2010 MATHEMATICS 2 SOLUTION
MATHEMATICS 2

1. **(a)** $(m + n)(2x - y) - x(m + n)$

Method 1

$= (m + n)[2x - y - x]$ — Factorizing (m + n) out

$= (m + n)(2x - x - y)$

$= \underline{(m + n)(x - y)}$ — **NB: $(m + n)(x - y) = (x - y)(m + n)$**

Method 2

$(m + n)(2x - y) \quad - \; x(m + n)$ — Expanding

$= 2mx - my + 2nx - ny - mx - nx$

$= 2mx - mx + 2nx - nx - my - ny$ — Grouping like terms & simplifying

$= mx \quad + \quad nx \qquad - my - ny$ — Factorizing

$= x(m + n) - y(m + n)$ — Factorizing (m+n) out

$= \underline{(m + n)(x - y)}$

1 (b) (i)

$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

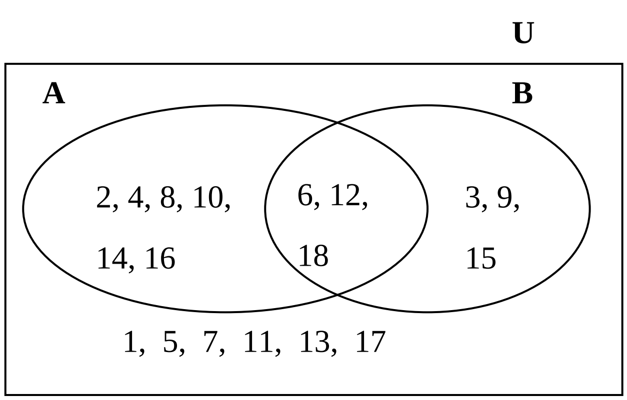
$B = \{3, 6, 9, 12, 15, 18\}$

$A \cap B = \{6, 12, 18\}$

$A \cup B = \{2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

$(A \cup B)' = \{1, 5, 7, 11, 13, 17\}$

1(b) (ii)



1 (c) $\begin{pmatrix} 5 \\ 3 \end{pmatrix} + 2\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ -7 \end{pmatrix} = 0$

Using the horizontal (x) component, we have

$$5 + 2x - 1 = 0$$

$$\Rightarrow \quad 2x = 1 - 5$$

$$\Rightarrow \quad \frac{2x}{2} = \frac{-4}{2}$$

$$\Rightarrow \quad \underline{\underline{x = -2}}$$

Solving for x

Using the vertical (y) component, we have

$$3 + 2y - (-7) = 0$$

$$\Rightarrow \quad 3 + 2y + 7 = 0$$

$$\Rightarrow \quad 2y + 10 = 0$$

$$\Rightarrow \quad 2y = -10$$

$$\Rightarrow \quad y = \frac{-10}{2}$$

$$\Rightarrow \quad \underline{\underline{y = -5}}$$

Solving for y

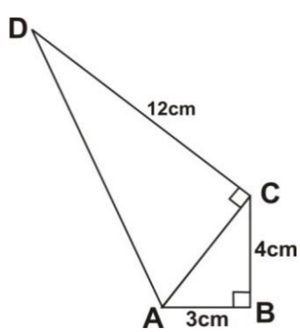
2. **(a)** 2,483.65 + 701.532 + 102.7

$$\begin{array}{r} 2483.65 \\ 701.532 \\ +\ 102.7 \\ \hline 3287.882 \\ \hline \end{array}$$

Ensure that the places of the addends are in line

$\approx$ $\underline{\underline{3287.9}}$ (one decimal place)

2 (b)



