac{1}{s} = \frac{K/(s+a)(s+b)}{1 + K/(s+a)(s+b)} \cdot \frac{1}{s}$$

TakingH(s)=1,theblockdiagramofcanberedrawnasinFig.physicalvariablescanbe speculated as x1=y, output, x2 =w= θ the angular velocity x_3=Ia the armature current in a position-control system.



($a = R_a/L_a$, $b = f/j$ and $K = 1/JL_a$ in Prob. 8)

Block diagram with physical variables

Where

$$x_1 = y,\; s x_1 = x_2,\; v = (s + a)\, x_3$$

Thestatespacerepresentationcan beobtained by

$$\dot{x}_1 = x_2,\; \dot{x}_2 = -bx_2 + Kx_3,\; \dot{x}_3 = -ax_3 - x_2 + u,\; y = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b & K \\ 0 & -1 & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

And

$$y(t) = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

SolutionofStateequations

Considerthestateequationn oflineartimeinvariantsystem as,

$$\dot{X}(t) = AX(t) + BU(t)$$

ThematricesAandBareconstantmatrices.Thisstateequationcan beoftwo types,

1. Homogeneousand
2. Non homogeneous

Homogeneous Equation

If Aisaconstantmatrixandinputcontrolforcesarezerothenthe equationtakesthe form,

$$\dot{X}(t) = A\,X(t)$$

Such an equation is called homogeneous equation. The obvious equation is if input is zero, In such systems, the driving force is provided by the initial conditions of the system to produce the output. For example, consider a series RC circuit in which capacitor is initially charged to V volts. The current is the output. Now there is no input control force i.e. external voltage appliedto the system. But the initial voltage on the capacitor drives the current through the system and capacitor starts discharging through the resistance R. Such a system which works on the initial conditions without any input applied to it is called homogeneous system.

NonhomogeneousEquation

If A is a constant matrix and matrix U(t) is non-zero vector i.e. the input control forcesare applied to the system then the equation takes normal form as,

$$\dot{X}(t) = A\,X(t) + B\,U(t)$$

Such an equation is called non homogeneous equation. Most of the practical systems require inputs to dive them. Such systems arc non homogeneous linear systems. The solution of the state equation is obtained by considering basic method of finding the solution of homogeneous equation.

Controllabilityand Observability

More specially, for system of Eq.(1), there exists a similar transformation that will diagonalize the system. In other words, There is a transformation matrix Q such that

$$\dot{X} = AX + Bu \quad ; \quad y = CX + Du \quad ; \quad X(0)=X_0 \qquad (1)$$

$$\hat{X} = QX \quad \text{or} \quad X=Q^{-1}\hat{X} \qquad (2)$$

$$\dot{\hat{X}} = \Lambda\hat{X} + \hat{B}u \qquad y = \hat{C}\hat{X} + \hat{D}u \qquad (3)$$

$$\text{Where } \Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ & & \ddots & \\ 0 & & \cdots & \lambda_n \end{bmatrix} \qquad (4)$$

Notice that by doing the diagonalizing transformation, the resulting transfer function between u(s) and y(s) will not be altered.

LookingatEq.(3),if $\hat{b}_k = 0$, then $x_k(t)$ isuncontrollablebytheinputu(t),since,x_k(t)is characterizedbythe mode $e^{-\lambda_k t}$ bythe equation.

$$x_k(t) = e^{\lambda_k t} x_k(0\_)$$

The lake of controllability of the state x_k(t) is reflect by a zero kth row of B . i.e. b_k . Which would cause a complete zero rows in the following matrix (known as the controllability matrix), i.e.:

$$C(A,b)=\left[\hat{B}\ \ \hat{A}\hat{B}\ \ \hat{A}^2\hat{B}\ \ \hat{A}^3\hat{B}\ \ \cdots\hat{A}^{n-1}\hat{B}\right]\cdots=\begin{bmatrix}\hat{b}_1 & \lambda_1\hat{b}_1 & \lambda_1^2\hat{b}_1 & \cdots & \lambda_1^{n-1}\hat{b}_1\\ \hat{b}_2 & \lambda_2\hat{b}_2 & \lambda_2^2\hat{b}_2 & \cdots & \lambda_2^{n}\\ \cdots & \cdots & \cdots & \cdots & \\ \hat{b}_k & \lambda_k\hat{b}_k & \lambda_k^2\hat{b}_k & & \lambda_1^{n-1}\hat{b}_1\\ \cdots & \cdots & \cdot & & \cdots\\ \hat{b}_n & \lambda_n\hat{b}_r & & & \lambda_n^{n-1}\hat{b}_n\end{bmatrix}$$

