PART 1 - Machine Scored

1. An angle in standard position θ has reference angle of $30°$ with $sin\theta < 0$ and $tan\theta < 0$. A possible measure of θ, in radians, is:

A. $\frac{13\pi}{6}$ B. $-\frac{7\pi}{6}$ C. $-\frac{5\pi}{6}$ D. $\frac{23\pi}{6}$

2. An angle in standard position θ has $cos\theta < 0$ and $cot\theta < 0$. The best estimate for the value of θ, in radians, is:

A. 1.05 B. 2.62 C. 3.92 D. 5.24

3. An angle in standard position θ has terminal arm that passes through a point $P(5, -4)$. The value of $sec\theta$ is approximately:

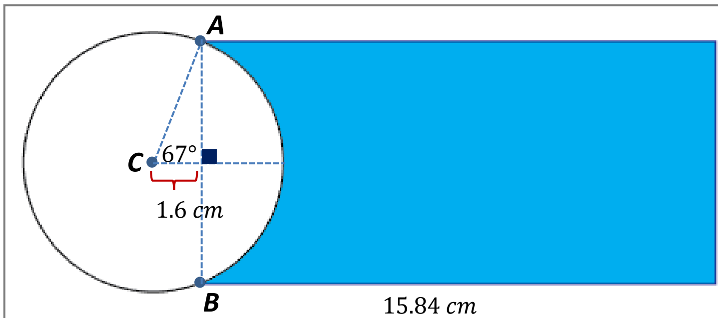
A. 1.28 B. -0.62 C. 0.78 D. -1.60

NR #1 An angle in standard position θ, where $0 \le \theta \le 2\pi$, has $sin\theta = -1/3$. Correct to the nearest tenth of a radian, the smallest possible value of θ is ______.

4. A point on the unit circle has coordinates $P(-\frac{5}{13}, m)$ and forms a principal angle in standard position, θ. If $tan\theta < 0$, then m is equal to:

A. $\frac{8}{13}$ B. $-\frac{8}{13}$ C. $\frac{12}{13}$ D. $-\frac{12}{13}$

Use the following information to answer question 5:

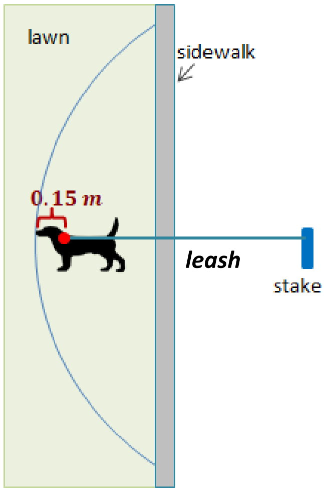


A rectangle with a base 15.84 cm in length intersects a circle with a centre **C**, at two points **A** and **B** as shown.

5. Correct to the nearest tenth, the **perimeter** of the shaded portion of the rectangle is:

A. $44.0\ cm$ B. $46.8\ cm$ C. $48.8\ cm$ D. $49.5\ cm$

Use the following information to answer NR #2:

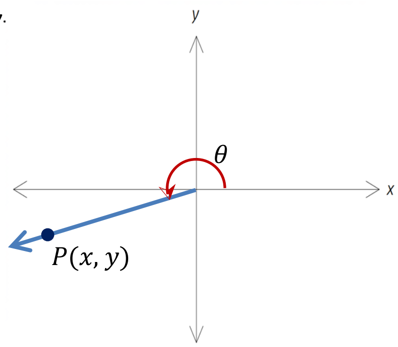


A dog has a leash tied to a stake in the ground such that she can reach a lawn on other side of a sidewalk as shown.

The dog is able to reach a horizontal distance of 0.15 m beyond the length of the leash, to form an arc on the lawn approximately 8.93 metres in length. The angle formed by lines connecting the stake and each end of the lawn arc is 110 degrees.

NR #2 The length of the leash, correct to the nearest tenth of a metre, is _____.

6. An angle in standard position θ passes through a point P as shown, where the x-coordinate is double the value of the y.



The value of $sec\theta$ can be written in the form $-\frac{\sqrt{a}}{b}$, where $a, b > 0$.

The value of $a + b$ is:

A. 3 B. 4 C. 6 D. 7

Use the following information to answer question 7:

A point on the unit circle has coordinates $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, through which a terminal arm passes forming an angle θ. A second point has coordinates $Q\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ forming an angle β. Consider the following statements:

Statement 1: The smallest angle formed between the two terminal arms through P and Q is $\frac{5\pi}{12}$

Statement 2: One possible measure for θ is $-\frac{4\pi}{3}$

Statement 3: The value of $sin\theta$ is $\frac{\sqrt{3}}{2}$ and the value of $cos\beta$ is $-\frac{\sqrt{2}}{2}$

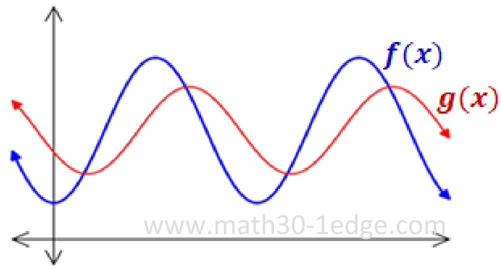
Statement 4: The value of $tan\theta$ is $-\sqrt{3}$ and the value of $cot\beta$ is 1

7. The number of true statements is:

A. 1 B. 2 C. 3 D. 4

8. The graph of a function $f(x)$ is shown, which can be expressed in the form $f(x) = a\sin[b(x - c)] + d$.

The graph of $g(x)$ is obtained from $f(x)$ by changing the two parameters:



A. a and b

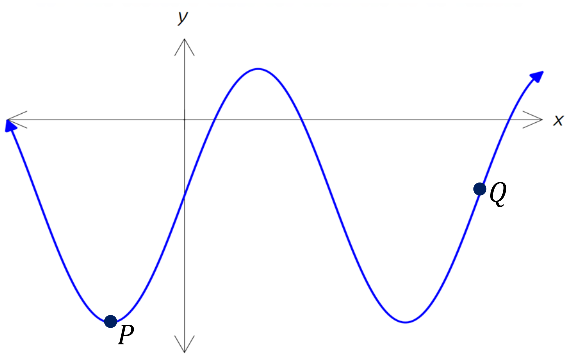
B. a and c

C. b and d

D. c and d

Use the following information to answer NR #3:

The graph shown models a sinusoidal function in the form $f(x) = a\sin x - d$, where $a > 0$ and $d > 0$. The point P is at



Consider the following statements:

Statement 1: The angle represented by the point P is $-\pi/2$

Statement 2: The difference between the angle represented by the point Q and the angle represented by the point P is greater than 2π

Statement 3: The value of a is less than the value of d

Statement 4: The value of the y-intercept is $a - d$

Statement 5: If the function were transformed to $f(x) = a\sin(bx) + d$, where a and b are unchanged and $b < 1$, the horizontal distance between P and Q would increase.

At least two of the statements are true; that is two, three, four, or all five of the statements may be true.