(i) Side **AC** is the hypotenuse of triangle ABC.
From the Pythagorean theorem,

$$|AC|^2 = |AB|^2 + |BC|^2$$

$$\Rightarrow \quad |AC|^2 = (3cm)^2 + (4cm)^2$$

$$\Rightarrow \quad |AC|^2 = 9cm^2 + 16cm^2$$

$$\Rightarrow \quad |AC|^2 = 25cm^2$$

$$\Rightarrow \quad |AC| = \sqrt{25cm^2}$$

$$\Rightarrow \quad |AC| = 5\ cm$$

Now, side **AD** is the hypotenuse of triangle ACD
From the Pythagorean theorem,

$$|AD|^2 = |AC|^2 + |CD|^2$$

$$\Rightarrow \quad |AD| = \sqrt{(5cm)^2 + (12cm)^2}$$

$$\Rightarrow \quad |AD| = \sqrt{169 cm^2}$$

$$\Rightarrow \quad |AD| = 13 cm$$

Hence the perimeter of ABCD

$$= |AB| + |BC| + |CD| + |DA|$$

$$= 3cm + 4cm + 12cm + 13cm$$

$$= \underline{\underline{32 \text{ cm}}}$$

<u>The perimeter of ABCD is 32 cm</u>

2 (b) (ii) Area of (ABCD = ΔABC + ΔACD)

$$= \tfrac{1}{2}(b_1h_1) + \tfrac{1}{2}(b_2h_2)$$

$$= \tfrac{1}{2}(3cm)(4cm) + \tfrac{1}{2}(5cm)(12cm)$$

$$= 6cm^2 + 30cm^2$$

$$= \underline{\underline{36 cm^2}}$$

<u>The area of ABCD is 36 cm^2</u>

3. **(a)**

$$\frac{2^7 \times 3^4 \times 5^3}{2^3 \times 3^2 \times 5^2}$$

$$= \frac{2^7}{2^3} \times \frac{3^4}{3^2} \times \frac{5^3}{5^2}$$

$$= 2^4 \times 3^2 \times 5$$

$$= 8 \times 9 \times 5$$

$$= 360$$

$$= \underline{\underline{3.6 \times 10^2}}$$

3 (b) Distance ridden = x km

Distance walked = $\frac{1}{2}$ h × 6km/h = 3km

Total distance = 10 km

Dist. ridden + dist. walked = total dist.

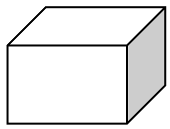
⇒ x km + 3 km = 10 km

⇒ x km = 10 km – 3km

⇒ x km = 7 km

<u>The distance Kwame covered by bicycle is 7 km</u>

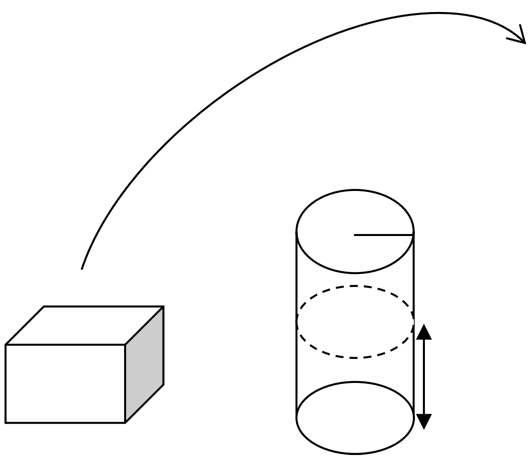
3 (c) (i)



Volume = length × width × height

= $22cm \times 9cm \times 16cm$

= <u>3168cm³</u>



Let d = the depth of water in the cylinder

3 (c) (ii) **<u>Method 1</u>** <u>(Using calculated volume of rectangular tank)</u>

Vol. of water in rectangular tank = Vol. of water in cylinder

⇒ $3168cm^3 = \pi r^2 \times d$

⇒ $3168cm^3 = {}^{22}/_7 \times (6cm)^2 \times d$ Substituting

⇒ $3168cm^3 = {}^{22}/_7 \times 36cm^2 \times d$

⇒ $\frac{3168\ cm^3}{\frac{22}{7} \times 36cm^2} = d$ Dividing both sides by ${}^{22}/_7 \times 36cm^2$

⇒ $\frac{3168}{792/7}\ cm = d$ Simplifying

$\Rightarrow \quad 3168 \div \frac{792}{7}\text{ cm} = d$

$\Rightarrow \quad 3168 \times \frac{7}{792}\text{ cm} = d$

$\Rightarrow \quad 28\text{ cm} = d$

You may avoid the tedious simplification here by using Method 2 below

Hence the depth of water in the cylindrical container = 28cm

3(c)(ii) **Method 2** (Using the given dimensions of rectangular tank)

