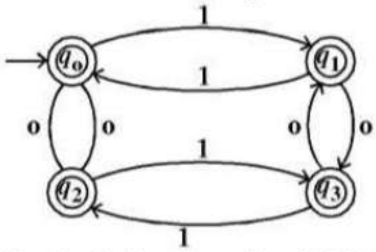
ASSIGNMENT NO.- 1

CSE251/Theory of Computation

1. Consider the following transition diagram



Check whether the string 110101 is accepted by the finite automata represented by above transition diagram. Show the entire sequence of states traversed.

2. Give DFA accepting the following languages over the alphabet {0, 1}:

(i) The set of all strings with three consecutive zeros.

(ii) The set of all strings such that every block of 05 consecutive symbols contains al least two zeros.

3. Construct DFA equivalent to the NFA ({p, q, r, s}, {0, 1}, δ, p, {s}), where δ is given by

	0	1
p	p, q	p
q	r	r
r	s	–
s	s	s

By using subset construction algorithm.

4. Differentiate L* and L+.

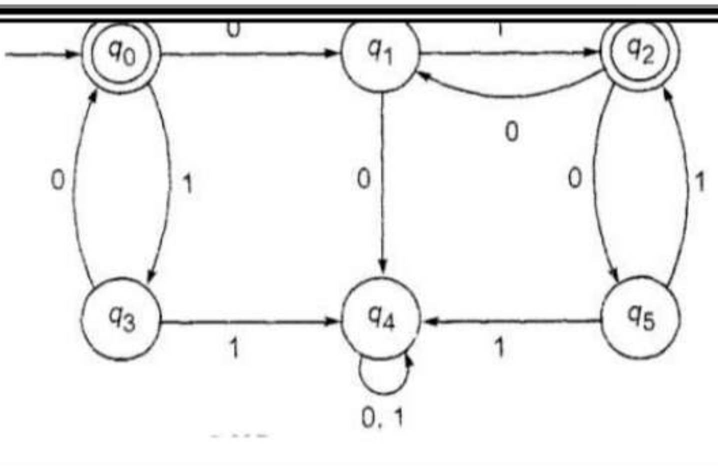
5. Give the state transition diagram for a FA for accepting :

(i) $L1 = \{x \in \{a, b\}^* \mid |x|_a = 3k$ for some $k \geq 0$ and also x ends with "ab"$\}$

(ii) $L2 = \{x \in \{a, b\}^* \mid |x|_a = 3k$ for some $k \geq 0$ or x ends with "ab"$\}$.

6. Define Deterministic Finite Automata (DFA) and design a DFA that accepts the binary number whose equivalent is divisible by 5.

7. Construct the minimum state automata equivalent to DFA described by following state diagram:



8. Explain Myhill-Nerode Theorem using suitable example.

9. Prove that for every NDFA accepting a language L there exist an equivalen accepting the same language L.

10. Explain the applications and limitations of finite automata.

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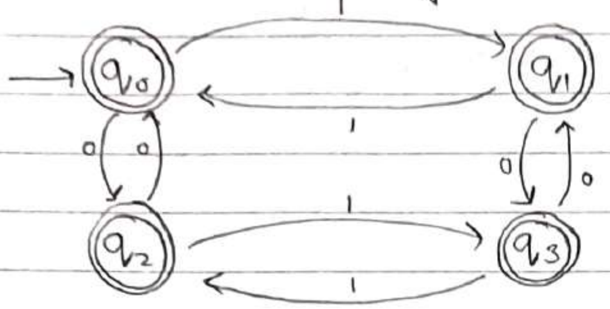
Q1. Consider the following transition diagram



Check whether the string 11010l is accepted by the finite automata represented by above transition diagram. Show the entire sequence of states traversed.

Sol- Given string s = 110101

$\Sigma = \{0,1\}$

$$\rightarrow q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \xrightarrow{0} q_1 \xrightarrow{1} q_0$$

Since this string ends in final state q_0.