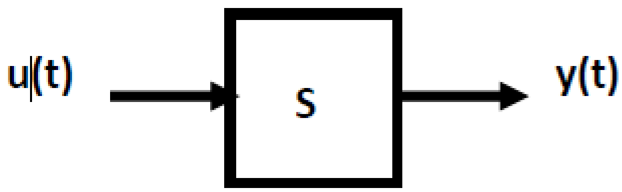
A C(A,b) matrix with all non-zero row has a rank of n

In fact, $\hat{B}=Q^{-1}B$ or $B=Q\hat{B}$. Thus, a non-singular C(A,b) matrix implies a non-singular matrix of C(A,b)of th following:

$$C(A,b)=\left[B\ \ AB\ \ A^2B\ \ \cdots A^{n-1}B\right]$$

TransferfunctionfromStateVariableRepresentation

A simple example of system has an input and output as shown in Figure 1. This class of system has general form of model given in Eq.(1).



$$\frac{d^n y}{dt^n}+a_{n-1}\frac{d^{n-1}y}{dt^{n-1}}+\ldots+a_0y(t)=b_{m-1}\frac{d^{m-1}u}{dt^{m-1}}+\ldots+b_0u(t)$$

Models of this form have the property of the following:

$$u(t)=\alpha_1u_1(t)+\alpha_2u_2(t)\quad\Rightarrow\quad y(t)=\alpha_1y_1(t)+\alpha_2y_2(t)\qquad(2)$$

where, (y1,u1)and(y2,u2)eachsatisfiesEq,(1).

Model of the form of Eq.(1) is known as linear time invariant (abbr. **LTI**) system. Assume the system is at rest prior to the time t0=0, and, the input u(t) (0 t <∞) produces the output y(t) (0 t < ∞), the model of Eq.(1) can be represented by a transfer function in term of Laplace transform variables, i.e.:

$$y(s)=\frac{b_ms^m+b_{m-1}s^{m-1}+\cdots+b_0}{a_ns^n+a_{n-1}s^{n-1}+\cdots+a_0}u(s)\qquad(3)$$

Then applying the same input shifted by any amount □ of time produces the same output shifted by the same amount q of time. The representation of this fact is given by the following transfer function:

$$y(s)=\left(\frac{b_ms^m+b_{m-1}s^{m-1}+\cdots+b_0}{a_ns^n+a_{n-1}s^{n-1}+\cdots+a_0}\right)e^{-\theta s}u(s)\qquad(4)$$

Models of Eq.(1) having all $b_i = 0\ (i > 0)$, a state space description arose out of a reduction to a system of first order differential equations. This technique is quite general. First, Eq.(1) is written as:

$$y^{(n)} = f\left(t, u(t), y, \dot{y}, \ddot{y}, \cdots, y^{(n-1)}\right); \quad \text{with initial conditions: } y(0)=y_0, \dot{y}(0) = y_1(0), \cdots, y^{(n-1)}(0) = y_{n-1}(0) \qquad (5)$$

Consider the vector $x \in R^n$ with $x_1 = y, x_2 = \dot{y}, x_3 = \ddot{y}, \cdots, x_n = y^{(n-1)}$, Eq.(5) becomes:

$$\frac{d}{dt}X = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \\ f\left(t, u(t), y, \dot{y}, \ddot{y}, \cdots, y^{(n-1)}\right) \end{bmatrix} \qquad (6)$$

In case of linear system, Eq.(6) becomes:

$$\frac{d}{dt}X = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ & & \ddots & & & \\ 0 & 0 & \cdots & \ddots & & 1 \\ -a_0 & -a_1 & & \cdots & & -a_{n-1} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u(t); \qquad y(t) = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} X \qquad (7)$$

It can be shown that the general form of Eq.(1) can be written as

$$\frac{d}{dt}X = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ & & \ddots & & & \\ 0 & 0 & \cdots & \ddots & & 1 \\ -a_0 & -a_1 & & \cdots & & -a_{n-1} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} u(t); \qquad y(t) = \begin{bmatrix} b_0 & b_1 & \cdots & b_m & 0 & \cdots & 0 \end{bmatrix} X \qquad (8)$$

and, will be represented in an abbreviation form:

$$\dot{X} = AX + Bu \ ; \ y = CX + Du; \ \ D=0 \qquad (9)$$

Eq.(9) is known as the controller canonical form of the system.