Vol. of water in cuboid = Vol. of water in cylinder

$\Rightarrow \quad l \times w \times h_{\text{cuboid}} = \pi r^2 \times d_{\text{cylinder}}$

$[22\text{cm} \times 9\text{cm}] \times 16\text{cm} = [{}^{22}/_{7} \times (6\text{cm})^2] \times d$ *Substituting and solving for d*

$\Rightarrow \quad [22\text{cm} \times 9\text{cm}] \times 16\text{cm} = (\frac{22}{7} \times 6\text{cm} \times 6\text{cm}) \times d$

$\Rightarrow \quad \frac{22\text{cm} \times 9\text{cm} \times 16\text{cm}}{\frac{22}{7} \times 6\text{cm} \times 6\text{cm}} = d$ Simplifying

$\Rightarrow \quad \frac{22\text{cm} \times 9\text{cm} \times 16\text{cm} \times 7}{22 \times 6\text{cm} \times 6\text{cm}} = d$ Simplifying (by 'cancellation')

$\Rightarrow \quad 28\text{ cm} = d$

∴ The depth (d) of water in the cylindrical container = 28cm.

4. (a)

Method 1 (Evaluating whole number and fractions separately)

$7\frac{2}{3} - 4\frac{5}{6} + 2\frac{3}{8}$

$\Rightarrow \quad 7 - 4 + 2 + \frac{2}{3} - \frac{5}{6} + \frac{3}{8}$

$\Rightarrow \quad 5 + \frac{16 - 20 + 9}{24}$

$\Rightarrow \quad 5 + \frac{5}{24}$

$= \quad 5\frac{5}{24}$

4 (a) **Method 2** (Changing mixed fractions to improper fractions)

$$7\tfrac{2}{3} - 4\tfrac{5}{6} + 2\tfrac{3}{8}$$

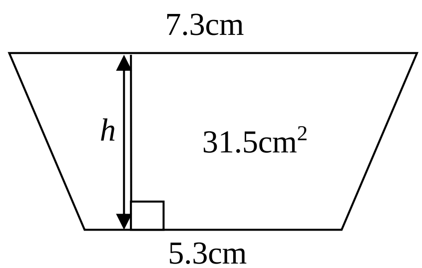
$$\Rightarrow \quad \frac{23}{3} - \frac{29}{6} + \frac{19}{8}$$

$$\Rightarrow \quad \frac{184 - 116 + 57}{24}$$

$$\Rightarrow \quad \frac{125}{24}$$

$$= \quad 5\tfrac{5}{24}$$

4 (b)



Let h = perpendicular dist. between parallel sides

Method 1 (Substituting first)

Area = ½ (sum of parallel sides) × h

$$\Rightarrow \quad 31.5\text{cm}^2 = \tfrac{1}{2}(7.3\text{cm} + 5.3\text{cm}) \times h$$

$$\Rightarrow \quad 2 \times 31.5\text{ cm}^2 = 2 \times \tfrac{1}{2}(7.3\text{cm} + 5.3\text{cm}) \times h$$

Mulitiplying both sides by 2 to remove fraction

$$\Rightarrow \quad 63\text{ cm}^2 = 12.6\text{cm} \times h$$

$$\Rightarrow \quad \frac{63\text{cm}^2}{12.6\text{cm}} = \frac{12.6\text{cm} \times h}{12.6\text{cm}}$$

$$\frac{630\text{cm}^2}{126\text{cm}} = h$$

$$\Rightarrow \quad 5\text{ cm} = h$$

4 (b) **Method 2** (Making h the subject first)

Let A = Area of trapezium

c + d = sum of parallel sides

h = perpendicular dist. between parallel sides

$\Rightarrow \quad A = \frac{1}{2}(c+d)\,h$

$\Rightarrow \quad 2 \times A = 2 \times \frac{1}{2}(c+d)\,h$

Mulitiplying both sides by 2 to remove fraction

$\Rightarrow \quad 2A = (c+d)\,h$

$\Rightarrow \quad \frac{2A}{c+d} = h$

Dividing both sides by 'c+d' to make 'h' the subject

$\Rightarrow \quad h = \frac{2A}{c+d}$

Switching positions

$\Rightarrow \quad h = \frac{2 \times 31.5\,\text{cm}^2}{7.3\,\text{cm} + 5.3\,\text{cm}}$

Substituting values to find 'h'

$h = \frac{63\,\text{cm}^2}{12.6\,\text{cm}} = \underline{\underline{5\,\text{cm}}}$

The perpendicular distance between the parallel sides is 5 cm.