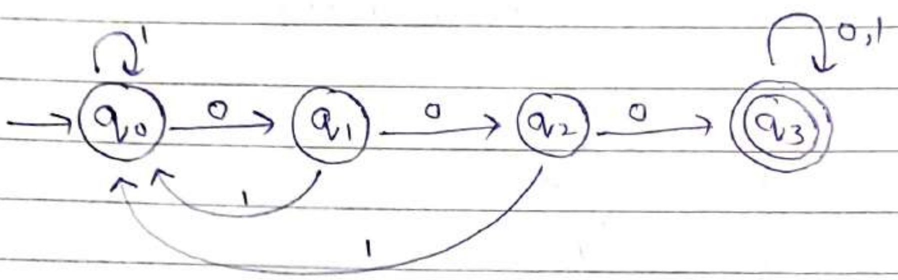
$\therefore$ It is accepted.

Q2. Give DFA accepting the following languages over the alphabet $\{0,1\}$:

i) The set of all strings with three consecutive zeros.

ii) The set of all strings such that every block of 05 consecutive symbols contain at least two zeros.

Sol = i) Three consecutive zero's.



$Q = q_0, q_1, q_2, q_3$

$\Sigma = \{0, 1\}$

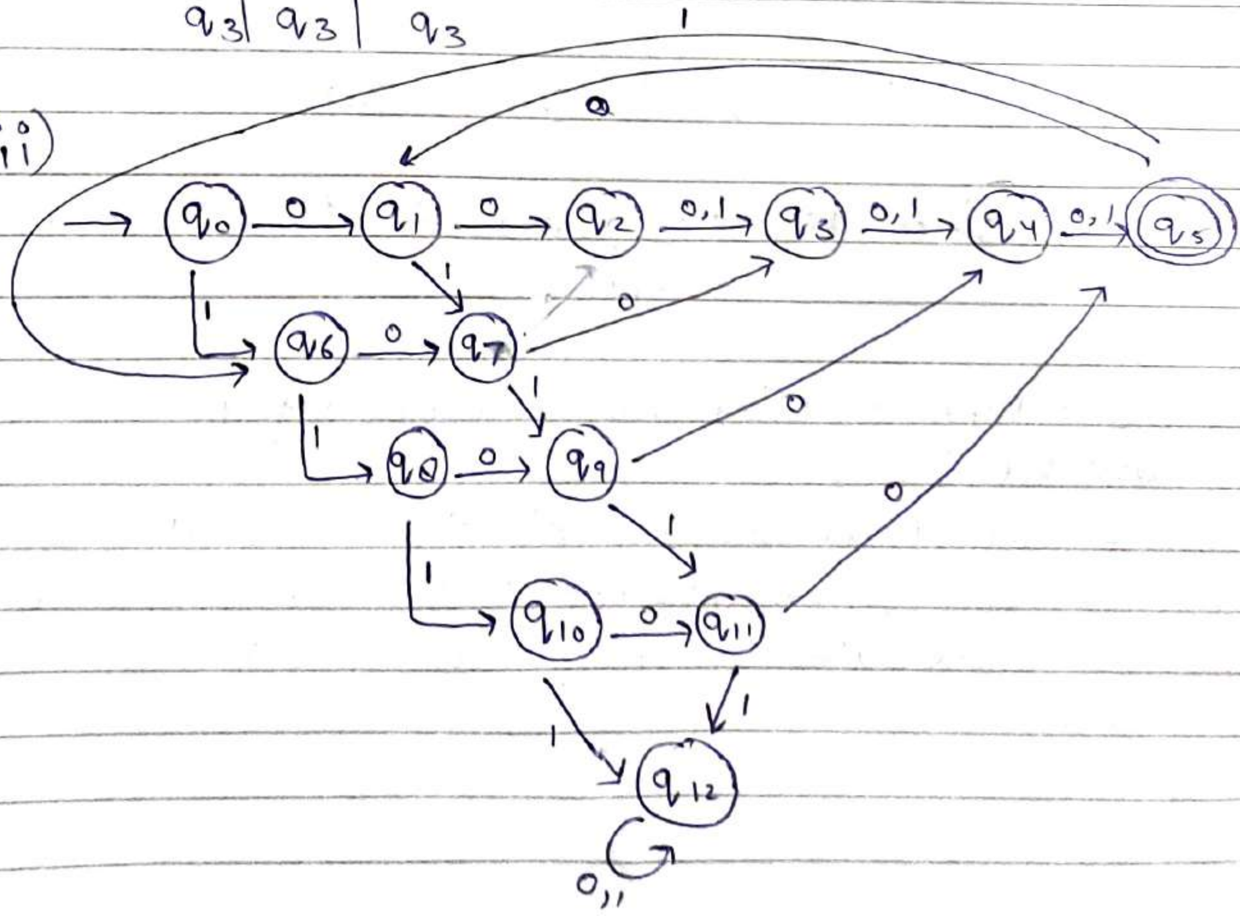
$q_0 = q_0$

$F = q_3$

$\delta =$

	0	1
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_3	q_0