NR #3 The true statements are statements number: *Enter in any order.*

9. A sinusoidal function has an equation $y = 5\sin(4x + \pi)$. The value of the **period** and the **horizontal phase shift** are, respectively:

A. $\frac{\pi}{2}, \frac{\pi}{4}$

B. $\frac{\pi}{2}, \pi$

C. $4, \pi$

D. $4, \frac{\pi}{4}$

NR #4

A sinusoidal function has an f

The **period** of the resulting graph, correct to the nearest whole number, is a two-digit number ab (*a and b are the first two digits of your answer*)

The **maximum value** of the function, correct to the nearest tenth, is $c.d$

(*c and d are the last two digits of your answer*)

The values of a, b, c and d are:______

10. The function $f(x) = \tan(4x)$ has a domain, where $n \in I$, of:

A. $x \neq \frac{\pi}{4} + \frac{n\pi}{2}$

B. $x \neq \frac{\pi}{4} + \frac{n\pi}{4}$

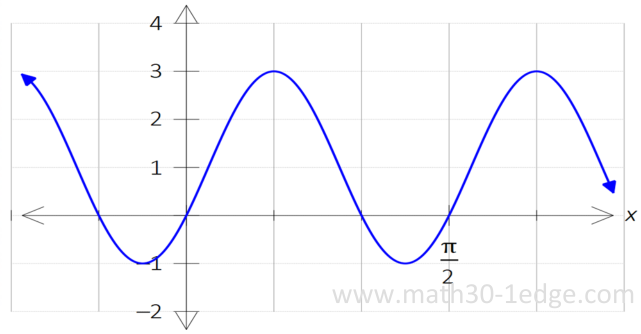
C. $x \neq \frac{\pi}{8} + \frac{n\pi}{2}$

D. $x \neq \frac{\pi}{8} + \frac{n\pi}{4}$

Use the following information to answer NR#5:

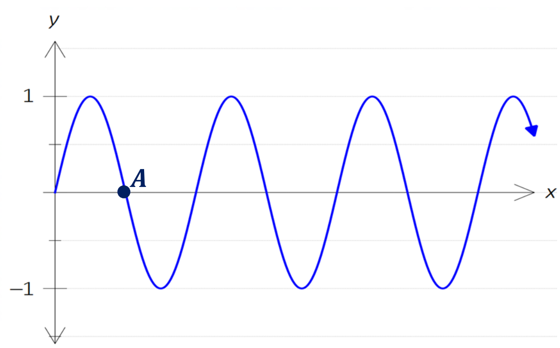
The partial graph of a cosine function with a max value of 3 and a min value of -1 is shown on the right.

The equation of the function can be written in the form $y = a\cos\,[b(x - c)] + d$, where a, b, and d are $\in I$.



NR #5 The value of b is ______ *first digit of your answer* and the value of d is ______. *second digit of your answer*

Use the following information to answer question 11:



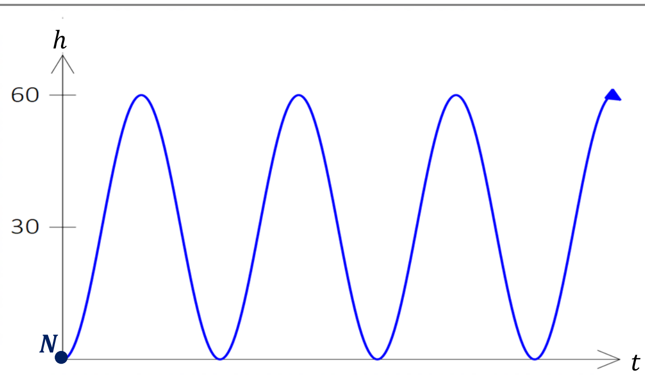
A particular musical note can be modeled by a function $g(x) = \sin(2\pi f x)$, where x is the time in seconds, and f is the frequency of the note, measured in number of cycles per second, or hertz.

The graph of $g(x)$ is shown. It has an x-intercept after 0.00125 seconds, at a point A.

11. The frequency of the note modeled by the function above is:

A. $800\pi\ Hz$ B. $400\pi\ Hz$ C. $800\ Hz$ D. $400\ Hz$

Use the following information to answer question 12:



The height of a nail caught in a tire rotating at a constant speed can be modeled by a sinusoidal function

$h = a\sin[b(t - c)] + d$

Where h is the height of a nail, in cm, after t seconds.

The graph shown models the height of a nail that starts at on the ground at a lowest position N at $t = 0$. The nail completes 20 rotations each minute.

12. The value of a and value of the phase shift c in the equation are, respectively:

A. 30 *cm*, *0.75 sec* B. 30 *cm*, *0.5 sec* C. 60 *cm*, *0.75* D. 60 *cm*, *0.5 sec*

13. Which of the following steps could lead to a correct solution of the equation $2cos^2\theta + 3cos\theta - 2 = 0$?

A. $cos\theta = \frac{1}{2}$ or $cos\theta =- 2$

B. $cos\theta = \frac{1}{2}$ or $cos\theta =- 1$

C. $cos\theta = \frac{-1}{2}$ or $cos\theta = 2$

D. $cos\theta = \frac{-1}{2}$ or $cos\theta = 1$

14. The solution, on $\{0 \leq x \leq 2\pi\}$, to $3csc^2\theta - 4 = 0$ is θ equal to:

A. $\frac{\pi}{3}, \frac{2\pi}{3}$

B. $\frac{\pi}{3}, \frac{2\pi}{3} \frac{4\pi}{3}, \frac{5\pi}{3}$

C. $\frac{\pi}{6}, \frac{5\pi}{6}$

D. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

15. A general solution of the equation $sec^2x - secx - 2 = 0$, where $n \in I$ is:

A. $x = \frac{\pi}{3}n$

B. $x = \frac{\pi}{3} + 2\pi n,\ x = \frac{5\pi}{3} + 2\pi n,\ x = \pi n$

C. $x = \frac{\pi}{3}n + \frac{2\pi n}{3}$

D. $x = \frac{\pi}{3} + 2\pi n,\ x = \frac{5\pi}{3} + 2\pi n,\ x = 2\pi n$

16. The solution to the equation $log_2(tanx) + log_2(cosx) + 1 = 0$, where $\{0 \leq x \leq 2\pi\}$ is:

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$

B. $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

C. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

D. $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

NR #6 The exact value of the trig ratio $cos\left(\frac{7\pi}{12}\right)$ can be determined to be an irrational expression in the form $\frac{\sqrt{a} - \sqrt{b}}{c}$ where a, b, c are positive integers.

The value of a is ______ *first digit*, the value of b is ______ *second digit* and the value of c is ______. *third digit*

17. A point $P(3, -5)$ lies on the terminal arm of an angle θ in standard position. The value of $\sin(\pi - \theta)$ is:

A. $\frac{3}{\sqrt{34}}$

B. $-\frac{3}{\sqrt{34}}$

C. $\frac{5}{\sqrt{34}}$

D. $-\frac{5}{\sqrt{34}}$

18. The non-permissible values of the expression $\frac{tanx}{1 + sinx}$ can be best written, where $n \in I$, as:

A. $x \neq \frac{3\pi}{2} + 2\pi n$

B. $x \neq \pi n,\ x \neq \frac{3\pi}{2} + 2\pi n$

C. $x \neq \frac{\pi}{2} + \pi n$

D. $x \neq \pi n,\ x \neq \frac{\pi}{2} + 2\pi n$

Use the following information to answer NR#7:

The simplified form of each of the following trigonometric expressions can be expressed using the indicated codes.