4 (c)

(i) The mean mark $= \underline{\underline{\frac{92+85+65+x}{4}}}$

(ii) If the mean is less than 80 then $\underline{\underline{\frac{92+85+65+x}{4} < 80}}$, $\{x: x \geq 0\}$

(iii) $\frac{92+85+65+x}{4} < 80$

$\Rightarrow \quad \frac{242+x}{4} < 80$

$\Rightarrow \quad 4 \times \left(\frac{242+x}{4}\right) < 4 \times 80$

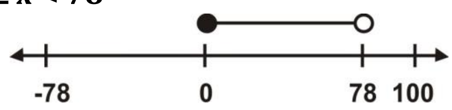
$\Rightarrow \quad 242 + x < 320$

$\Rightarrow \quad x < 320 - 242$

$\Rightarrow \quad x < 78 \quad \{x: x < 78,\ x \geq 0\}$

The possible marks (x) that Efe scored in the test

$= \quad \mathbf{0 \leq x < 78}$



5. **(a)**

Method 1 (Clearing fractions first)

$$\frac{4x-3}{2} = \frac{8x-10}{8} + 2\tfrac{3}{4}$$

$$\Rightarrow \quad \frac{4x-3}{2} = \frac{8x-10}{8} + \frac{11}{4}$$

Changing mixed fraction $2\frac{3}{4}$ to improper fraction

$$8\times\left(\frac{4x-3}{2}\right) = 8\times\left(\frac{8x-10}{8}\right) + 8\times\frac{11}{4}$$

Multiplying through by 8 (to clear fractions):

$$\Rightarrow \quad 4(4x-3) = 8x-10 + 2(11)$$

Expanding and Simplifying

$$\Rightarrow \quad 16x-12 = 8x-10 + 22$$

$$\Rightarrow \quad 16x-8x = 12-10 + 22$$

Grouping like terms on one side

$$\Rightarrow \quad 8x = 24$$

Dividing both sides by 8 to find x

$$\Rightarrow \quad x = \frac{24}{8} = \underline{\underline{3}}$$

5 **(a)**

Method 2 (Grouping and simplifying terms containing the variable first)