q_3	q_3	q_3

ii)



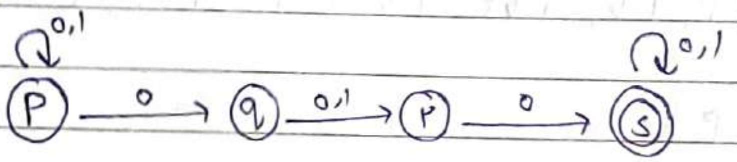
Q3. Construct DFA equivalent to the NFA ({p, q, r, s}, {0,1}, δ, p, {s}),

where δ is given by :-

	0	1
p	p,q	p
q	r	r
r	s	—
s	s	s

by using subset construction algorithm.

Sol:- NFA :-



Given transition table of NFA is :-

δ° =

	0	1
p	{p,q}	p
q	r	r
r	s	—
s	s	s

Now, the corresponding DFA is :-

δ' =

	0	1
p	pq	p
pq	pqr	pr
pqr	pqrs	pr
pr	pqs	p
pqrs	pqrs	prs
pqs	pqrs	prs
prs	pqs	prs

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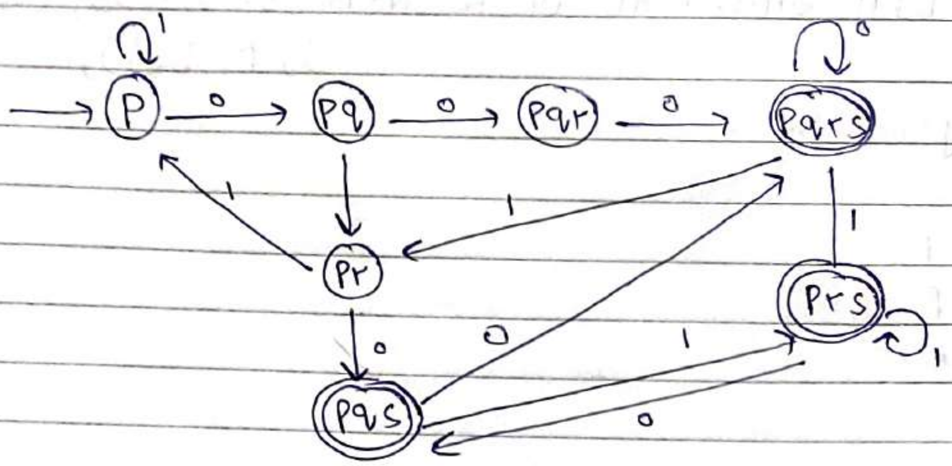