A $tanx + \dfrac{cosx}{1 + sinx}$ **B** $\dfrac{cscx}{sinx} - \dfrac{cotx}{tanx}$

Use the following codes (in bold) to complete the sentence below:

1 1 **2** $cosx$ **3** $sinx$ **4** $secx$ **5** $cscx$ **6** $tanx$

NR #7 Expression **A** simplifies to _____ *first code* and expression **B** simplifies to _____ *second code*.

19. Given an angle θ in standard position where $cot\theta = -\frac{4}{5}$ and $sin\theta > 0$, the value of $sin\left(\frac{\pi}{3} + \theta\right)$ is:

A. $\dfrac{4\sqrt{3} - 5}{2\sqrt{41}}$ B. $\dfrac{5 - 4\sqrt{3}}{2\sqrt{41}}$ C. $\dfrac{5\sqrt{3} - 4}{2\sqrt{41}}$ D. $\dfrac{4 - 5\sqrt{3}}{2\sqrt{41}}$

A student is working on the left side of a proof for the identity $\dfrac{1 + sinA - cos2A}{cosA + sin2A} = tanA$ when she makes a mistake. Her work is shown below:

Step 1 $\dfrac{1 + sinA - cos^2A - sin^2A}{cosA + 2sinAcosA}$

Step 2 $\dfrac{1 - cos^2A + sinA - sin^2A}{cosA + 2sinAcosA}$

Step 3 $\dfrac{sinA}{cosA + 2sinAcosA}$

Step 4 $\dfrac{1}{3cosA}$

20. The first recorded mistake occurs in:

A. *Step 1* B. *Step 2* C. *Step 3* D. *Step 4*

Use the following information to answer NR#8:

An angle in standard position θ terminates in quadrant II, with $cos\theta = -4/5$.

NR #8 The expression $tan2\theta$ simplifies to $-\dfrac{a}{b}$, where a, b are positive integers, a can be expressed in two digits and b is one. The three digits representing the values of a and b are _____.

PART 2 - Written Response

Use the following information to answer WR#1:

An angle in standard position θ passes through a point $P(-5, 1)$ and a second angle in standard position β passes through a point $Q(-3, -4)$.

❖ ***Written Response Question 1***

- Fully **sketch** each angle in the correct quadrant labeling all sides of the triangle, and **determine** the value of each angle, correct to the nearest degree. ***(3 marks)***

- **Determine** the exact value of $sin\,(\theta + \beta)$, written in the form $\frac{p}{q}$ ***(2 marks)***

❖ ***Written Response Question 2***

- Using a trigonometric identity, **simplify** the equation $2sin^2x - cosx - 1 = 0$ to express in terms of one trig function, where the lead coefficient is positive. ***(2 marks)***

- **Algebraically solve** the resulting equation on $\{0 \leq x < 2\pi\}$, and state a general solution. ***(3 marks)***

❖ ***Written Response Question 3***

- **Prove** the equation $\frac{cscx\ cosx}{tanx + cotx} = cos^2x$ is an identity using an algebraic approach. ***(3 marks)***

- **Determine** each of the possible non-permissible values, in radians. ***(2 marks)***

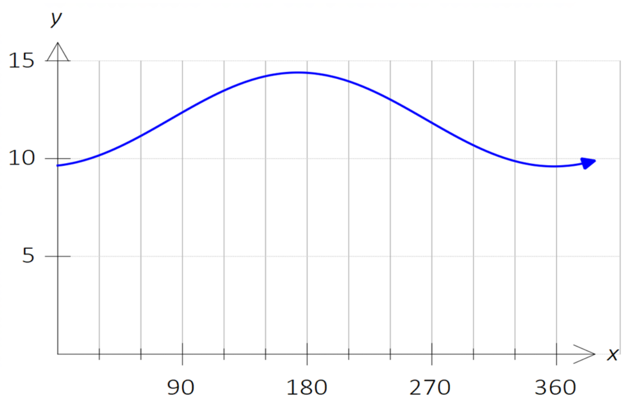
Use the following information to answer WR#4:

In San Diego, California the number of hours of daylight follows a sinusoidal pattern where the maximum hours of sunlight is 14.4 hours on day 173 (June 27th), and the minimum hours of sunlight is 9.6 hours on day 356 (Dec 22nd).

The function below is for a particular leap year of 366 days.

The hours of sunlight (H) can be modeled as a cosine function of day number (x):

$H = a\,cos\,[b(x - c)] + d$



❖ ***Written Response Question 4***

- **Determine** the values of a, b, c, and d in the equation $H = a\,cos\,[b(x - c)] + d$ ***(3 marks)***

- The daily high temperature in San Diego can be modeled by the function $T = 5.1sin\,[0.524(d - 2.75)] + 23.9$, where T is the temperature in degrees Celsius, and m is the number of months from the start of the year.

  Use a graphing approach to **determine** the approximate total number of months, correct to the nearest tenth, where the daily high temperature would be above $26°\text{C}$. ***(2 marks)***

- A function of similar form to the last bullet is constructed for Calgary Alberta, where the temperatures are much cooler. **Explain** which of the two parameters a, b, c, and d would be different, and how. **Justify** your reasoning.
  (Note, on the actual diploma exam each WR question will have exactly two bullets)

Answers

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Multiple Choice

1. D **2.** B **3.** A **4.** C **5.** C **6.** D **7.** C **8.** B **9.** A **10.** D **11.** D

12. A **13.** A **14.** B **15.** C **16.** C **17.** D **18.** C **19.** B **20.** A

Numerical Response

1. 3.5 **2.** 4.5 **3.** 125 **4.** 5281 **5.** 41 **6.** 264 **7.** 41 **8.** 247

Written Response

1. *First bullet* $\theta = 169° \ \ \beta = 233°$ *Second bullet* $\frac{17}{5\sqrt{26}}$

2. *First bullet* $2cos^2x + cosx - 1 = 0$ *Second bullet* $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ (any order) $x = \frac{\pi}{3} + \frac{2\pi}{3}n \ \ n \in I$ (general sol.)

3. *First bullet* See full solutions on math30-1edge.com *Second bullet* $x \neq \frac{\pi}{2}n\ , n \in I$

4. *First bullet* $a = 2.4,\ b = \frac{\pi}{183},\ c = 173,\ d = 12$ *Second bullet* **4.4** total months above 26°C.

