RTD EXAM PREP

Study Guide for the Math 30-1 Diploma Exam

Part 1 (pages 1 – 63)

- ❖ Function Operations and Transformations
- ❖ Polynomial Functions
- ❖ Exponential and Logarithmic Functions

Your next math course could be with us!

School website:

rtdacademy.com

Learn more here:

Video

RTD ACADEMY
Online School

*Earn **High School Credits** - just as you would in any public school - at no cost!*

(if you are in class now – turn down your volume!)

RTD Math Academy is a new provincially accredited online high school from by Rock the Diploma. Learn from exceptional online resources and amazing teachers, with a program **customized** to your schedule.

*Math 10C, 20-1, 20-2, **30-1**, **30-2**, 31 (**Calculus**), Physics 20, 30, and Coding ... With new courses in development*

FLEXIBILITY: We'll build a **custom schedule** for you to start and complete your course **anytime**. *You pick the dates!*

NO COST: We are a provincially funded independent high school, so most students can sign up for free.

Registration Is Now Open for Semester II

Upgrade your Math mark (or work ahead), or earn credits for CTS-CODING *Start as early as **tomorrow!***

The Math 30-1 Diploma Exam

Mathematics 30-1 is designed to provide students with the knowledge base, mathematical understandings, and critical thinking skills identified for entry into post-secondary programs that require the study of calculus.

In Mathematics 30-1, **algebraic**, **numerical**, and **graphica**l approaches are used to solve problems. Technology is used to enable students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Students are expected to communicate solutions clearly and effectively when **solving problems**. Students are also expected to develop both **conceptual** and **procedural** understandings of mathematics and apply them to real-life problems. It is important for students to realize that it is acceptable to solve problems in different ways, using a variety of strategies.

Exam Breakdown *The Mathematics 30-1 Exam comprised of 32* ***machine scored*** *questions and 3* ***written response****:*

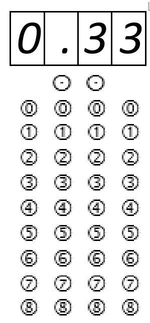
Question Format	Number of Questions	Emphasis
Machine Scored		75%
o Multiple Choice	24 formerly 28	
o Numerical Response	8 formerly 12	
Written Response	3	25%

❖ There are various types of ***numerical response questions*** that students should be familiar with…

❶ ***Calculation Question and Solution – Decimal***

Numerical Response: A function $P(x) = x(x+3)(x-1)$ is transformed to $y = P(3x)$. The largest positive x-intercept, correct to the nearest hundredth, is:_______.

Answer: $1/3 = \mathbf{0.33}$



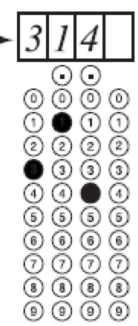
❷ *Any-Order Question and Solution*

The zeros of the polynomial function $P(x) = x^3 - 8x^2 + 19x - 12$ are ___, ___, and ___.

(Record all **three digits** of your answer **in any order** in the numerical-response section on the answer sheet.)

Digits to be recorded: 314

Record 314 on the answer sheet →



Correct-Order Question and Solution

Four Expressions

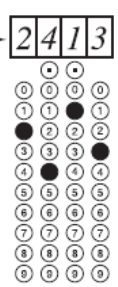
1 $5 \times 4 \times 3$
2 ${}_5C_2$
3 $5!$
4 ${}_5P_2$

When the expressions above are arranged in ascending order, their order is ___, ___, ___, and ___.

(Record the answer in the numerical-response section on the answer sheet.)

Value to be recorded: 2413

Record 2413 on the answer sheet →



Calculation Question and Solution

If $f(x) = 2x^2 + 3x + 5$ and $g(x) = x^2 + 2x - 3$, then $f(x) + g(x)$ can be expressed in the form $ax^2 + bx + c$.

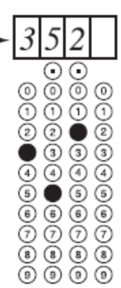
In the expression above, the value of

a is _______ (Record in the **first** column)
b is _______ (Record in the **second** column)
c is _______ (Record in the **third** column)

(Record the answer in the numerical-response section on the answer sheet.)

Value to be recorded: 352

Record 352 on the answer sheet →



Exam Breakdown – Question Emphasis

Mathematical Understanding	*Emphasis*
Conceptual	34%
Procedural	30%
Problem Solving	36%

If y is replaced by $\frac{1}{2}y$ in the equation $y = f(x)$, then the graph of $y = f(x)$ will be stretched:

A. horizontally about the y-axis by a factor of $\frac{1}{2}$
B. horizontally about the y-axis by a factor of 2
C. vertically about the x-axis by a factor of
D. vertically about the x-axis by a factor of 2

Numerical Response

In the equation $3^{2x+1} = 7$, the value of x, to the nearest hundredth, is ________.

The population of a city was 173 500 on Jan 1, 1998, and it was 294 000 on Jan 1, 2012. If the growth rate of the city can be modelled as an exponential function, then the average annual growth rate of the city, to the nearest tenth of a percent, was

A. 1.0% **B.** 3.8% **C.** 6.9% **D.** 12.1%

Answers *(Mult choice examples above) -* Conceptual: **D** Procedural: **0.39** Problem Solving: **B**

Exam Breakdown – Breakdown of Content

Diploma Exam Content	*Percentage Emphasis*	
Relations and Functions	55%	*18*
Trigonometry	29%	*9*
Permutations, Combinations, and Binomial Theorem	16%	*5*

Possible # of (machine scored) exam questions

Relations & Functions Topics	**Exam Weighting** *adds to 55*	
Function Operations	5%	*1 or 2*
Transformations	14%	*4 or 5*
Exp. Functions / Logs	18%	*5 or 6*
Polynomial Functions	10%	*3 or 4*
Radical & Rational	8%	*2 or 3*

Poss. # of (machine scored) exam questions

NEW – Written Response

➔ Your exam will have 3 written response questions, each composed of two parts (bullets).
The written-response component is designed to assess the degree to which students can draw on their mathematical experiences to solve problems, explain mathematical concepts, and demonstrate their algebraic skills. A written-response question may cover more than one specific outcome and will require students to make connections between concepts. Each written-response question will consist of two parts and will address multiple cognitive levels. Students should be encouraged to try to solve the problems in both parts as an attempt at a solution may be worth partial marks.

Students may be asked to solve, explain, or prove in a written-response question. Students are required to know the definitions and expectations of directing words such as **algebraically**, **compare**, **determine**, **evaluate**, **justify**, and **sketch**.

The following instructions will be included in the instructions pages of all mathematics diploma exam booklets:

- Write your responses in the test booklet as neatly as possible.
- For full marks, your responses must address **all** aspects of the question.
- All responses, including descriptions and/or explanations of concepts, must include pertinent ideas, calculations, formulas, and correct units.
- Your responses must be presented in a well-organized manner. For example, you may organize your responses in paragraphs or point form.

In **scoring** the written-response questions, markers will evaluate how well students

- demonstrate their understanding of the problem or the mathematical concept
- correctly apply mathematical knowledge and skills
- use problem-solving strategies and explain their solutions and procedures
- communicate their solutions and mathematical ideas

General Scoring Guide for a 2-mark Part

Score	General Description
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.
1.5	
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.

General Scoring Guide for a 3-mark Part

Score	General Description
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.
1.5	
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.
2.5	
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.

Specific Scoring Guides for each written-response question will provide detailed descriptions to clarify expectations of student performance at each benchmark score of 0, 1, 2, and 3. A student response that does not meet the performance level of a benchmark score may receive an augmented score of 0.5, 1.5, or 2.5.

Descriptions of these augmented scores will be determined with teachers at each marking session and are not an exhaustive list. Each part will be scored separately and the scores will be combined for a total of 5 marks.

<u>Time Limits on Diploma Examinations</u>

All students may now use extra time to write diploma exams. This means that all students now have up to 6 hours to complete the Mathematics 30–1 Diploma Examination, if they need it. **The examination is still designed so that the majority of students can comfortably complete it within 3 hours.** The examination instructions state both the original time and the total time now available.

Extra time is available for diploma examinations in all subjects, but the total time allowed is not the same in all subjects.

Function Operations / Transformations 1

Course weighting: **19%** Approx. Number of (Machine Scored) Exam Questions: 5 to 7

1.1 – Operations on Functions **1.2** – Function Transformations **1.3** – Inverse Functions and Relations

1.1 Operations on Functions

i – Domain of a Function

For domain of a function we do not need to consider the graph. Just ask yourself: **Are there any restrictions?** That is, values of x that would result in ....

❶ Dividing by 0 ❷ Taking the square root of a negative ❸ Taking the LOG of 0 or a negative *None of these things are "allowed"!*

Worked Example *Without graphing, state the domain of each of the following*

(a) $f(x) = \dfrac{x^2+x-2}{x^2-3x+2}$ | **(b)** $g(x) = \sqrt{4-2x}$ | **(c)** $h(x) = log_2(2x-3)$

Solutions:

FACTOR the bottom (denom.) $= \dfrac{x^2+x-2}{(x-1)(x-2)}$

Set each factor to 0 to find NPVs $x-1=0 \quad x-2=0$

Can't divide by 0, so .... $\{x|x \neq 1, 2, x \in \mathbb{R}\}$

This domain is expressed in **set notation**

Using interval notion is not ideal!

Set what's under the $\sqrt{\ }$ *sign greater than (or equal to) 0.* $g(x) = \sqrt{\underbrace{4-2x}}$

$4-2x \geq 0$

Solve resulting inequality $\dfrac{4}{2} \geq \dfrac{2x}{2}$

Can't $\sqrt{\ }$ *negatives, so ....* $\{x|x \leq 2, x \in \mathbb{R}\}$

Alternatively, in **INTERVAL NOTATION:** $(-\infty, 2]$

Read as: "From $-\infty$ (not including endpoint) to 2 (including endpoint)"

Set what's being LOGGED greater than 0. $h(x) = log_2(\underbrace{2x-3})$

$2x-3 > 0$

Solve resulting inequality $\dfrac{2x}{2} > \dfrac{3}{2}$

Can't LOG zero or negatives, so .... $\{x|x > 3/2, x \in \mathbb{R}\}$

➔ $\sqrt{0} = 0$ *, but* $log(0)$ *is undfined.* $10^{\square} = 0$

Since there's no exp we can take base 10 (or any base) to, to get zero

Alternatively, in **INTERVAL NOTATION:** $(3/2, \infty)$

Class Example 1 *Obtaining the Domain of a Function from its Equation*

State the domain of each function, using either set notation or interval notation.