Here :-

$Q' = [P, Pq, Pqr, Pqrs, Pr, Pqs, Prs]$

$q_0 = P$

$F' = \{Pqrs, Pqs, Prs\}$

$\Sigma = \{0, 1\}$

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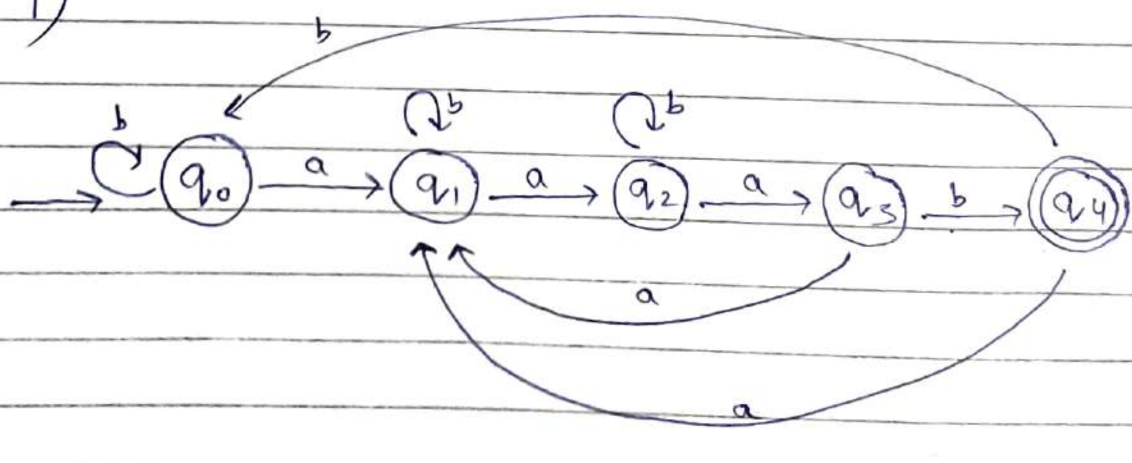
Q4. Differentiate L^* and L^+.

Sol= L^* denotes kleene closure and is given by
$L^* = \bigcup L^i$
for $i=0$
example: $0^* = \{\epsilon, 0, 00, 000, \ldots\}$
Language includes empty words also.

L^+ denotes positive closure and is given by
$L^+ = \bigcup L^i$
$i=1$
example: $0^+ = \{0, 00, 000, \ldots\}$

Q5. Given the state transition diagram for a FA for accepting:
i) $L1 = \{x \in (a,b)^* \mid |x|_a = 3k$ for some $k \geq 0$ and also x ends with "ab"$\}$
ii) $L2 = \{x = \{a,b\}^* \mid |x|_a = 3k$ for some $k \geq 0$ or x ends with "ab"$\}$.

Sol= i)



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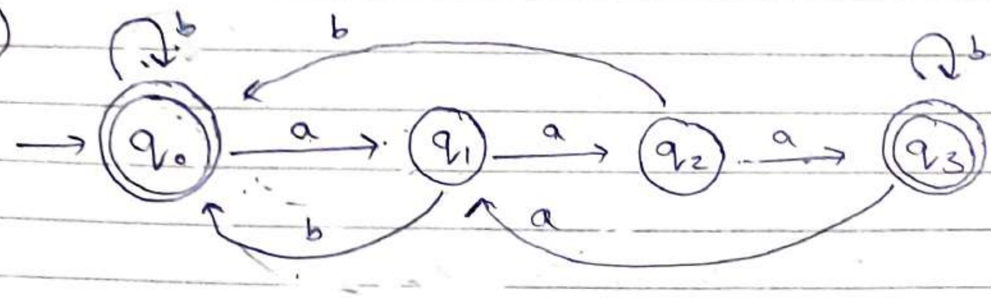
ii)



$\Sigma = \{a, b\}$

$Q = \{q_0, q_1, q_2, q_3\}$

$q_0 = q_0$

$F = \{q_0, q_3\}$

δ :-

	a	b
q_0	q_1	q_0
q_1	q_2	q_0
q_2	q_3	q_0
q_3	q_1	q_3

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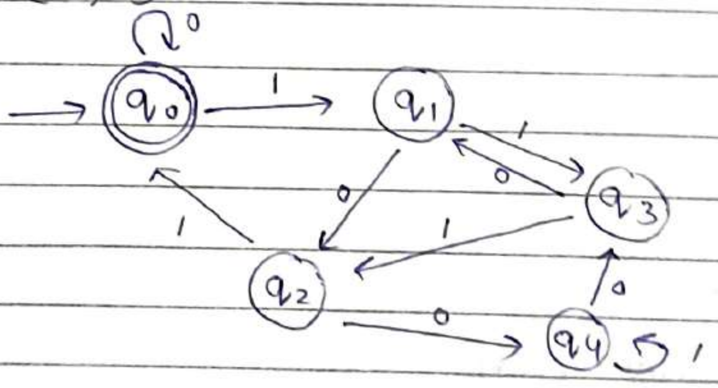
Q6. Define DFA and design a DFA that accepts the binary number whose equivalent is divisible by 5.