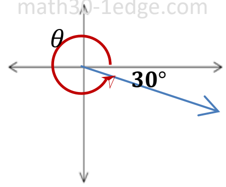
Third bullet $\boldsymbol{a}$ would be **higher**, as the range of Calgary temperatures (between min and max) would be greater

$\boldsymbol{d}$ would be **lower**, as the median temperature for Calgary (represented by d) would be lower

Also…. (not needed in your answer)

$\boldsymbol{b}$ would be **unchanged**, as the period for each city would be the same (12 months). Similarly, $\boldsymbol{c}$ would be essentially unchanged, as the number of months after which the min / max temperature occurs would be approximately the same as both cities are in the northern hemisphere.

1 First determine quadrant θ terminates in. Since sin is negative in Quad III and IV, and tan is neg. in II and IV, θ is in **quad IV**.



So. $\theta = 360° - 30°$

$= 330°$ **or** $\dfrac{11\pi}{6}$ *(not one of the choices)*

So, consider co-terminal angles of $330°$ *by adding / subtracting* $360°$ *(and converting to radians), or use* $\frac{11\pi}{6}$ *and add/subtract* 2π*...*

$= 330° + 360° = 690°$, converts to $\dfrac{23\pi}{6}$

ANSWER: **D**

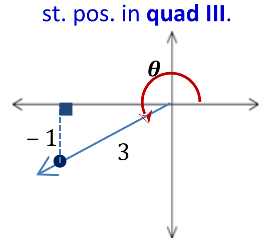
2 This is a reasoning question, not a calculation question! Using CAST rule, since cos is negative in Quad II and III, and tan is neg. in II and IV, θ is in **quad II.**

Which means, θ is between $\frac{\pi}{2}$ and π. (1.57 rads and 3.14 as decimals)

So, the only possible option there is **2.62** (Note, all answers are presumed to be in radians as no unit is specified!)

ANSWER: **B**

NR#1 Since sin is negative in Quad III and IV, and we want the *smallest* option for θ draw an angle in st. pos. in **quad III.**



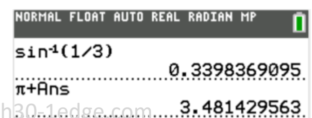
Given:

$sin\theta = \dfrac{-1}{3}$

First find reference angle, let's call it α *(inside the triangle)*

$\boldsymbol{\alpha} = sin^{-1}\left(\dfrac{1}{3}\right)$

NOTE: When finding reference angles (by definition less than $90°$ so all trig ratios are positive) **drop any negatives**

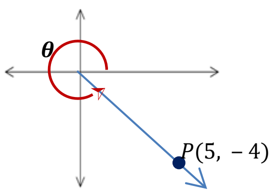


(radian mode)

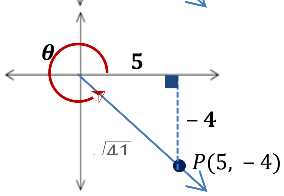
$\boldsymbol{\theta \approx 3.5}$

ANSWER: **3.5**

3 First determine quadrant θ terminates in. Since sin is negative in Quad III and IV, and tan is neg. in II and IV, θ is in **quad IV**.



Next, plot a point P in quad IV and sketch angle θ



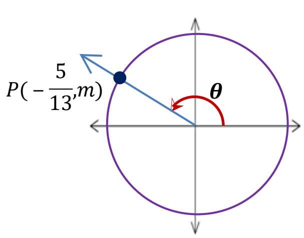
Then make a triangle by connecting P to the x-axis. Label the sides of the triangle from the coordinates of P ... use pyth. theorem to get the hyp.

Now use $sec\theta = \frac{1}{cos\theta}$ where $cos\theta = \frac{adj}{hyp}$....so $sec\theta = \frac{hyp}{adj}$

$sec\theta = \dfrac{\sqrt{41}}{5}$ ➡ $\approx \mathbf{1.28}$

ANSWER: **A**

4 There are two options for $P(-\frac{5}{13}, m)$, it can be drawn in quadrant II or III. *(as the x-coord is negative)* However, it is given that $\boldsymbol{tan}$ is negative, we know we should draw P in **quad II.** At this point we can pause to consider how fun it is to reason things out like that. *(pause for 10 to 15 seconds)*



🡐 *Diagram of information given*

We can now solve for m by either drawing a triangle and label the hypotenuse 1 (since – unit circle), or by going straight to the unit circle formula:

$$x^2 + y^2 = 1$$

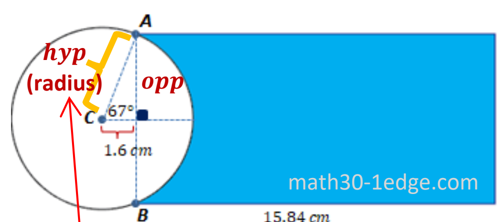
$\left(-\frac{5}{13}\right)^2 + m^2 = 1$ ➡ $m^2 = 1 - \frac{25}{169}$ ➡ $m^2 = \frac{144}{169}$ ➡ $m = \pm\sqrt{\frac{144}{169}}$

➡ $m = \pm\frac{12}{13}$ ➡ $m = \mathbf{\frac{12}{13}}$

Since quad II, use the + version!

ANSWER: **C**

5



First find the length of :

$cos67° = \dfrac{1.6}{AC}$ ➡ $AC = \dfrac{1.6}{cos67°}$ ➡ $\boldsymbol{AC \approx 4.095}$ cm *(radius)*

Next find the length of : *(half of AB is the opp side ...)*

$tan67° = \dfrac{opp}{1.6}$ ➡ $opp = 1.6tan67°$ ➡ $opp \approx 3.769$

➡ $AB \approx 2 * 3.769$ ➡ $\boldsymbol{AB \approx 7.539}$

Finally find the arc length:

$a = r\theta$ ➡ $a = 4.095 * \dfrac{(67° * 2)\pi}{180°}$ ➡ $\boldsymbol{arc \approx 9.577}$ **cm**

θ in radians

So **perimeter** is: $2 * 15.84 + 7.54 + 9.58$

Rectangle top / bottom

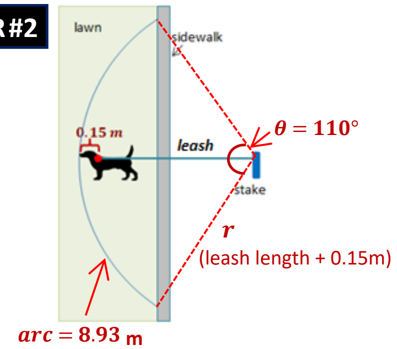
Third rectangle side, AB

Final "side", the arc length

Perimeter $\approx$ **48.8 cm**

ANSWER: **C**

NR#2



$\theta = \frac{a}{r}$

$r = \frac{a}{\theta}$ re-arrange

$r \approx \frac{8.93}{1.92}$ convert θ to radians: $\frac{110°\pi}{180°}$

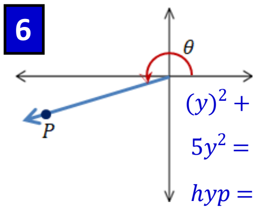
$r \approx 4.65$

So, leash+0.15 $\approx$ **4.65**

Remember, dog's extra reach is included here!