(a) $y = \dfrac{2x-5}{4x^2-9}$ (b) $y = \dfrac{x(x-2)}{x^2+1}$ (c) $y = log(10-4x)$

(d) i $y = 5(2)^{x-1} - 3$

ii $y = 2x^3 - 7x + 8$

iii $y = 3\,sin(x-\pi) + 1$

(e) $g(x) = \sqrt{5-6x} + 3$ (f) $g(x) = \sqrt{ax+b}$

RTD LEARNING

ii – Operations on Functions

Functions (defined by equations, graphs, charts, etc.) can be **added**, **subtracted**, **multiplied**, or **divided**. (Combined)

Worked Example

Given $f(x) = x^2 - 4$ and $g(x) = x - 2$,

(a) Evaluate $(f \times g)(3)$

(b) Determine a simplified expression for $h(x) = \left(\frac{g}{f}\right)(x)$

(c) State the domain of $h(x)$.

Solutions:

(a) Read as: $f(3) \times g(3)$

Value, or y-coord of $f(x)$ when $x = 3$

Value of $g(x)$ when $x = 3$

$= [(3)^2 - 4] \times [(3) - 2]$

$= 5 \times 1$ ➔ $= \mathbf{5}$

$f(3)$ $g(3)$

(b) Simplify:

$$h(x) = \frac{x-2}{x^2-4}$$

$$= \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}}$$

$$= \frac{\mathbf{1}}{\mathbf{x+2}} \;; x \neq \pm 2$$

(c) Refer to the "pre-canceled" equation:

$$h(x) = \frac{x-2}{(x+2)(x-2)}$$

$$\{\mathbf{x} \mid \mathbf{x} \neq \pm\mathbf{2}, \mathbf{x} \in \mathbb{R}\}$$

The **domain** of combined functions is like being a good host cooking a meal for two friends:

- One can't have gluten
- The other can't have meat

*So your meal must honor **both restrictions** – and not have gluten and not have meat!*

However when **dividing**, as we saw above, we have an additional restriction as the bottom function.

Class Example 2 — *Adding and Subtracting Functions*

Given functions $f(x) = x^2 + x - 6$ and $g(x) = x + 3$, determine:

(a) A simplified expression for the function $h(x) = (f + g)(x)$

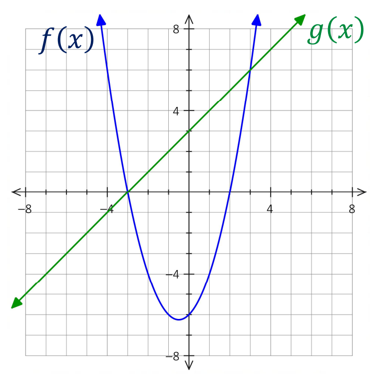
*Also state the **domain** of $h(x)$*

(b) The value of $(f - g)(2)$

(c) A simplified expression for $k(x) = \dfrac{f(x)}{g(x)}$

*Also state the **domain** of $k(x)$*

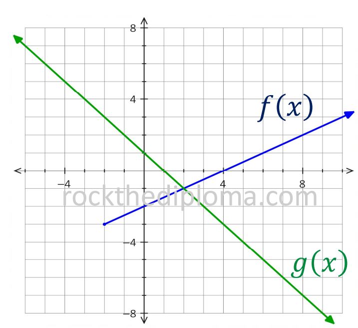
(d) Sketch the graph of $y = k(x)$ ➔ on the same grid



Class Example 3 — *Obtaining the Graph of a Combined Function*

Given the graphs of $f(x)$ and $g(x)$ on the right,

Sketch the graph of $h(x) = (f - g)(x)$, and state the domain.



RTD LEARNING

Worked Example Given $f(x) = \dfrac{x}{x+5}$ and $g(x) = 3x + 1$, a combined function is defined $h(x) = (f \times g)\,(x)$

(a) state the domain of $h(x)$

(b) Evaluate $h(-4)$

Solutions:

(a) Domain of $y = f(x)$ is $\{x \neq -5\,, x \in \mathbb{R}\}$

Domain of $y = g(x)$ is $\{x \in \mathbb{R}\}$

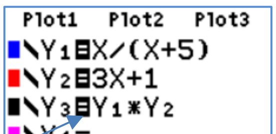
The domain of $h(x)$ must be "include" the domain of either $f(x)$ or $g(x)$

The domain of $h(x)$ is $\{x | \boldsymbol{x \neq -5}\,, x \in \mathbb{R}\}$

(b) Find $f(-4) \times g(-4)$

$= \underset{f(-4)}{-4} \times \underset{g(-4)}{-11} = \mathbf{44}$

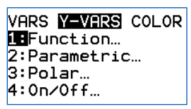
CALC TIP You can do combined functions on your graphing calculator!

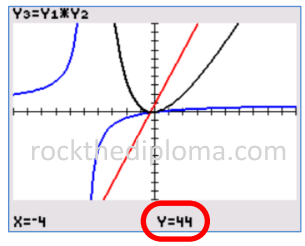


Now you can compare the graphs, and TRACE on the graph of $h(x)$, then key in -4 for x to get your answer for (b) above.

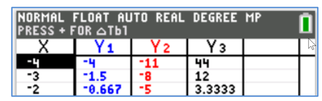
To enter "y_1", key in vars, select #1 Function

Then choose from there





Alternatively, you can look through the TABLE:



Class Example 4 *Domain of a Combined Function*

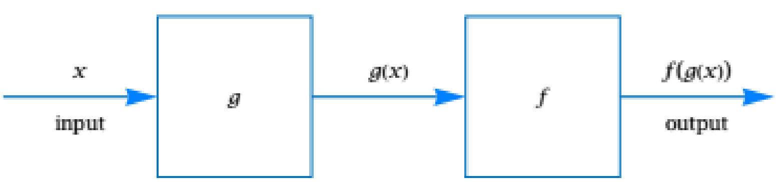
Given $f(x) = 2x^2 - 1$ and $g(x) = \sqrt{4x+5}$,

(a) State the domain of $k(x) = (f + g)\,(x)$

(b) Evaluate $k(5)$

iii – Composition of Functions

The composition $\boldsymbol{f \circ g}$ as the combination of two machines.



Here $g(x)$ is the output of the $\boldsymbol{g}$ machine and $g(x)$ is also the input of the $\boldsymbol{f}$ machine.

note

Equivalent notation: $(f \circ g)(x)$ is the same as $f(g(x))$, read "f of g of x"

Worked Example

Given $g(x) = 2x - 2$ and $h(x) = x^2 - 1$,

(a) Evaluate $g(h(3))$ (a) Determine the function $y = (h \circ g)(x)$

Solutions:

THINK: since $h(x) = x^2 - 1$

First, $h(3) = (\mathbf{3})^2 - 1$ ➧ $= \mathbf{8}$

This becomes the input into function "g"

THINK: since $g(x) = 2x - 2$

$g(8) = 2(\mathbf{8}) - 2$ ➧ $\underline{= \mathbf{14}}$

(b) *Substitute the expression for $g(x)$ into $h(x)$*

" $2x - 2$ "

$h(g(x)) = (2x - 2)^2 - 1$

$= \underline{\mathbf{4x^2 - 8x + 3}}$

Class Example 5 *Composite Functions involving multiple steps*

Given $f(x) = \sqrt{x-2}$, $g(x) = 2x - 1$ and $h(x) = x^2$ determine: (include any restriction on domain)

(a) the value of $f(g(6))$ (b) the function $y = (g \circ g)(x)$ (c) the function $y = (h \circ f \circ g)(x)$

To find the **domain** of a composite function, such as $f \circ g(x)$, first find and simplify the composite function, and then consider any restrictions. *(Dividing by zero, square rooting negatives, etc)*

Worked Example

Given $f(x) = \sqrt{x+3}$, $g(x) = 2x + 1$ and $h(x) = x^2$, determine the domain of $y = (h \circ f \circ g)(x)$

Solution:

Expression for $f(g(x))$ is $\sqrt{\mathbf{2x+1}+3}$

So, expression for $h(f(g(x)))$ is $(\sqrt{\underbrace{\mathbf{2x+4}}})^{\mathbf{2}}$

Can't sq root negatives, so this must be ≥ 0

➧ $2x + 4 \geq 0$ ➧ $\underline{\{x | x \geq -2, x \in \mathbb{R}\}}$

NOTE: Do not use the simplified form of the composite function, $y = 2x + 4$ *... you must remember where the function came from!*

Class Example 6 *Domain of a Composite Function*

This question may not be taken up in class **You Try**

Scan to check your work ➔

solution

Given $f(x) = \dfrac{x}{x+5}$ and $g(x) = 2x - 1$, determine the domain of $y = (f \circ g)(x)$

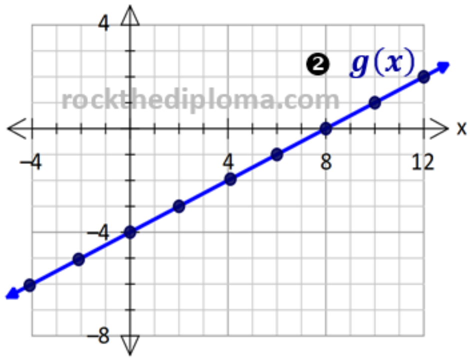
RTD LEARNING

Function Operations / Transformations Practice Question 1

Check as you go! ***Solutions for all "Practice Questions" are in your BOOK of ANSWERS.***

Three functions are defined as follows:

❶ $f(x) = \sqrt{x+4}$



x	$h(x)$
-1	0
0	-1
1	0
2	3
3	8

(a) Determine the value of $(f - g - h)(0)$

(b) Determine the value of $(f \circ g \circ h)(3)$

(c) State the domain of $y = (f + g)(x)$

Function Operations / Transformations Practice Question 2

Given function $f(x) = \sqrt{3 - 2x}$ and $g(x) = 3x + 1$, determine an expression for each function (don't simplify), and state the domain.