Sol= DFA stands for deterministic finite automata.
A finite automata is called DFA if the machine is read an input string one symbol at a time.

- In DFA there is only one path for specific input from current state to next state.
- DFA does not accept null move.
i.e It can't change the state without any input character.

Now, DFA that accepts binary number div. by 5.

$\Sigma = \{0,1\}$



$Q = q_0, q_1, q_2, q_3, q_4$

$q_0 - q_0$

$F - q_0$

$\delta =$

	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2

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Q8. Explain Myhill-Nerode Theorem using suitable example.

Sol= Myhill Nerode theorem is a fundemental result coming down to the theory of languages.
This theory is used to prove whether or not a language L is reguler and it is used in minimization of states in DFA.

This theorm states that :-
A language L is reguler if and only if $\equiv_L$ partitions Σ^* into finitely many equivalence classes.
If $\equiv_L$ partitions Σ^* into n equivalence classes, then minimal DFA ~~according~~ recognizing L has exactly n states.

example:-
To prove that $L = \{a^n b^n \mid n \geq 0\}$ is not reguler.

Proof:- we can show that L has infinitely many equivalence classes by showing that a^k and a^i are distinguished by L whenever $k \neq i$. Thus, for $x = a^k$ and $y = a^i$ we let $z = b^k$. Then $xz = a^k b^k$ is in the language but $yz = a^i b^k$ is not. Thus, each equivalence class of L can contain at most one string of the form a^i so there must be ~~indifinitely~~ infinitely many equivalence classes. That means L is not a reguler language by Myhill Nerode theorem.

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Q9. Prove that for every NFA accepting a language L there exist an equivalent DFA accepting the same language L.

Sol- we have to prove that $\hat{\delta}'(q_0', x) = \hat{\delta}(q_0, x)$ for every x. i.e, $L(D) = L(N)$.

using the method of induction :-

→ Base step :

let x be the empty string ϵ.

$$\therefore \hat{\delta}'(q_0', x) = \hat{\delta}'(q_0', \epsilon)$$
$$= q_0' \quad (\because (q_0, \epsilon) = q_0)$$
$$= \hat{\delta}(q_0, \epsilon)$$
$$= \hat{\delta}(q_0, x)$$

→ inductive step :-

Assume that for any Y with $|Y| \geq 0$

$$\hat{\delta}'(q_0', Y) = \hat{\delta}(q_0, Y).$$

if we let $n = |Y|$, then we need to prove that for a string z with $|z| = n+1$, $\hat{\delta}(q_0', z) = \hat{\delta}(q_0, z)$.

we can represent the string z as a concatenation of string Y ($|Y| = n$) and symbol 'a' from the alphabet Σ ($a \in \Sigma$)

$\therefore$ so, $z = Ya$.

$\therefore$

$$\hat{\delta}'(q_0', z) = \hat{\delta}'(q_0', Ya)$$
$$= \delta'(\hat{\delta}(q_0', Y), a)$$
$$= \delta'(\hat{\delta}(q_0, Y), a) \quad \text{(by assumption)}$$
$$= \bigcup_{P \in \hat{\delta}(q_0, Y)} \hat{\delta}(P, a) \quad \text{(by def. of } \delta')$$
$$= \hat{\delta}(q_0, aY)$$
$$= \hat{\delta}(q_0, z)$$

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DFA D accepts a string x iff $\hat{\delta}'(q_0', x) \in F'$. From the ~~above~~ previous conclusions it follows that D accepts x iff $\hat{\delta}(q_0, x) \cap F \neq \phi$.

So, a string is accepted by DFA D iff it is accepted by NFA N.

$$\therefore L(D) = L(N)$$

Hence proved.

Q10. Explain the applications and limitations of finite automata.

Sol= Applications:-

I. For the designing of lexical analysis of a compiler.
II. For recognizing the pattern using regular expressions.
III. used in text editors.
IV. used in decision making.
V. Designing of sequential circuits.
VI. Can parse text to extract information and structure data.

Limitations:-

I. FA can only count finite input.
II. Head movement is only in one direction.
III. limited memory.
IV. There is no FA that can find and recognize set of binary string of equal 0's and 1's.
V. It can have only string pattern.

Spiral

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