ANSWER: **4.5**

6



$(y)^2 + (2y)^2 = (hyp)^2$

$5y^2 = (hyp)^2$

$hyp = \sqrt{5}y$

$sec\theta = \frac{\sqrt{5}y}{-2y}$ ← hyp ← adj

$\boldsymbol{sec\theta = -\frac{\sqrt{5}}{2}}$

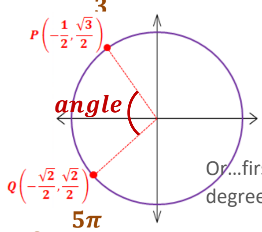
$a = 5 \quad b = 2$

ANSWER: **D**

Add: $5 + 2 = \mathbf{7}$

7

$\theta = \frac{2\pi}{3}$



$\beta = \frac{5\pi}{4}$

angle between $= \frac{5\pi}{4} - \frac{2\pi}{3}$

$= \frac{15\pi}{12} - \frac{8\pi}{12}$

$= \mathbf{\frac{7\pi}{12}}$

Or...first convert to degrees: $= 95°$

Converts to $\frac{7\pi}{12}$

Statement 1 is FALSE

$\theta = \frac{2\pi}{3}$ *Principal angle*

$\theta = \frac{2\pi}{3} - \mathbf{2\pi}$ *Is co-terminal*

$\theta = -\frac{4\pi}{3}$

Statement 2 is TRUE

y-coord at $\frac{2\pi}{3}$ is $sin\theta$

x-coord at $\frac{5\pi}{4}$ is $cos\theta$

Statement 3 is TRUE

$\boldsymbol{tan\frac{2\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$ ← y ← x

$= \frac{\sqrt{3}}{2} * \frac{-2}{1}$

$= -\sqrt{3}$

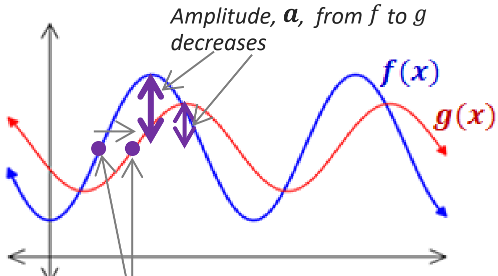
$\boldsymbol{cot\frac{5\pi}{4}} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$ ← x ← y

$= \mathbf{1}$

Statement 4 is TRUE

ANSWER: **C**

8



ANSWER: **B**

9

First factor b for horiz. phase shift:

$y = 5\,sin\left[4\left(x + \frac{\pi}{4}\right)\right]$

Period is $\frac{2\pi}{b}$ Phase shift

$= \frac{2\pi}{4}$ ➡ $= \frac{\boldsymbol{\pi}}{\mathbf{2}}$ *(to the left, but not relevant)*

ANSWER: **A**

NR#4

$y = 2.3sin\,(0.1208x - 0.3624) + 5.8$

Period is $\frac{2\pi}{b}$

$= \frac{2\pi}{0.1208}$ ➡ $= \mathbf{52}$

MAX is **AMPL + Vert. Shift**

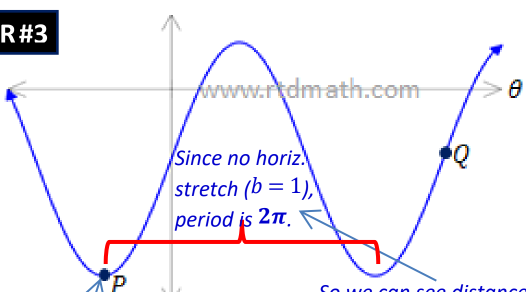
$= a + d$

$= 2.3 + 5.8$

$= \mathbf{8.1}$

ANSWER: **5281**

NR#3

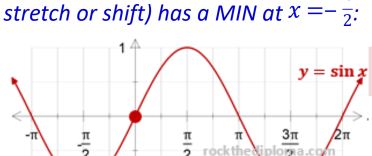


Since no horiz. stretch ($b = 1$), period is $\mathbf{2\pi}$.

So we can see distance between P and Q is greater than 2π

Statement 2 is TRUE

Basic sine curve (with no horizontal stretch or shift) has a MIN at $x = -\frac{\pi}{2}$:



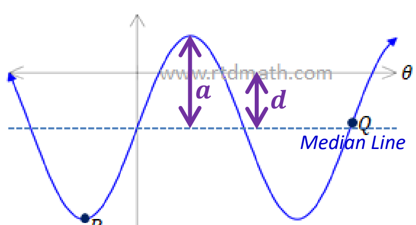
Statement 1 is TRUE

For y-int, set $x = 0$:

But $sin\,(0) = 0$, so...

$y = a * sin\,(\mathbf{0}) - d \qquad \boldsymbol{y = -d}$

Statement 4 is FALSE



$\boldsymbol{a}$ can be visualized as dist. from MAX (or MIN) to median line

$\boldsymbol{d}$ as dist. from median line to x-intercept

Statement 3 is FALSE

For $b < 1$, for example....

$y = a\,sin\,(2x) - d$

The horiz. str. would be the reciprocal, or here **0.5**

For horiz str < 1, all points move closer to the x-axis ...

Statement 5 is TRUE

ANSWER: **125**

Note: *Mistake on some answer keys*

10.

Domain of $y = tanx$ is $x \neq \frac{\pi}{2} + n\pi$

That is, where $cosx = 0$, since $tanx = \frac{sinx}{cosx}$

... at the top / bottom of the unit circle

So for $y = tan4x$ ← Hor. Str. of $\frac{1}{4}$

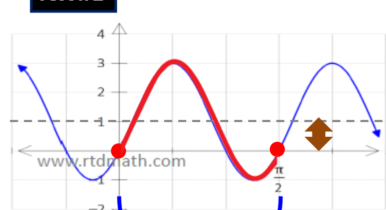
Domain is:

$x \neq \mathbf{\frac{1}{4}} * \frac{\pi}{2} + \mathbf{\frac{1}{4}} * n\pi$

$x \neq \mathbf{\frac{\pi}{8}} + \mathbf{\frac{n\pi}{4}}$

ANSWER: **D**

NR#5



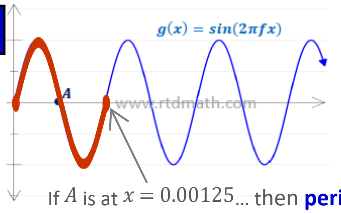
$\boldsymbol{d = 1}$

Period is $\frac{\pi}{2}$ ➡ $b = \frac{2\pi}{\frac{\pi}{2}}$ ➡ $b = 2\pi * \frac{2}{\pi}$

$\boldsymbol{b = 4}$

ANSWER: **41**

11.



If A is at $x = 0.00125$... then **period** is double, $\mathbf{0.0025\ s}$

Use $period = \frac{2\pi}{b}$ formula to solve for f: $0.0025\ s = \frac{2\pi}{2\pi f}$ ➡ $\frac{1}{f} = 0.0025\ s$ ➡ $f = \frac{1}{0.0025\ s}$

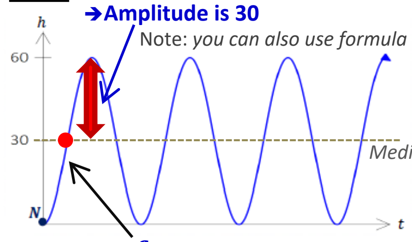
$f = 400\ Hz$

ANSWER: **D**

12.