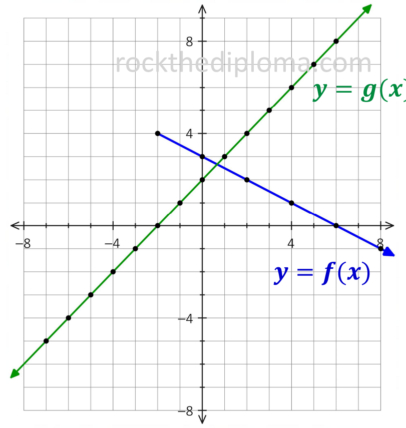
(a) $y = f(x)g(x)$ *Do not simplify combined function*

(b) $y = \left(\frac{f}{g}\right)(x)$ *Do not simplify combined function*

(c) $y = (f \circ g)(x)$ *Simplify*

(d) $y = g(f(x))$

Function Operations / Transformations Practice Question 3



Given the graphs of the functions $y = f(x)$ and $y = g(x)$,

(a) Sketch the graph of $k(x) = f(x) \times g(x)$, on the same grid.

(b) State the domain and range of $y = k(x)$

Function Operations / Transformations Practice Question 4

Diploma Example

Alex is given the following list of functions, where $b > 1$. She is asked to determine a new function, $h(x)$, which is the quotient of two different functions below and where the domain of $h(x)$ is $\{x | x \in \mathbb{R}\}$

Function 1 $y = x - b$

Function 2 $y = x^2 + b$

Function 3 $y = x^3 + b$

Function 4 $y = log_b x$

Function 5 $y = b^x$

Function 6 $y = \sqrt{x + b}$

NR If $h(x) = \dfrac{f(x)}{g(x)}$ and Alex selects Function 1 for $f(x)$, then the two functions that she could select for $g(x)$ are numbered _____ and _____.

Function Operations / Transformations Practice Question 5

Two functions are defined as: $f(x)$: $\{(1,4), (2,6), (3,5), (5,8)\}$ $g(x)$: $\{(2,5), (3,2), (5,8), (6,3)\}$

The value of $(f \circ g \circ g)(3)$ is _____.

Function Operations / Transformations Practice Question 6

Given that $f(x) = \sqrt{x - 2}$ and $g(x) = \dfrac{x^2}{x^2 - 9}$, state the domain of each of the following functions:

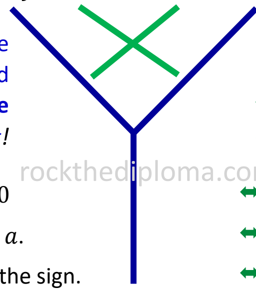
(a) $y = (f - g)(x)$

(b) $y = g(f((x)$

1.2 Transformations of Functions

Let's review the possible transformations on a function $y = f(x)$, using parameters you will be familiar with. (a,b,h,and k)

$$y = af[b(x-h)] + k$$



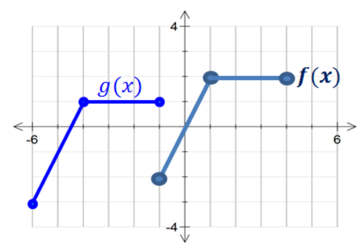
Outside values affect "y" only. These are the vertical stretches, reflections, and translations. **Vertical transformations are honest** – *what you see is what you get*!

Inside values affect "x" only. These are the horizontal stretches, reflections, and translations. **Horizontal transformations are deceptive** – *you must decipher what the say!*

- ⬍ Vertical reflection about the x-axis when $a < 0$
- ⬍ Vertical stretch about the x-axis, by a factor of a.
- ⬍ Vertical translation k units - same direction as the sign.

- ⬌ Horizontal reflection about the y-axis when $b < 0$
- ⬌ Horizontal stretch about the y-axis, by a factor of $1/b$.
- ⬌ Horizontal translation h units - opposite direction as the sign.

i – Horizontal and Vertical Translations



Translations involve adding / subtracting parameters. $y = f(x) \rightarrow y = f(x - \boldsymbol{h}) + \boldsymbol{k}$

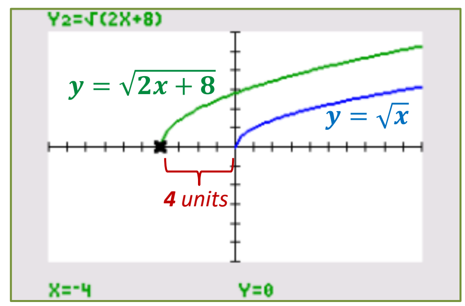
State the equation of $g(x)$, in terms of $f(x)$. ➡ ____________________

Be careful with horizontal translations

What is the horizontal translation of $y = \sqrt{2x+8}$, compared to the basic radical function $y = \sqrt{x}$?

We may be tempted to say "8 units to the left". But if we inspect the graph ➔ we can in fact see the correct answer is "**4 units to the left**".

For this example we are reminded that we need to first factor out any "$\boldsymbol{b}$" value in an equation before describing a horizontal translation.



First factor to write $\boldsymbol{y = \sqrt{2x+8}}$ as $\boldsymbol{y = \sqrt{2(x+4)}}$, to correctly identify the horizontal translation as "**4 units right**".

ii – The Mapping Rule

A mapping rule provides another way to describe the transformations of a function – by providing a formula for what happens to each coordinate.

The most important thing to remember is: *The mapping rule goes hand in hand with the transformation descriptions.*

To illustrate this, consider a function $\boldsymbol{y = f(x)}$ transformed to $\boldsymbol{y = -5f\left(\frac{1}{3}x - 2\right) + 7}$

Function ("cleaned up"):	**Description** of transformations	**Mapping Rule:**
➡ ____________________	*or – "about the line y=0"* *- A vertical reflection about the x-axis* *a vertical stretch by a factor of 5* *and a vertical translation 7 units up* *- A horizontal stretch by a factor of _____* *and a horizontal translation _____ units _______*	➡ *All points* $(x, y) \rightarrow$ ____________________

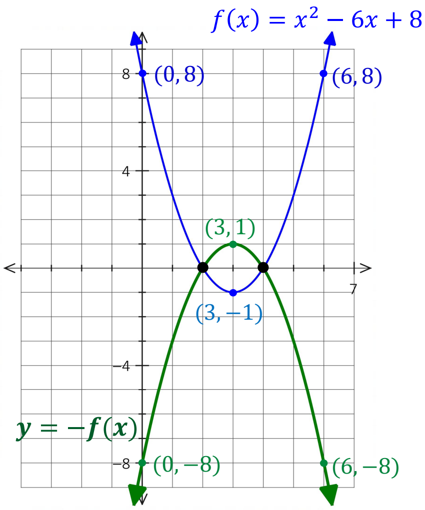
Fill in the blanks!

ii – Reflections

In this course we consider three types of reflections:

Vertical Reflections

About the line $y = 0$ (x-axis)



Mapping Rule: $(x, y) \rightarrow (x, -y)$

The **y-coordinates** are made **negative**

Invariant Point(s): On the **x-axis**

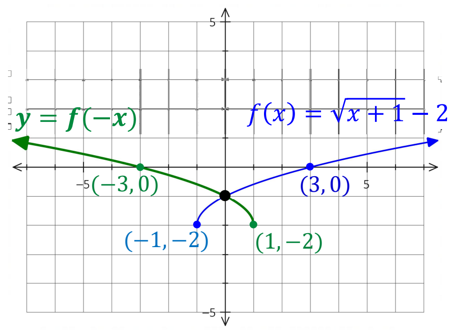
For **equation**, replace "y" with "$-y$"

Think of this as: ***Make the entire expression negative***

$y = -(x^2 - 6x + 8)$ ➧ $\boldsymbol{y = -x^2 + 6x - 8}$

Horizontal Reflections

About the line $x = 0$ / aka y-axis



Mapping Rule: $(x, y) \rightarrow (-x, y)$

The **x-coordinates** are made **negative**

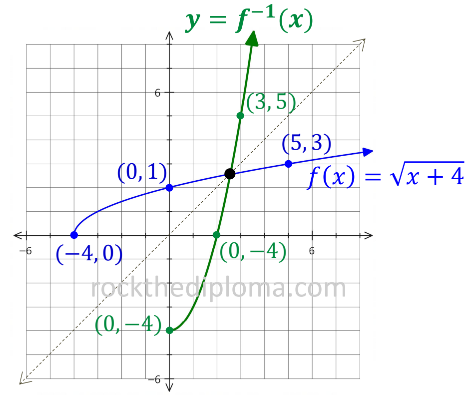
Invariant Point(s): On the **y-axis**

For **equation**, replace "x" with "$-x$"

$$\boldsymbol{y = \sqrt{-x+1} - 2}$$

Reflections About $y = x$

The **inverse,** $x = f(y)$



Mapping Rule: $(x, y) \rightarrow (y, x)$

The **coordinates** are **interchanged**

Invariant Point(s): On the line $\boldsymbol{y = x}$

For **equation**, interchange "x" and "y"

$$x = \sqrt{y+4}$$

$$x^2 = y + 4$$

$$\boldsymbol{y = x^2 - 4} \;;\, x \geq 0$$

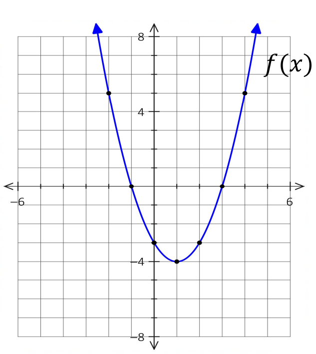
Be careful with horizontal translations: What is the horizontal translation of $y = (-x - 5)^3$, compared to the basic radical function $y = x^3$?

Class Example 7 *Obtaining the Graph and Equation of a Reflected Function*

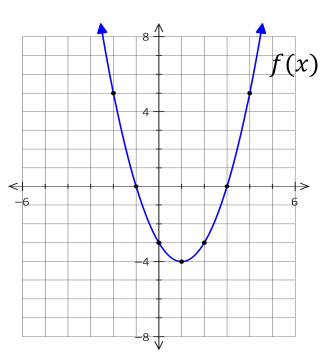
A function $f(x) = x^2 - 2x - 3$ is sketched below. On the same grid, **(i)** sketch the resulting graph when the graph of $f(x)$ is reflected as described. Be sure to indicate any invariant points, and state all indicated characteristics.