$$\frac{4x-3}{2} = \frac{8x-10}{8} + 2\tfrac{3}{4}$$

$$\Rightarrow \quad \frac{4x-3}{2} - \frac{8x-10}{8} = 2\tfrac{3}{4}$$

Grouping the terms containing the variable

$$\Rightarrow \quad \frac{4(4x-3)-(8x-10)}{8} = \frac{11}{4}$$

Simplifying

$$\Rightarrow \quad \frac{16x-12-8x+10}{8} = \frac{11}{4}$$

$$\Rightarrow \quad \frac{16x-8x-12+10}{8} = \frac{11}{4}$$

$$\Rightarrow \quad \frac{8x-2}{8} = \frac{11}{4}$$

$$\Rightarrow \quad 4(8x-2) = 8(11)$$

Cross multiplying

$$\Rightarrow \quad 32x-8 = 88$$

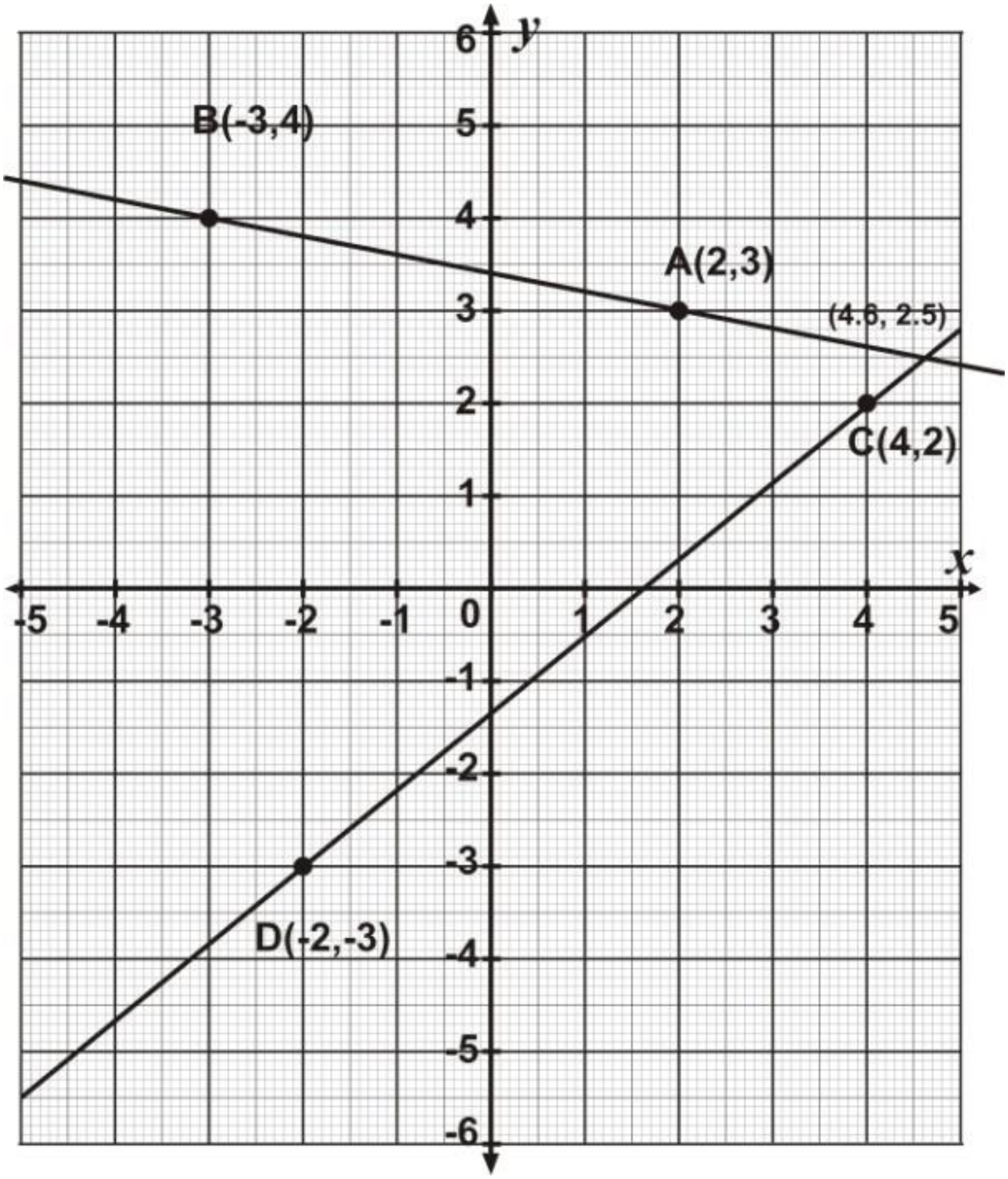
$$\Rightarrow \quad 32x = 88 + 8$$

Simplifying and finding *x*

$$\Rightarrow \quad \frac{32x}{32} = \frac{88+8}{32} = \frac{96}{32}$$

$$\Rightarrow \quad x = 3$$

5 (b)



5 (b) (iii) The acute angle between lines AB and CD $\approx$ 50°

5(b) (iv) The coordinates of the point at which the lines through AB and CD meet

$\approx$ (4.6, 2.5)

6.

(a) (i) The Mode = The most-occurring number of letters

(the number of letters with the highest frequency)

$\Rightarrow$ The mode = 7 letters

(ii)

No. of letters *(x)*	No. of students *(f)*	fx

4	7	28
5	3	15
6	2	12
7	8	56
8	5	40
9	3	27
10	1	10
	$\Sigma f = 29$	$\Sigma fx = 188$

The mean $= \dfrac{\Sigma f x}{\Sigma f} = \dfrac{188}{29}$

$= 6\frac{14}{29} \approx \underline{\underline{6.483}}$

6 (b) P(surname has more than 7 letters)

$= \dfrac{\text{Number of students with more than 7 letters in surname}}{\text{Total number of students}}$