➔**Amplitude is 30**

Note: *you can also use formula* $a = \frac{max - min}{2}$



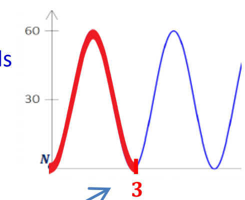
c_{sine} *Distance (from y-axis) of point where curve upswing intersects median line.*

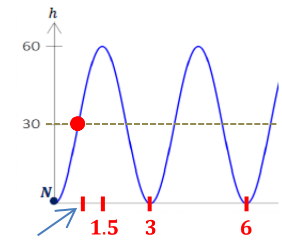
For c_{sine} determine x-coords of max / mins...

20 rotations in 60 sec.... (1 min)

1 rotation in $\frac{60}{20}$

$= \mathbf{3\ sec}$ (this is the **period**)





$c_{sine} = \mathbf{0.75s}$

Halfway between $\mathbf{0}$ *(where min is) and* $\mathbf{1.5\ s}$ *(where max is)*

ANSWER: **A**

13. Solve by factoring: $2cos^2\theta + 3cos\theta - 2 = 0$

$(2cos\theta \quad)(cos\theta \quad) = 0$

Two numbers must mult. to -2

$(2cos\theta - 1)(cos\theta + 2) = 0$

Now set each factor to zero ...

$2cos\theta - 1 = 0 \quad cos\theta + 2 = 0$

$cos\theta = \frac{1}{2}$ or $cos\theta = -2$

ANSWER: **A**

14. First isolate trig term: $3csc^2\theta - 4 = 0$

$3csc^2\theta = 4$

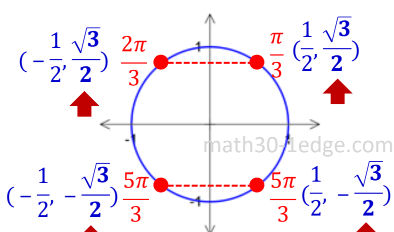
$csc^2\theta = \frac{4}{3}$

Sq. root both sides $\quad csc\theta = \pm\sqrt{\frac{4}{3}}$

Since $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\quad csc\theta = \pm\frac{2}{\sqrt{3}}$

Since $\boldsymbol{csc\theta}$ is reciprocal of $\boldsymbol{sin\theta}$ $\quad sin\theta = \pm\frac{\sqrt{3}}{2}$

Find where on unit circle the y-coord is $\pm\frac{\sqrt{3}}{2}$



$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

ANSWER: **B**

15. Factor $sec^2x - secx - 2 = 0$

$(secx + 1)(secx - 2) = 0$

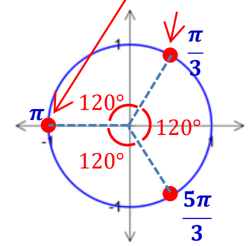
Set each factor to zero:

$secx = -1$ or $secx = 2$

Since $\boldsymbol{secx}$ is reciprocal of :

$cosx = -1$ or $cosx = \frac{1}{2}$

Find where on unit circle the x-coord is -1 *or* $1/2$



Note that all three solutions are 120°, or $\frac{2\pi}{3}$, apart

$\theta = \frac{\pi}{3} + \frac{2\pi}{3}n$; $n \in I$

first solution — diff between sols

ANSWER: **C**

16. First isolate and use log laws on LS

$log_2(tanx) + log_2(cosx) = -1$

$log_2(tanx * cosx) = -1$

Use trig identities to simplify

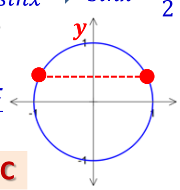
$log_2\left(\frac{sinx}{cosx} * cosx\right) = -1$

$log_2(sinx) = -1$

Convert to log form

$2^{-1} = sinx$ ➡ $sinx = \frac{1}{2}$

Find where on unit circle the y-coord is $\frac{1}{2}$



$x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

ANSWER: **C**

NR#6 First re-write in degrees: $cos\ (105°)$

Next find any two standard unit circle angles that add (or subtract) to $105°$ *Such as:* $cos\ (45° + 60°)$

Note: There are many options, another is $cos\ (135° - 30°)$

$cos\ (\alpha + \beta) = cos\alpha\ cos\beta - sin\alpha\ sin\beta$

$= cos45°cos60° - sin45°sin60°$

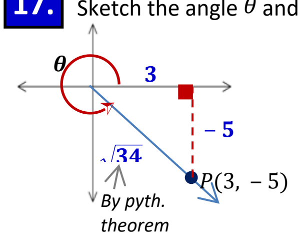
Finally refer to unit circle and simplify

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$ ➡ $= \frac{\sqrt{2} - \sqrt{6}}{4}$

ANSWER: **264**

17. Sketch the angle θ and draw a triangle to determine trig ratios:



Now, for $sin\ (\pi - \theta)$ ...

$= sin\pi\ cos\theta - cos\pi\ sin\theta$

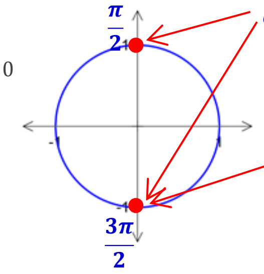
$= (0)\left(\frac{3}{\sqrt{34}}\right) - (-1)\left(\frac{-5}{\sqrt{34}}\right)$ ➡ $= -\frac{5}{\sqrt{34}}$

From unit circle — From diagram $cos = \frac{adj}{hyp}$

ANSWER: **D**

18. $\dfrac{tanx}{1+sinx}$ Simplifies to … $\dfrac{\frac{sinx}{cosx}}{1+sinx}$

So, NPVs … $cosx \neq 0$; $1 + sinx \neq 0$



$cosx \neq 0$
Where on the unit circle is the x-coord 0?

$1 + sinx \neq 0$
$sinx \neq -1$
Where on the unit circle is the y-coord equal to -1? (already covered above!)