Then, **(ii)** determine an equation for each reflected function, both in terms of $f(x)$ and in terms of x.

(a) The graph of $y = f(x)$ is reflected about the line $y = 0$.



(b) The graph of $y = f(x)$ is reflected about the line $x = 0$.



ii – Stretches

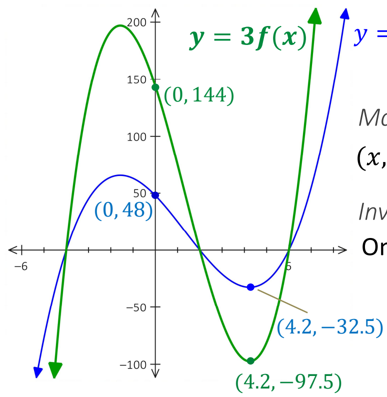
We also consider two types of stretches.

When the graph of $y = f(x)$ is transformed to $\boldsymbol{y = af(bx)}$, there is a:

- **Vertical stretch** about the x-axis by a factor of $\boldsymbol{a}$
- **Horizontal stretch** about the y-axis by a factor of $\boldsymbol{\frac{1}{b}}$

For example, consider the graph of $f(x) = (x+4)(x-2)(x-6)$, shown below, transformed by both a vertical and then horizontal stretch. *Notice the effect on the coordinates of each graph, and the equation!*

Vertical Stretch About the line $y = 0$, **by a factor of 3**:



Mapping Rule:

$(x, y) \to (x, 3y)$

Invariant Points:

On the x-axis

Notice that all points on the graph move 3 times as far from the x-axis

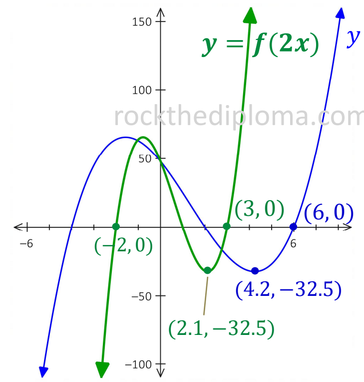
For the **equation**, multiply the entire function by 3

Alternatively, replace "y" with "$\frac{1}{3}y$"

$$\boldsymbol{y = 3(x+4)(x-2)(x-6)}$$

Equation of stretched function in terms of x

Horizontal Stretch About the line $x = 0$, **by a factor of $1/2$**:



Mapping Rule:

$(x, y) \to (\frac{1}{2}x, y)$

Invariant Points:

On the y-axis

Here all points on the graph move 1/2 the distance from the y-axis

For the **equation**, replace all x's with "$2x$"

$$\boldsymbol{y = (2x+4)(2x-2)(2x-6)}$$

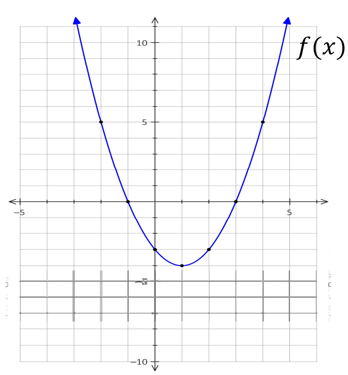
Equation in terms of x

Class Example 8 *Obtaining the Graph and Equation of a Stretched Function*

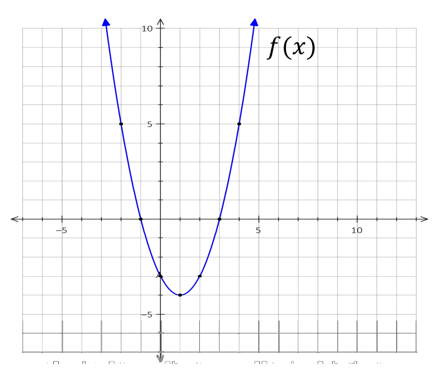
A function $f(x) = x^2 - 2x - 3$ is sketched below. On the same grid, (i) sketch the resulting graph when the graph of $f(x)$ is stretched as described. Be sure to indicate any invariant points, and state all indicated characteristics.

Then, (ii) determine an equation for each stretched function, both in terms of $f(x)$ and in terms of x.

(a) The graph of $y = f(x)$ is stretched about the line $y = 0$, by a factor of 2.



(b) The graph of $y = f(x)$ is stretched about the line $x = 0$, by a factor of 3.



For (c), do not sketch, but provide an equation for the transformed function, in terms of $f(x)$ and in terms of x.

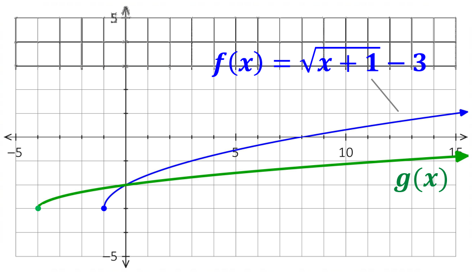
(c) The graph of $y = f(x)$ is stretched about the line $y = 0$, by a factor of 3 and vertically translated 2 units down.

Class Example 9 *Determining the Equation of a Function that's been Stretched*

For both (a) and (b) below, the graph of $y = g(x)$ is obtained by transforming the graph of $y = f(x)$ through stretching and / or reflecting.

For each – Describe the transformation from $f(x)$ to $g(x)$, and determine an equation for $g(x)$ in terms of $f(x)$ and in terms of x.

(a)



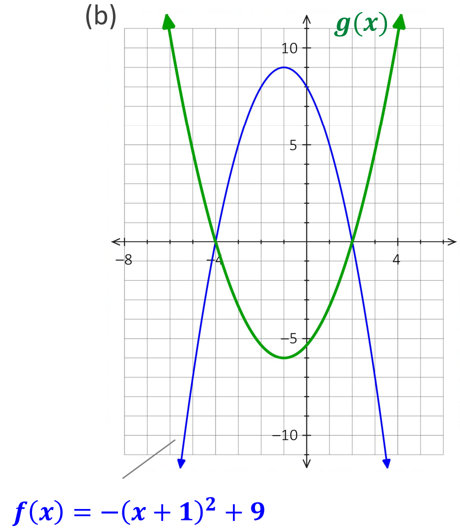
Transformation in words:

Mapping rule:

Equation in terms of $f(x)$:

Equation in terms of x

(b)



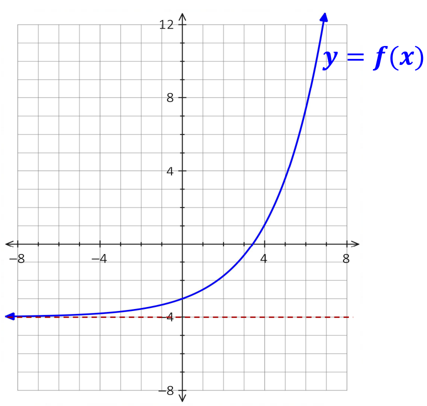
Transformation in words:

Mapping rule:

Equation in terms of $f(x)$:

Equation in terms of x

Class Example 10 *Applying a Stretch and Translation*



(a) The graph of $y = f(x)$ is stretched about the line $y = 0$ by a factor of 3/2, and vertically translated 5 units up. Determine the range and y-intercept of the transformed graph.

Mapping rule:

Range:

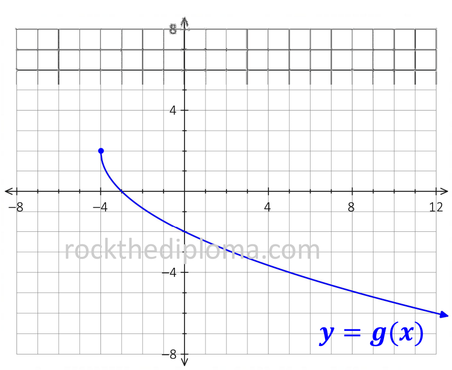
y-intercept:

Part (b) may not be taken up in class – try on your own!

You Try

Scan to check your work →

Solution



(b) The graph of $y = g(x)$ is reflected about the line $x = 0$ and horizontally translated 5 units right. Determine the domain, range, and x-intercept of the transformed graph.

Mapping rule:

Range:

Domain:

x-intercept:

RTD LEARNING

iii – Combining Transformations

When describing *multiple transformations*,

First consider…	*And then…*
Stretches ***Reflections***	***Translations***

So given the transformation from $y = f(x)$ to $\boldsymbol{y = af[b(x-h)] + k}$

The mapping rule is: $(x, y) \rightarrow (\frac{1}{b}x + h, ay + k)$

Note that the horizontal translation, "$+h$", is applied after all x-coordinates are multiplied by the horizontal stretch factor.

…and same goes for the vertical translation "k"!

- The *vertical stretch about the x-axis factor* is $|\boldsymbol{a}|$
- If $a < 0$, there is a *vertical reflection* in the x-axis
- The *horizontal stretch about the y-axis factor* is $\frac{\boldsymbol{1}}{|\boldsymbol{b}|}$
- If $b < 0$, there is a *horizontal reflection* in the y-axis
- The *horizontal translation* is $\boldsymbol{h}$ units, and the *vertical translation* is $\boldsymbol{k}$ units.

Analyzing the Horizontal Translation in a Function with Multiple Translations

A related concept is identifying the horizontal translation in a transformed function, then there is also a horizontal reflection or stretch.

Recall that the transformed function is in the form $\boldsymbol{y = af[b(x-h)] + k}$, that is, the b value must be factored out.

For example, if we wished to describe the transformations from the graph of y_1 to the graph of y_2

$$y_1 = \sqrt{x}$$

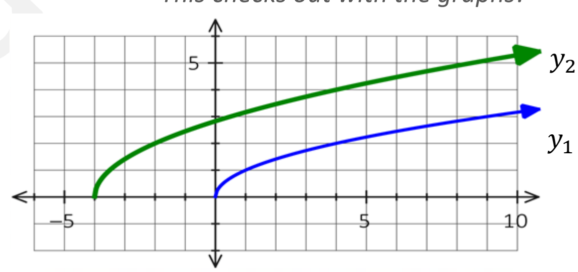
$$y_2 = \sqrt{2x + 8}$$

…we must first factor out the horizontal stretch!

➧ $y_2 = \sqrt{2(x+4)}$

- *Horizontal stretch, factor of 1/2*
- *Horizontal translation, 4 units left*

This checks out with the graphs!