$= \dfrac{5+3+1}{7+3+2+8+5+3+1}$

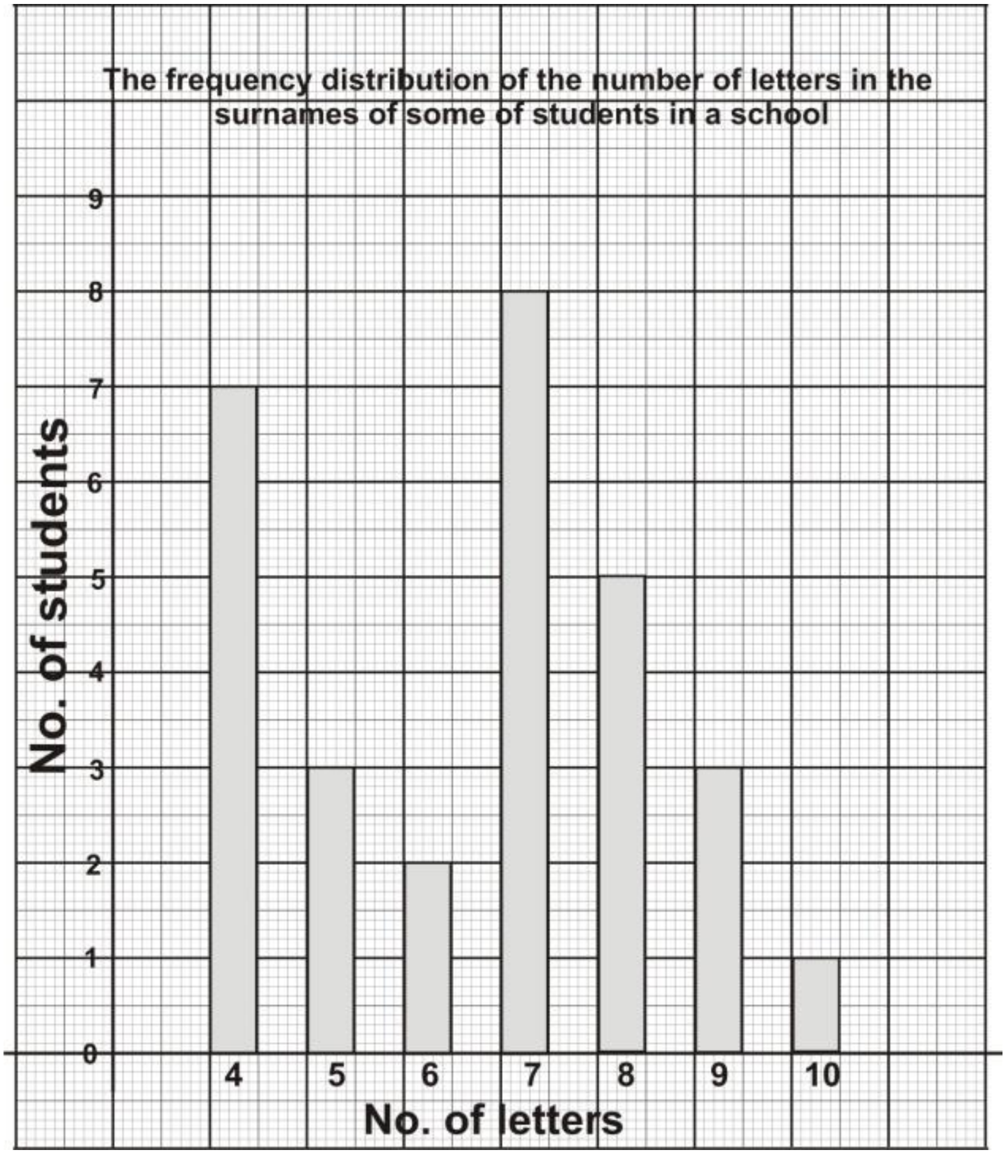
$= \dfrac{9}{29}$

The probability that a student selected at random has a surname with more than 7 letters

$= \underline{\underline{9/29}}$

6 (c)

Vertical AxisScale: 2cm to 1 student


The frequency distribution of the number of letters in the surnames of some of students in a school
No. of students
0
1
2
3
4
5
6
7
8
9
4
5
6
7
8
9
10
No. of letters