So, NPV at the top / bottom of the unit circle

$x \neq \frac{\pi}{2}$ then every π

Which we write as:

$x \neq \frac{\pi}{2} + n\pi$ where $n \in I$

ANSWER: **C**

NR #7 **Simplify:** $tanx + \dfrac{cosx}{1+sinx}$

$= \dfrac{sinx}{cosx} + \dfrac{cosx}{1+sinx}$

$= \dfrac{sinx(1+sinx)}{cosx(1+sinx)} + \dfrac{cosx(cosx)}{1+sinx(cosx)}$

Pyth. identity, this is "1"

$= \dfrac{sinx + sin^2x + cos^2x}{cosx(1+sinx)}$

$= \dfrac{sinx+1}{cosx(1+sinx)}$

$= \dfrac{1}{cosx}$

$= secx$ Code: **4**

Simplify: $\dfrac{cscx}{sinx} + \dfrac{cotx}{tanx}$

$= \dfrac{\frac{1}{sinx}}{sinx} + \dfrac{\frac{cosx}{sinx}}{\frac{sinx}{cosx}}$

$= \dfrac{1}{sinx} * \dfrac{1}{sinx} + \dfrac{cosx}{sinx} * \dfrac{cosx}{sinx}$

$= \dfrac{1}{sin^2x} + \dfrac{cos^2x}{sin^2x}$

$= \dfrac{1+cos^2x}{sin^2x}$

$= \dfrac{sin^2x}{sin^2x}$ ➡ $= 1$ Code: **1**

ANSWER: **41**

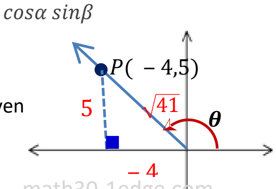
19. Start with appropriate addition / subtraction formula:

$sin(\alpha+\beta) = sin\alpha\, cos\beta + cos\alpha\, sin\beta$

$= sin\frac{\pi}{3}cos\theta + cos\frac{\pi}{3}sin\theta$

Use unit circle for this and $cos\frac{\pi}{3}$

Draw θ using info given



To sketch θ, use: $cot\theta = -\frac{4}{5}$ ← adj ← opp

In **Quad II** since adj side is neg and $sin\theta > 0$

Use pyth. theorem for hyp., $(-4)^2 + 5^2 = hyp^2$

so… $cos\theta = \dfrac{-4}{\sqrt{41}}$ $sin\theta = \dfrac{5}{\sqrt{41}}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-4}{\sqrt{41}}\right) + \left(\frac{1}{2}\right)\left(\frac{5}{\sqrt{41}}\right)$

$= \dfrac{-4\sqrt{3}}{2\sqrt{41}} + \dfrac{5}{2\sqrt{41}}$

$= \dfrac{-4\sqrt{3}+5}{2\sqrt{41}}$

ANSWER: **B**

20. ➡ **Step 1** The mistake is here!

Correct steps:

$\dfrac{1 + sinA - (cos^2A - sin^2A)}{cosA + 2sinAcosA}$

$= \dfrac{1 - cos^2A + sinA + sin^2A}{cosA + 2sinAcosA}$

$= \dfrac{sin^2A + sinA + sin^2A}{cosA + 2sinAcosA}$

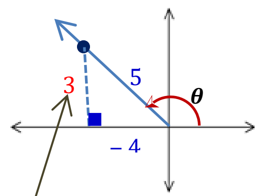
$= \dfrac{2sin^2A + sinA}{cosA + 2sinAcosA}$

$= \dfrac{sinA(2sinA+1)}{cosA(1+2sinA)}$

$= tanA$

ANSWER: **A**

NR #8 Sketch:



By pyth. theorem

So, $tan\theta = -\frac{3}{4}$

Formula sheet: $\tan(2\alpha) = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$= \dfrac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}$

$= \dfrac{-\frac{6}{4}}{1-\frac{9}{16}}$

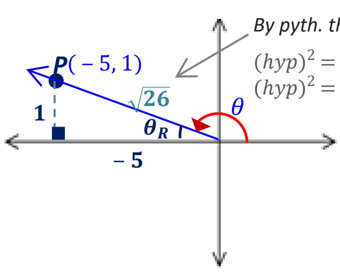
$= \dfrac{-\frac{3}{2}}{\frac{7}{16}}$

$= -\frac{3}{2} * \frac{16}{7}$ ➡ $= -\frac{24}{7}$

ANSWER: **247**

Written

First bullet



By pyth. theorem:
$(hyp)^2 = (-5)^2 + (1)^2$
$(hyp)^2 = 26$

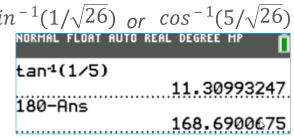
To find θ, first find reference angle θ_R (inside the triangle)

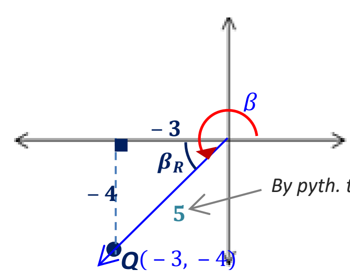
$\theta_R = tan^{-1}(1/5)$ or $sin^{-1}(1/\sqrt{26})$ or $cos^{-1}(5/\sqrt{26})$

so …

$\theta \approx 180° - 11°$

$\theta \approx 169°$





By pyth. theorem:

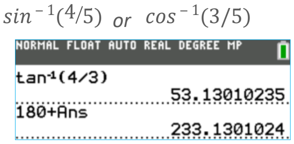
To find β, first find reference angle β_R (inside the triangle)

$\beta_R = tan^{-1}(4/3)$ or $sin^{-1}(4/5)$ or $cos^{-1}(3/5)$

so …

$\theta \approx 180° + 53°$

$\theta \approx 233°$



WR #1

Second bullet

Refer to your formula sheet: $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

We'll need all this: (leave exact)

$\sin\theta=\frac{1}{\sqrt{26}}$ ← opp / ← hyp $\cos\theta=\frac{-5}{\sqrt{26}}$ ← adj / ← hyp

similarly… $\sin\beta=\frac{-4}{5}$ $\cos\beta=\frac{-3}{5}$

So…

$$\sin(\theta+\beta)=\left(\frac{1}{\sqrt{26}}\right)\left(\frac{-3}{5}\right)+\left(\frac{-5}{\sqrt{26}}\right)\left(\frac{-4}{5}\right) \Rightarrow =\frac{-3}{5\sqrt{26}}+\frac{20}{5\sqrt{26}} \Rightarrow =\frac{17}{5\sqrt{26}}$$

***Note:** There was a mistake in some answer keys!*

Written

First bullet

Use $\sin^2\theta+\cos^2\theta=1$, *which re-arranges to* $\sin^2\theta=1-\cos^2\theta$ *, to re-write equation in terms of* $\cos\theta$ *only*

$2(1-\cos^2\theta)-\cos x-1=0$

$2-2\cos^2\theta-\cos x-1=0$

$0=2\cos^2\theta+\cos x-1$ ➡ $2\cos^2\theta+\cos x-1=0$

2nd bullet

$(2\cos\theta\ \ \ )(\cos\theta\ \ \ )=0$ *Factor*

Find two #s that mult to -1*, and expression will expand back out to* $2\cos^2\theta+\cos x-1$