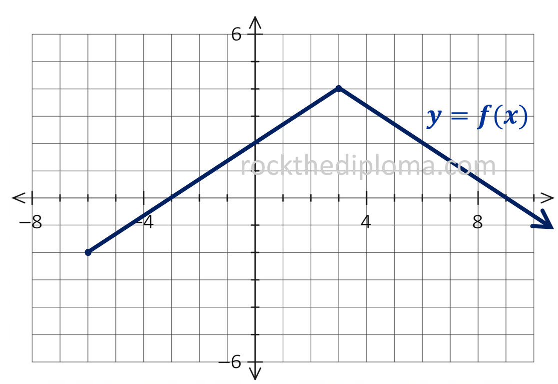


Class Example 11 *Sketching graphs involving Multiple Transformations*

The graph of $y = f(x)$ is shown on the right.

A function $y = g(x)$ is defined as $g(x) = \frac{1}{2}f(-x + 2) - 4$

(a) Construct a mapping rule for the transformation from $y = f(x)$ to $y = g(x)$.



(b) Sketch the graph of $y = g(x)$ on the same grid.

(c) State the domain and range of $y = g(x)$

Function Operations & Transformations Practice Questions *Group 2*

Complete the following table. *For example, for the first row, use the transformation (in words) to determine the equation of **f** and the mapping rule. For the second row – start with the equation to express the transformation in words and mapping rule. And for the bottom row – start with the mapping notation!*

*The solution is **NOT in the book of answers**, scan the QR code instead!*

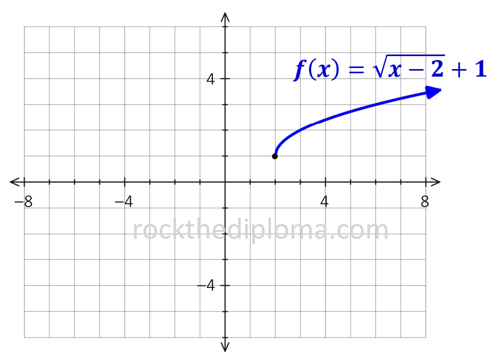
Equation in terms of *f*	**Transformations** (in words)	**Mapping Notation**
	- A vertical stretch by a factor of 1/4 about the x-axis - A reflection about the y-axis - A horizontal translation 3 units left	
$3y = f\left(-\frac{1}{2}x + 8\right) + 6$		
		$(x, y) \rightarrow (3x + 6, -y + 2)$

Function Operations / Transformations Practice Question 7

A function $f(x)$, as shown, is reflected about the y axis, and vertically translated four units down.

(a) State the domain and range of the resulting function.

(b) State the equation of the resulting function, both in terms of $f(x)$ and in terms of x.



Function Operations / Transformations Practice Question 8

Diploma Example

Given the function $y = -x^2 + 4$ is reflected about the y-axis, the new equation will be

A. $y = x^2 + 4$ **C.** $y = x^2 - 4$

B. $y = -x^2 + 4$ **D.** $y = -x^2 - 4$

Function Operations / Transformations Practice Question 9

Given a function $y = x^2 - 6x + 11$ with range $y \geq 2$, determine the resulting equation (in terms of x) and resulting range, after each transformation:

(a) Reflection in the line $y = 0$, and vertical translation one unit down.

(b) Reflection in the line $y = 0$, vertical translation one unit down, and horizontal translation 2 units left.

Function Operations / Transformations Practice Question 10

A function $f(x) = \sqrt{x-3} + 1$ has a domain $x \geq 3$ and a range $y \geq 1$.
A function $k(x)$ is obtained by reflecting $f(x)$ about the line $y = 0$ and the line $x = 0$.

Determine the equation of $k(x)$ (in terms of x), and state the domain and range.

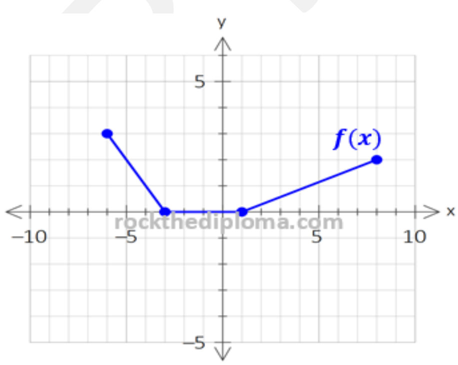
Function Operations / Transformations Practice Question 11

A function defined by the equation $f(x) = x^2 - 8x + 11$ has a range $y \geq -5$.
A function $g(x)$ is obtained by stretching in terms $f(x)$ about the line $y = 0$ by a factor of 3.
A function $p(x)$ is obtained by horizontally stretching $f(x)$ by a factor of $1/2$.

(a) Determine the equation **and range** of $y = g(x)$
in terms of x

(b) Determine the equation of $y = p(x)$
in terms of x

Function Operations / Transformations Practice Question 12



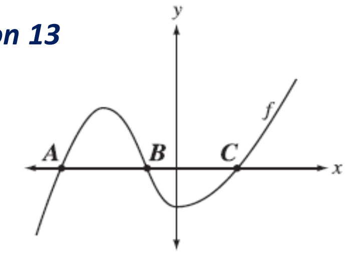
Given the function $f(x)$ as shown,

Determine the values of b and k in $h(x) = f(bx) + k$, if the domain of $h(x)$ is $[-9, 12]$ and the range is $[4, 7]$.

Function Operations / Transformations Practice Question 13

Diploma Example

The partial graph of $y = f(x)$ is shown.



Which of the following transformations will **always** create invariant points at A, B, and C?

A. $y = f(bx)$ **B.** $y = af(x)$ **C.** $y = f(x) + k$ **D.** $y = f(x - h)$

Function Operations / Transformations Practice Question 14

The transformation of a function $f(x)$ to $g(x)$ is given by a mapping rule $(x, y) \rightarrow (5x - 1, -y + 3)$.
State the equation of $g(x)$, in terms of f.

Function Operations / Transformations Practice Question 15

Diploma Example

The graph of $y = f(x)$ is reflected in the x-axis, stretched vertically about the x-axis by a factor of $\frac{1}{3}$ and stretched horizontally about the y-axis by a factor of 4 to create the graph of $y = g(x)$.

For the point $(-3, 6)$ on the graph of $y = f(x)$, the corresponding point on the graph of $y = g(x)$ is

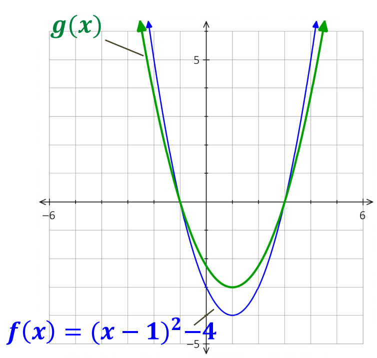
A. $(9, 24)$ **C.** $(1, 24)$

B. $(-12, -18)$ **D.** $(-12, -2)$

Function Operations / Transformations Practice Question 16

The graphs below show a function $f(x)$ and a second transformed function, $g(x)$ obtained by stretching or stretching and reflecting the graph of $f(x)$. State the equation of $g(x)$ for each, as directed.

(a)

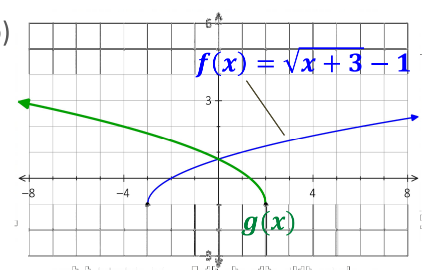


(i) ______________
Equation in terms of f…

(ii) ______________
Equation in terms of x…

(iii) ______________
y-intercept of $g(x)$

(b)

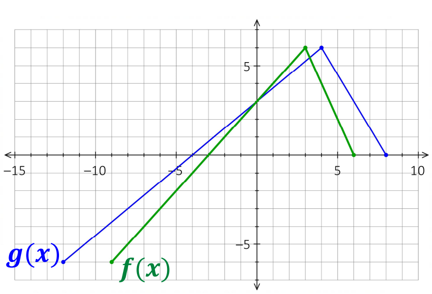


(i) ______________
Equation in terms of f…

(ii) ______________
Equation in terms of x…

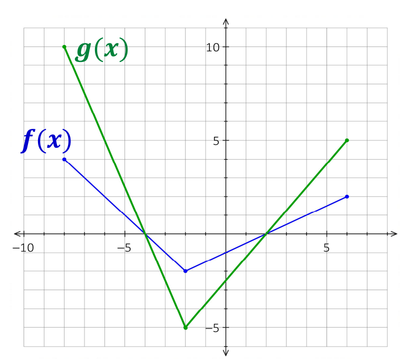
(iii) ______________
x-intercept of $g(x)$

(c)



____
Equation in terms of $f(x)$

(d)



____
Equation in terms of $f(x)$

RTD LEARNING

Function Operations / Transformations Practice Question 17

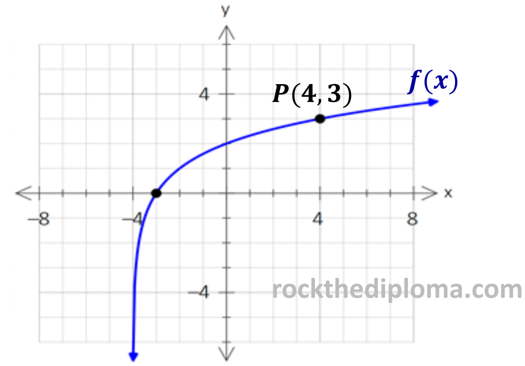
Written Response The format of this next question is similar to what you'll find on your exam

Use the following information to answer the following question

The graph of a logarithmic function $y = f(x)$ is shown on the right. There is a vertical asymptote at $x = -4$, and an x-intercept at -3.

A new function $\boldsymbol{g(x)}$ is defined by $g(x) = f(-\frac{1}{2}x - 1)$.