Statespacerepresentationfordiscretetime systems

The dynamics of a linear time (shift)) invariant discrete-time system may be expressed in terms state (plant) equation and output (observation or measurement) equation as follows

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k),$$
$$\mathbf{y}(k) = C\mathbf{x}(k) + D\mathbf{u}(k)$$

Where x(k) an n dimensional slate rector at time t =kT. an r-dimensional control (input) vector y(k). an m-dimensional output vector ,respectively, are represented as

$$\mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T, \quad \mathbf{u}(k) = [u_1(k), u_2(k), \ldots, u_r(k)]^T, \quad \mathbf{y}(k) = [y_1(k), y_2(k), \ldots, y_m(k)]^T.$$

The parameters (elements) of A, an *nX n* (plant parameter) matrix. B an *n*X *r* control (input)matrix, and C An *m* X*r* output parameter, Dan m Xr parametricmatrix areconstants for the LTI system. Similar to above equation state variable representation of SISO (single output and singleoutput)discrete-rimesystem (with direct couplingofoutput with input)can bewritten as

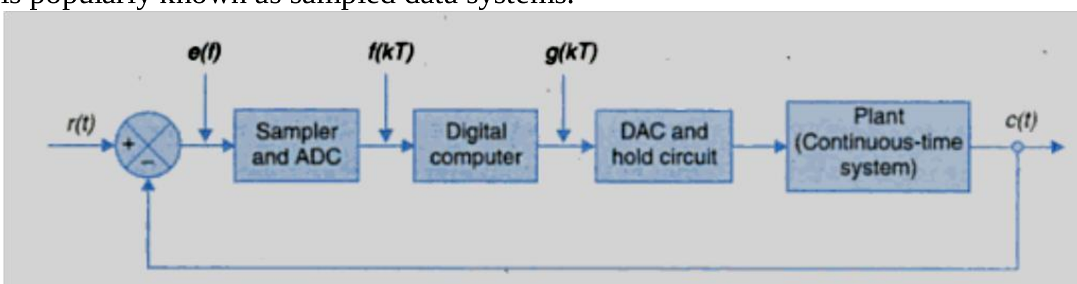
Wherethe input u, outputyandd.arescalars,andb andcaren-dimensional vectors.

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + \mathbf{b}u(k)$$
$$y(t) = \mathbf{c}^T \mathbf{x}(k) + du(k)$$

The concepts of controllability and observability for discrete time system are similar to the continuous-time system. A discrete time system is said to be controllable if there exists a finite integernandinputmu(k);*k*[0,*n*1]thatwilltransferanystate(0)x^0=*bx(0)*tothestatex^n*atk* =*n*n.

SampledData System

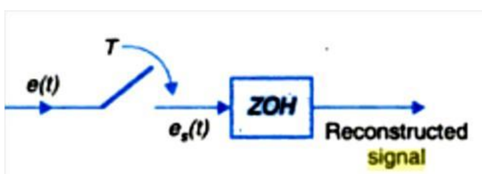
When the signal or information at any or some points in a system is in the form ofdiscretepulses.Thenthesystemiscalleddiscretedatasystem.Incontrolengineeringthediscrete data system is popularly known as sampled data systems.



SamplingTheorem

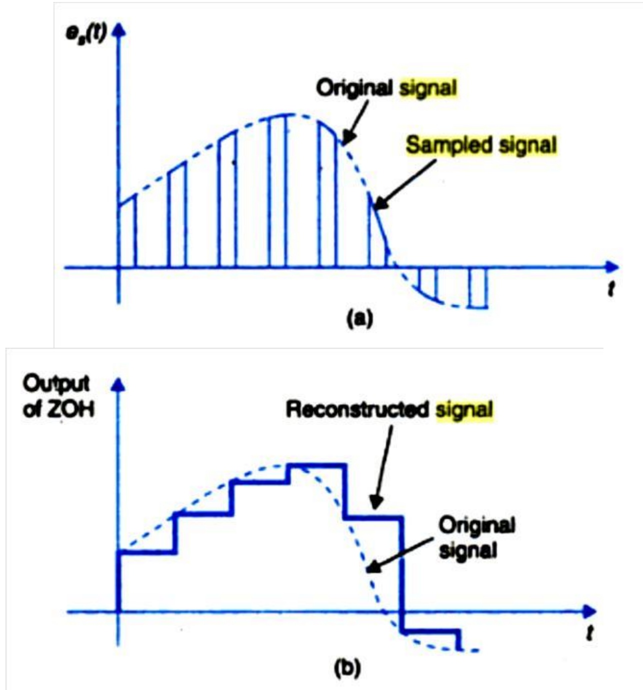
A band limited continuous time signal with highest frequency fm hertz can be uniquely recovered from its samples provided that the sampling rate Fs is greater than or equal to 2fm samples per seconds.

Sample&Hold



The Signal given to the digital controller is a sampled data signal and in turn the controller gives the controller output in digital form. But the system to be controlled needs an analog control signal as input. Therefore the digital output of controllers must be converters into analog form.

This can be achieved by means of various types of hold circuits. The simplest hold circuits are the zero order hold (ZOH). In ZOH, the reconstructed analog signal acquires thesame values as the last received sample for the entire sampling period.



The high frequency noises present in the reconstructed signal are automatically filtered out bythecontrolsystem component which behaves likelow pass filters. Inafirst orderhold the last two signals for the current sampling period. Similarly higher order hold circuit can be devised. First or higher order hold circuits offer no particular advantage over thezero order hold.