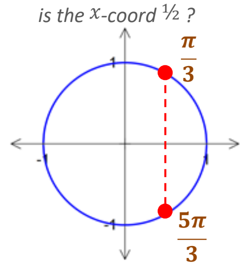
$(2\cos\theta-1)(\cos\theta+1)=0=0$

Set each factor to zero

$2\cos\theta-1=0$ *or* $\cos\theta+1=0$

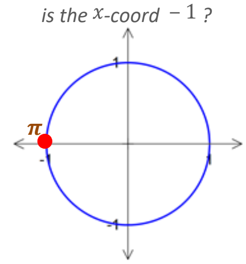
$\cos\theta=1/2$

Where on the unit circle is the x*-coord ½ ?*



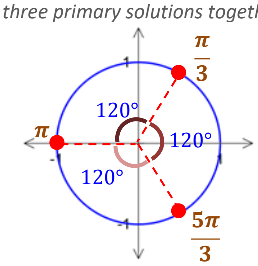
$\cos\theta=-1$

Where on the unit circle is the x*-coord* -1 *?*



PRIMARY SOLUTIONS: $\theta=\frac{\pi}{3},\pi,\frac{5\pi}{3}$

*For **GENERAL SOLUTION**, consider all three primary solutions together*



They're all $\frac{2\pi}{3}$ *(that is,* $120°$*) apart*

GENERAL SOLUTION: $\theta=\frac{\pi}{3}+\frac{2\pi}{3}n \quad n\in I$

Written

First bullet

Proceed on left side:

$$=\frac{\frac{1}{\sin x}*\cos x}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$$

Write each expression in terms of $\sin$ *and* $\cos$

$$=\frac{\frac{\cos x}{\sin x}}{\frac{\sin x*\sin x}{\cos x*\sin x}+\frac{\cos x*\cos x}{\sin x*\cos x}}$$

Re-write bottom expressions with a common denom.

$$=\frac{\frac{\cos x}{\sin x}}{\frac{\sin^2x+\cos^2x}{\cos x\sin x}}$$

← *Pyth. identity, this is "1" !*

$$=\frac{\cos x}{\sin x}*\frac{\cos x\sin x}{1}$$

Invert and multiply ➡ $=\cos^2x$

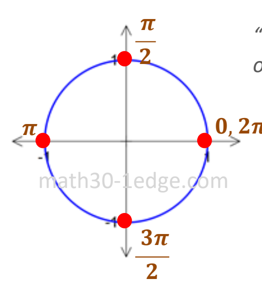
2nd bullet

$$=\frac{\frac{1}{\sin x}*\cos x}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$$

Go back to first step, restriction at any expression in the denominator. (Can't divide by 0)

$\sin x\neq 0$ $\cos x\neq 0$

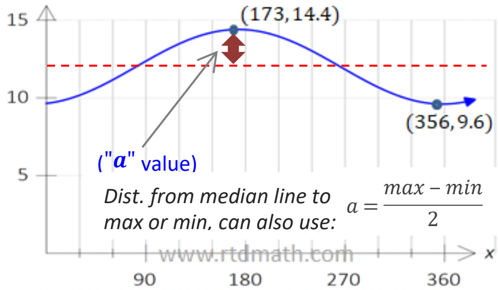
"where on the unit circle is the x *or* y *coord equal to 0 ?"*



$x\neq\frac{\pi}{2}n \quad n\in I$

Written

First bullet



("a" value)

Dist. from median line to max or min, can also use: $a = \frac{max - min}{2}$

$a = \frac{14.4 - 9.6}{2}$ $\boldsymbol{a = 2.4}$

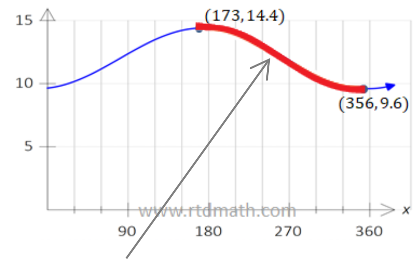
Median line ("d" value)

$d = \frac{max + min}{2}$

$d = \frac{14.4 + 9.6}{2}$

$\boldsymbol{d = 12}$

For $\boldsymbol{b}$*, determine the* ***period****:*



This shows half of the ***period*** *(dist from max to min)*

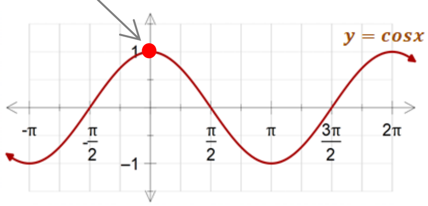
Half of Period = 183 *days* (356 − 173)

Period = 366 *days* *Actually, this was given!*

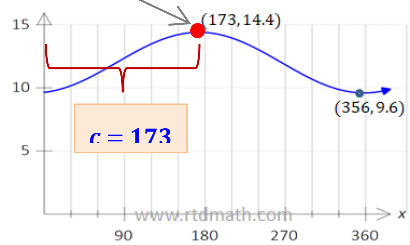
$b = \frac{2\pi}{366}$ ➡ $\boldsymbol{b = \frac{\pi}{183}}$

Finally for $\boldsymbol{c_{cos}}$*, determine the horiz. dist of the max from the* y*-axis*

Basic ***cos*** *curve "starts" at the MAX*



Here the max has ***shifted****, 173 to the right*



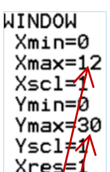
$c = 173$

$$H = 2.4\cos\left[\frac{\pi}{183}(x - 173)\right] + 12$$

2nd bullet

Graph $y_1 = 5.1\sin(0.524(x - 2.75)) + 23.9$

$y_2 = 26$



For x*-max, graph over 1 period*

$per = \frac{2\pi}{0.524}$

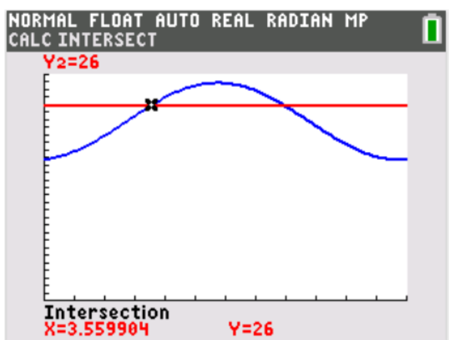
$\approx \boldsymbol{12}$

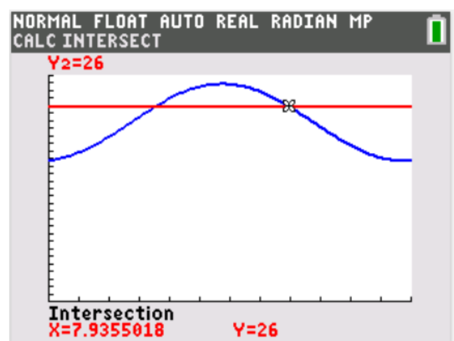
For y*-max, use* $\boldsymbol{MAX = a + d}$

$= 5.1 + 23.9$

$= \boldsymbol{29}$

And find the ***INTERSECTS***





So, total # months above 26°C *can be found by subtracting….*

$= 7.94 - 3.56$

4.4 months

Third bullet

$\boldsymbol{a}$ would be **higher**, as the range of Calgary temperatures (between min and max) would be greater

$\boldsymbol{d}$ would be **lower**, as the median temperature for Calgary (represented by d) would be lower

Also…. (not needed in your answer)

$\boldsymbol{b}$ would be **unchanged**, as the period for each city would be the same (12 months). Similarly, $\boldsymbol{c}$ would be essentially unchanged, as the number of months after which the min / max temperature occurs would be approximately the same as both cities are in the northern hemisphere.