A second function $\boldsymbol{k(x)}$ is defined by reflecting the graph of $f(x)$ about the line $y = x$.



a) **Describe** the characteristics of the graph of $y = g(x)$, in terms of the domain, range, x-intercept, and asymptote. [***3 marks***]

b) **State** the domain, range, and coordinates of P on the graph of $y = k(x)$. **Justify** your answers. [***2 marks***]

Function Operations / Transformations Practice Question 18

The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$, as shown below:

Exam Style

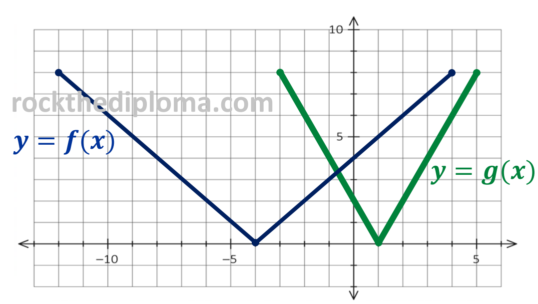
An equation for $g(x)$ in terms of $f(x)$ is:

A. $g(x) = f(-2x + 1)$

B. $g(x) = f(2x - 6)$

C. $g(x) = f(\frac{1}{2}x - 3)$

D. $g(x) = f(-\frac{1}{2}x + 1)$



Function Operations and Transformations Question 19

The point $P(3, 8)$ is on the graph of $y = f(x)$.

The point corresponding to P on the graph of $y + 2 = 2f(\frac{1}{3}x - 4)$ is:

Exam Style

A. $(21, 14)$

B. $(21, 12)$

C. $(13, 14)$

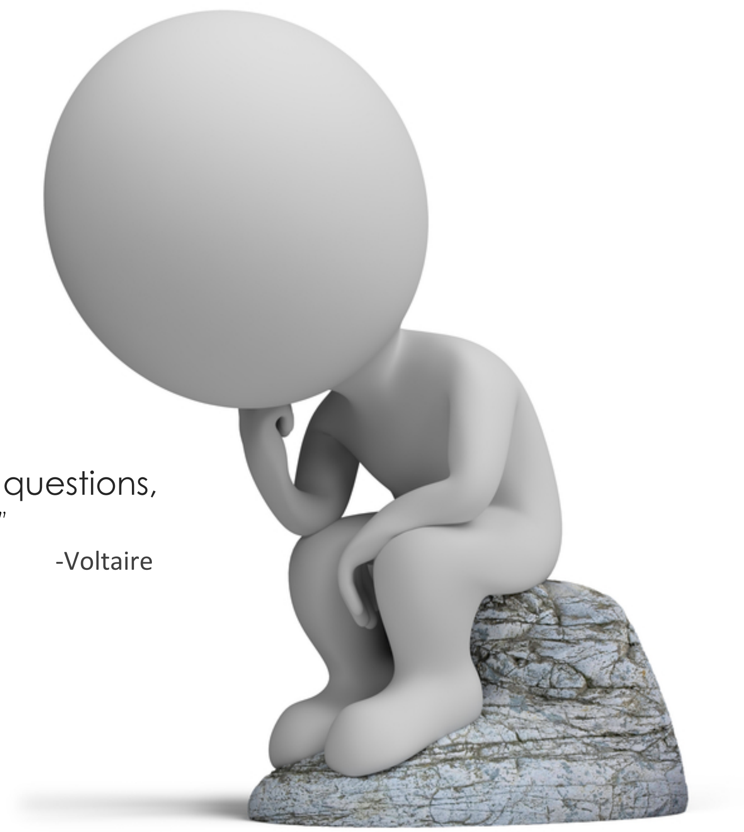
D. $(13, 12)$

RTD EXAM PREP

BOOK OF ANSWERS

For all Class Examples and Practice Questions in your Study Materials

Extra Questions *include detailed, step-by-step solutions*



"Judge a person by their questions, rather than their answers"

-Voltaire

Visit ***www.rtdmath.com*** **for:**

- **Any updates / mistakes found in your study guides (or this answer key!)**
- **Additional Math 30-1 practice exams (yes!)**

Students Please Note

Thank you for choosing RTD, and congratulations on making it this far in one of the most challenging courses in high school! And for greeting the return of mandatory diploma exams with determination and cheer.

So first off – props to you for you studious and detail-oriented nature. I'm sure it will take you far in life.

With that, please note that the answers **appear in the same order as the questions in your study guides A and B**.

Which may seem obvious. But there are two types of questions:

- Answers to CLASS EXAMPLES are designated "**CE**"

Class Example 3 *Obtaining the Graph of a Combined Function*

Given the graphs of $f(x)$ and $g(x)$ on the right,

Sketch the graph of $h(x) = (f - g)(x)$, and state the domain.

So on the next page you'll find the answer for this class example:

CE 3 $h(x) = x^2 + 2x - 3$ D: $\{x \geq -2, x \in \mathbb{R}\}$

- While answers to PRACTICE QUESTIONS are designated "**PQ**"

Function Operations / Transformations Practice Question 18

The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$, as shown below:

Exam Style

An equation for $g(x)$ in terms of $f(x)$ is:

So on page 3 you'll find the answer for this practice question:

PQ 18 Ans: ***B*** From $f(x)$ to $g(x)$, we see: - a h... tch by a factor of 1/2 … **then moves vertex to** $(-2, 0)$

- a ... slation 3 units right … **moves vertex to** $(1, 0)$ ← *where we find it!*

Practice Question Answers include step-by-step solutions

Please take note of this as you check your answers! Otherwise it will be easy to go looking for the answer to "Practice Question 18" but get mixed up by instead finding the answer to "Class Example 18".

Each time you refer to this key – please take care in noting whether you are searching for a "CE" or "PQ" answer!!!

FUNCTION OPERATIONS AND TRANSFORMATIONS

Note 1: "U" means "union", which means "or", which combines the sets.

Note 2: While useful, "union" is not part of the Math 30-1 curriculum. *This particular domain is *far better* expressed in set notation!

Note 3: You can express your domains (and ranges) in either notations. *But you must be familiar with both!*

CE 1 (a) set notation: $\{x \neq \pm\frac{3}{2}, x \in \mathbb{R}\}$ alternatively, interval notation*: $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

(b) set notation: $\{x \in \mathbb{R}\}$ alternatively, interval notation: $(-\infty, \infty)$

(c) set notation: $\{x < \frac{5}{2}, x \in \mathbb{R}\}$ interval notation: $(-\infty, \frac{5}{2})$

(d) set: ALL are: $\{x \in \mathbb{R}\}$ interval: $(-\infty, \infty)$

(e) set: $\{x \leq \frac{5}{6}, x \in \mathbb{R}\}$ interval: $(-\infty, \frac{5}{6}]$ *Set notation could also look like this:* $\{x \in \mathbb{R} \,|\, x < \frac{5}{6}\}$ (pick your pleasure)

(f) set: $\{x \geq -\frac{b}{a}, x \in \mathbb{R}\}$ interval: $[-\frac{b}{a}, \infty)$

CE 2 (a) $h(x) = x^2 + 2x - 3$ *D:* $\{\boldsymbol{x \in \mathbb{R}}\}$

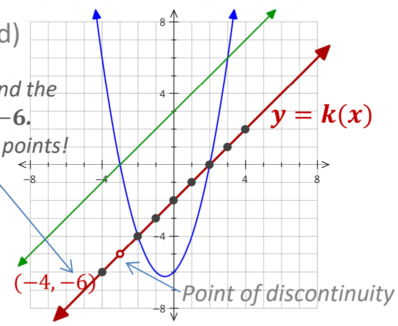
(b) $\mathbf{-5}$

(c) $k(x) = x - 2\,;\ x \neq -3$ *D:* $\{\boldsymbol{x \neq -3, x \in \mathbb{R}}\}$

Note: Domain restriction must be included with the function. (Because we canceled a term which gave a restriction)

*At $x = -4$, the value of $f(x)$ is 6 and the value of $g(x)$ is -1. **Divide to get -6.** Continue rightward to obtain more points!*

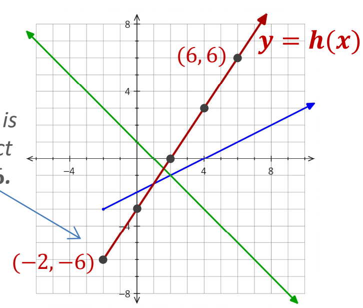
CE 2 (d)



CE 3 *D:* $\{\boldsymbol{x \geq -2, x \in \mathbb{R}}\}$

*At $x = -2$, the value (y-coord) of $f(x)$ is -3 and the value of $g(x)$ is 3. Subtract to get the **y-coord on $k(x)$ which is -6.***

Continue this process / moving right!



CE 4 (a) *D:* $\{\boldsymbol{x \geq -5/4, x \in \mathbb{R}}\}$

(b) $\mathbf{54}$

CE 5 (a) $\mathbf{3}$ (b) $\boldsymbol{y = 4x - 3}$

(c) $\boldsymbol{y = 2x - 3}\ ; x \geq 3/2$

CE 6 (a) $\mathbf{-9}$ (b) *D:* $\{\boldsymbol{x \neq -2, x \in \mathbb{R}}\}$

PQ 1 (a) **Ans: 7**

$= f(0) - g(0) - g(0)$

$= 2 - (-4) - (-1)$

(b) **Ans: 2**

$= f(g(h(3))$ *h(3)=8, giving* ➔ $f(g(8))$

g(8)=0, giving ➔ $f(0)$ ➔ *f(0)=**2***

(c) **Ans:** $\{\boldsymbol{x \geq -4, x \in \mathbb{R}}\}$

(Domain of $f(x)$)

PQ 2 (a) $(fg)(x) = \sqrt{(3 - 2x)} \times (3x + 1)$

For domain: $3 - 2x \geq 0$

$3 \geq 2x$

D: $\{\boldsymbol{x \leq 3/2, x \in \mathbb{R}}\}$

(b) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{3 - 2x}}{3x + 1}$

$x \leq 2/3$ *Takes care of "top"*

$x \neq -1/3$ *Takes care of bottom*

Combine these restrictions:

$\{\boldsymbol{x \leq 3/2, x \neq -1/3, x \in \mathbb{R}}\}$

Or, interval notation: $(-\infty, -1/3) \cup (-1/3, 3/2]$

(c) $f(g(x)) = \sqrt{3 - 2(3x + 1)}$

$\boldsymbol{f(g(x)) = \sqrt{-6x + 1}}$

$-6x + 1 \geq 0$

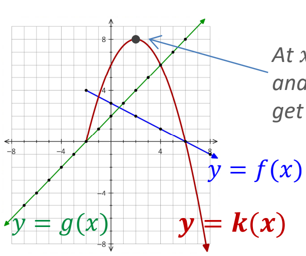
$1 \geq 6x$

Domain:

$\{\boldsymbol{x | x \leq 1/6, x \in \mathbb{R}}\}$

(d) $g(f(x)) = 3\sqrt{3 - 2x} + 1$ *D:* $\{\boldsymbol{x \leq 3/2, x \in \mathbb{R}}\}$

PQ 3 (a)



*At $x = 2$, the value (y-coord) of $f(2)$ is 2 and the value of $g(2)$ is 4. Multiply to get the **y-coord on $k(x)$ which is 8.***

Multiply all corresponding y-coords

(b) *Domain of $k(x)$:* $\{\boldsymbol{x | x \geq -2, x \in \mathbb{R}}\}$ *Range of $k(x)$:* $\{\boldsymbol{y | y \leq 8, y \in \mathbb{R}}\}$

PQ 4 Ans: **25** or **52**

Goal: Analyze each function to identify those which can never equal zero. (And therefore the denominator can never equal zero and the domain of $h(x)$ would be $\{x \in \mathbb{R}\}$.

- Cannot be *Function 1* is as then $h(x)$ would simplify to 1
- **Function 2** works, $h(x)$ would have a domain of $\{x \in \mathbb{R}\}$ as $x^2 + b$, when $b > 0$, can never equal zero.
- *Function 3* would not work as $x^3 + b$, when $b > 0$, CAN be zero as x^3 can equal $-b$
- *Function 4* would not work as $log_b x$, when CAN be zero at $b^0 = x$ .... $x = 1$
- **Function 5** works, as $y = b^x$ can never be equal to zero .... there is no x we can pick to make this true: $b^x = 0$

PQ 5 Ans: **8** $f(g(g(3))$ → $g(3)$ is 2 , giving $f(g(2))$ → $g(2)$ is 5 , giving $f(5)$ → $f(5)$ is 8

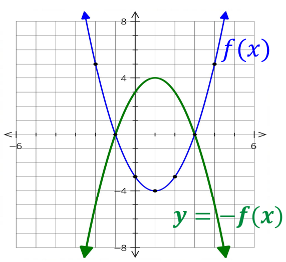
PQ 6 (a) Domain restriction for $f(x)$ is $x \geq 2$. For $g(x)$ it's $x^2 - 9 \neq 0$ → $x^2 \neq 9$ → $x \neq \pm 3$.

COMBINE these ... $x \geq 2$ takes care of $x \neq -3$, so we're left with: $\{\boldsymbol{x \geq 2, x \neq 3, x \in \mathbb{R}}\}$

(b) Simplify the composite function: $y = \dfrac{x^2}{(\sqrt{\boldsymbol{x-2}})^2 - 9}$ → $y = \dfrac{x^2}{x - 2 - 9}$ → $y = \dfrac{x^2}{x - 11}$

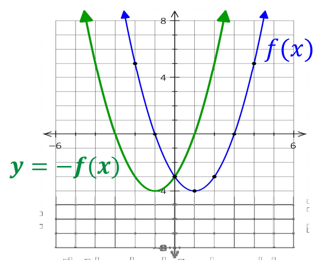
Domain: $x - 11 \neq 0$ as well as $x \geq 2$
(consider the "pre-simplified" function as well) → $\{\boldsymbol{x \geq 2, x \neq 11, x \in \mathbb{R}}\}$

CE 7 (a) $y = -x^2 + 2x + 3$



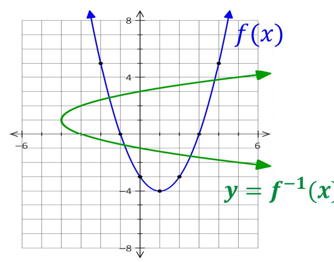
$(x, y) \to (\boldsymbol{x, -y})$

(b) $y = x^2 + 2x - 3$



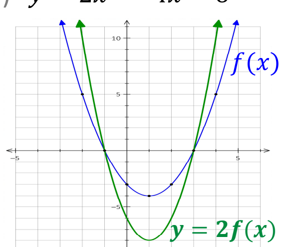
$(x, y) \to (\boldsymbol{-x, y})$

(c) $y = \pm\sqrt{x + 4} + 1$



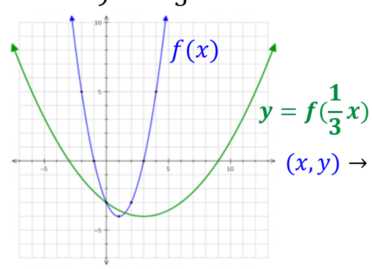
$(x, y) \to (\boldsymbol{y, x})$

CE 8 (a) $y = 2x^2 - 4x - 6$



$(x, y) \to (\boldsymbol{x, 2y})$

(b) $y = \dfrac{1}{9}x^2 - \dfrac{2}{3}x - 3$



$(x, y) \to (\boldsymbol{3x, y})$

(c) $y = 3x^2 - 6x - 11$

CE 9 (a) Horizontal stretch about y-axis, factor of 4 $(x, y) \to (\boldsymbol{4x, y})$

$y = f(\frac{1}{4}x)$ $y = \sqrt{(1/4)x + 1} - 3$

Equation in terms of f Equation in terms of x

(b) Vertical reflection about x-axis, plus a vertical stretch by stretch, factor of 3/2. $(x, y) \to (\boldsymbol{x, -\frac{2}{3}y})$

$y = -\dfrac{2}{3}f(x)$ $y = -\dfrac{2}{3}[-(x+1)^2 + 9]$ $y = \dfrac{2}{3}(x + 1)^2 - 6$

Equation in terms of f Equation in terms of x **^ Simplified form ^**

CE 10 (a) $(x, y) \to (\boldsymbol{x, \frac{3}{2}y + 5})$ $\{\boldsymbol{y > -1, y \in \mathbb{R}}\}$ $(\boldsymbol{0, 0.5})$

Mapping rule Range *y*-intercept

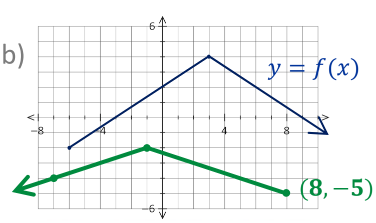
(b) $(x, y) \to (\boldsymbol{-x + 5, y})$ D: $\{\boldsymbol{x \leq 9, x \in \mathbb{R}}\}$ $(\boldsymbol{8, 0})$

Mapping rule R: $\{\boldsymbol{y \leq 2, y \in \mathbb{R}}\}$ *x*-intercept

CE 11 (a) $(x, y) \to (\boldsymbol{-x + 2, \frac{1}{2}y - 4})$

Mapping rule

(b)

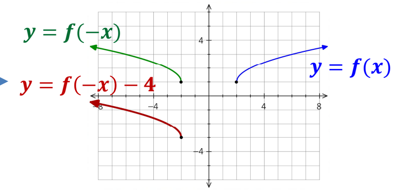


(c) Domain $\{\boldsymbol{x \leq 8, x \in \mathbb{R}}\}$

Range $\{\boldsymbol{y \leq -2, y \in \mathbb{R}}\}$

PQ 7 (a) Domain: $\{x|x \le -2, x \in \mathbb{R}\}$ Range: $\{y|y \ge -3, y \in \mathbb{R}\}$

(b) $y = f(-x) - 4$ / $y = \sqrt{-x-2} - 3$



PQ 8 Ans: B Replace x with $-x$ (for horiz reflection)
So, $y = -x^2 + 4$ becomes $y = -(-x)^2 + 4$.
Simplify by applying the exponent "2" (squaring a negative makes positive), we get $\boldsymbol{y = -x^2 + 4}$

PQ 9 (a) $y = -(x^2 - 6x + 11) - 1$ ➔ $\boldsymbol{y = -x^2 + 6x - 12}$ R: $\{y|y \le -3, y \in \mathbb{R}\}$

(b) $y = -(x+2)^2 + 6(x+2) - 12$ ➔ $\boldsymbol{y = -x^2 + 2x - 4}$ R: $\{y|y \le -3, y \in \mathbb{R}\}$

PQ 10 Replace x with $-x$ (for horiz reflection) and replace y with $-y$. (for vert reflection)
$-y = \sqrt{-x-3} + 1$ ➔ $k(x) = \boldsymbol{-\sqrt{-(x+3)} - 1}$ *Domain:* $\{x|x \le -3, x \in \mathbb{R}\}$ *Range:* $\{y \le -1, y \in \mathbb{R}\}$

PQ 11 (a) $g(x) = 3(x^2 - 8x + 11)$, simplifies to $\boldsymbol{g(x) = 3x^2 - 24x + 33}$ to range apply stretch: $\{y|y \ge -15, y \in \mathbb{R}\}$

(b) $p(x) = (2x)^2 - 8(2x) + 11$, simplifies to $\boldsymbol{g(x) = 4x^2 - 16x + 11}$

PQ 12 Ans: $\boldsymbol{b = 2/3, k = 4}$ Total width of $f(x)$ is 14 units (from furthest left x-coord -6 to furthest right, 8) $21 \div 14$
Total width of $h(x)$ is 21 units (from -9 to 12) So, $h(x)$ had a horiz. stretch, factor of **1.5**
➔ So the b value in $h(x) = f(bx) + k$ is the reciprocal of 3/2, that is, **2/3**
Range of $f(x)$ is $[0, 3]$, as range of $k(x)$ is $[4, 7]$, it's been vertically translated 4 units up

PQ 13 Ans: B Invariant points must be on the x-axis, meaning we cannot **horizontally** stretch or shift, (so it's not A or D).
For C we'd shift the entire graph up, which would move the x-intercepts.
Only **B** (vertical stretch) would not affect the x-ints, as all points would move a times further from the y-axis.

PQ 14 $g(x) = -f\left[\frac{1}{5}(x+1)\right] + 3$ *Mapping rule indicates horizontal stretch of 5, vertical reflection, and translations 1 unit left, 3 units up.*

PQ 15 Ans: D *Mapping rule:* $(x, y) \to (4x, -\frac{1}{3}y)$ *Put point (−3, 6) through:* $(-3, 6) \to (4(-3), -\frac{1}{3}(6)) \to \boldsymbol{(-12, -2)}$

PQ 16 (a) Vertical stretch, factor of 3/4 *Notice that the vertex of $f(x)$ is 4 units from x-axis, on $g(x)$ it's 3 units away*

$4 \times \square = 3$
vert. stretch factor

i $y = \frac{3}{4}f(x)$

ii $y = \frac{3}{4}[(x-1)^2 - 12]$ *Simplifies:* $\boldsymbol{y = \frac{3}{4}(x-1)^2 - 3}$

iii $(0, 2.25)$ *Mult. y-int of $f(x)$, **3**, by the vert. stretch factor 3/4*

(b) Horiz. stretch, factor of 2/3 *Notice the "start point" of $f(x)$ is 3 units from y-axis, on $g(x)$ it's 2 units away*
PLUS, horiz. reflection about y-axis

$3 \times \square = 2$
Horiz. stretch factor

i $g(x) = f(-\frac{3}{2}x)$

ii $g(x) = \sqrt{-\frac{3}{2}x + 3} - 1$ or $g(x) = \sqrt{-\frac{3}{2}(x-2)} - 1$

iii $(4/3, 0)$ *Mult. x-int of $f(x)$, **-2**, by 2/3*

(c) Horiz str, factor of **4/3** *Notice that the furthest left point on $f(x)$ is 9 units from y-axis, on $g(x)$ it's 12 units away*

➔ $\boldsymbol{y = f(\frac{3}{4}x)}$

$9 \times \square = 12$
horiz. stretch factor

(d) Vert. str, factor of **5/2** *Notice that the lowest point on $f(x)$ is 2 units from x-axis, on $g(x)$ it's 5 units away*

➔ $\boldsymbol{y = \frac{5}{2}f(x)}$

$2 \times \square = 5$
horiz. stretch factor

PQ 17 (a) $g(x) = f\left[-\frac{1}{2}(x+2)\right]$, transformations can be mapped by $(x, y) \to (-2x - 2, y)$.
For $g(x)$, D: $[-\infty, \boldsymbol{6})$, R: $\boldsymbol{y} \in \mathbb{R}$, $x - int$ is $(\boldsymbol{4}, \boldsymbol{0})$, VA is $\boldsymbol{x = 6}$

(b) $k(x)$ is the INVERSE of $f(x)$, so $P(4,3) \to (3, 4)$, domain & range interchange… D: $\{x \in \mathbb{R}\}$ and R: $\{y > -4, y \in \mathbb{R}\}$ on $k(x)$.

PQ 18 Ans: B From $f(x)$ to $g(x)$, we see: - a horizontal stretch by a factor of 1/2 … **then moves vertex to** $(\boldsymbol{-2}, \boldsymbol{0})$
- a horizontal translation 3 units right … **moves vertex to** $(\boldsymbol{1}, \boldsymbol{0})$ ← *where we find it!*

This one is a bit tricky! You might also treat this as a horiz. reflection (along with a horiz. stretch factor of 1/2), and then a horiz. translation 1 left. But that choice isn't available, we'd need to see: $g(x) = f[-2(x+1)]$ **So close to the "A" choice, but different!**

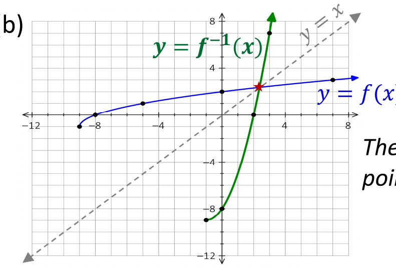
$g(x) = f[2(x-3)]$
Multiply 2 through to get "B" choice

PQ 19 Ans: A Re-write: $y = f\left[\frac{1}{3}(x-12)\right] - 2$ *Map. rule:* $(x, y) \to (3x + 12, 2y - 2)$ *so:* $(3, 8) \to (3(3) + 12, 2(8) - 2)$
$\to (21, 14)$

CE 12 (a) The given graph of $y = f(x)$ passes the V.L.T. (the vertical line test), so it **is a function**.

It also passes the H.L.T. (the horiz. line test), so its **INVERSE is also a function**.

(b)



There is one invariant point, marked ★

(c) $y = f(x)$ *D:* $\{\boldsymbol{x \geq -9}, x \in \mathbb{R}\}$ *R:* $\{\boldsymbol{y \geq -1}, y \in \mathbb{R}\}$

$y = g(x)$ *D:* $\{\boldsymbol{x \geq -1}, x \in \mathbb{R}\}$ *R:* $\{\boldsymbol{y \geq -9}, y \in \mathbb{R}\}$

(d) $y = f(x)$ *equation is:* $\boldsymbol{f(x) = \sqrt{x+9} - 1}$

For $g(x)$ equation, switch x & y ....

Isolate "y" / simplify:

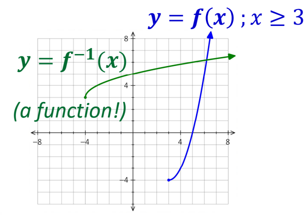
$y = \sqrt{x+9} - 1$ ➔ $x = \sqrt{y+9} - 1$ ➔ $\boldsymbol{y = (x+1)^2 - 9}$; $x \geq -1$

When stating equation, we must restrict the domain to be a half-parabola

CE 13 (a) We must restrict the domain of $y = f(x)$ so that it will pass the H.L.T. (horiz. line test)

The two "largest" options are: $\{\boldsymbol{x \geq 3}, x \in \mathbb{R}\}$ *-or-* $\{\boldsymbol{x \leq 3}, x \in \mathbb{R}\}$ ➔

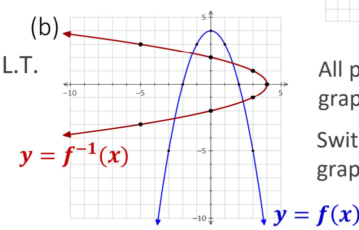
However there are infinite options: $\{x \geq 4, x \in \mathbb{R}\}$ *-or-* $\{x \leq 0, x \in \mathbb{R}\}$ *would also work!*



(b) The "largest" options is: $[\boldsymbol{-7, 2}]$ *another is:* $[\boldsymbol{5, 7}]$

PQ 20 (a) Inverse will NOT be a function, as graph of $y = f(x)$ fails the H.L.T.

(b)



All pts $(\boldsymbol{x}, \boldsymbol{y})$ on the graph of $f(x)$...

Switch to $(\boldsymbol{y}, \boldsymbol{x})$ on the graph of $y = f^{-1}(x)$

(c) $y = f(x)$ *D:* $\{\boldsymbol{x} \in \mathbb{R}\}$ *R:* $\{\boldsymbol{y \leq 4}, y \in \mathbb{R}\}$

$y = g(x)$ *D:* $\{\boldsymbol{x \leq 4}, x \in \mathbb{R}\}$ *R:* $\{\boldsymbol{y} \in \mathbb{R}\}$

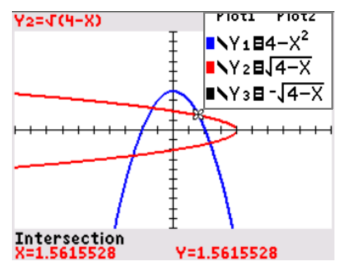
(d) For inverse equation, switch x & y ....

$y = 4 - x^2$ ➔ $x = 4 - y^2$ ➔ $y^2 = 4 - x$ ➔ $\sqrt{y^2} = \pm\sqrt{4-x}$ ➔ $\boldsymbol{f^{-1}(x) = \pm\sqrt{4-x}}$

(e) There are **two** invariant points, where the graph of $y = f(x)$ intersects with the line $y = x$.

To find these, graph $y_1 = f(x)$ and graph the inverse using y_1 and y_2 (must be done in two parts, the pos and neg of $\sqrt{4-x}$)

And find the two points of INTERSECTION



Invariant points (approx. coordinates) are: $(\boldsymbol{-2.56, -2.56})$ and $(\boldsymbol{1.56, 1.56})$

PQ 21 (a) **i** – NO invariant points, as the graph of $f(x)$ does not intersect with $y = x$

ii – Two possibilities are: $\{\boldsymbol{x \geq -2}, x \in \mathbb{R}\}$ *-or-* $\{\boldsymbol{x \leq -2}, x \in \mathbb{R}\}$

(b) **i** – YES, there an infinite number! (When $x = 4$ on the graph of $f(x)$, or when $x \leq -4$, graph intersects with $y = x$)

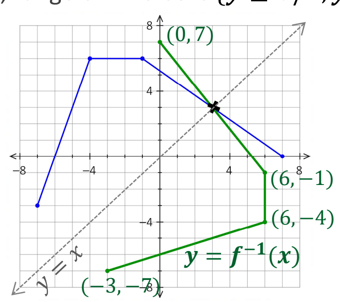
ii – Two possibilities are: $\{\boldsymbol{x \leq -3}, x \in \mathbb{R}\}$ (largest domain) *-or-* $\{\boldsymbol{x \leq - -4}, x \in \mathbb{R}\}$

PQ 22 See the domain of $f(x)$: ➔ $3 - 4x \geq 0$ ➔ $3 \geq 4x$ ➔ $3/4 \geq x$ So, range of inverse is $\{\boldsymbol{y \leq 3/4}, y \in \mathbb{R}\}$

PQ 23 (a) To sketch, transform all points $(x, y) \to (y, x)$

$(-3, -7) \to (\boldsymbol{-7, -3})$ Plot all points to sketch $y = f^{-1}(x)$ ➔

$(-6, -4) \to (\boldsymbol{-4, -6})$ *... and so on*

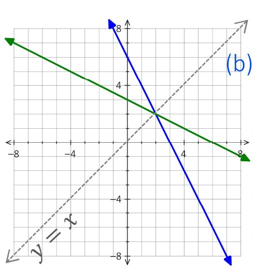


(b) Invariant point is on the line $y = x$, **so** $(\boldsymbol{3, 3})$

(c) $y = f(x)$, ***D:*** $[\boldsymbol{-7, 7}]$ ***R:*** $[\boldsymbol{-3, 6}]$ **Domain and Range interchange**

$y = f^{-1}(x)$, ***D:*** $[\boldsymbol{-3, 6}]$ ***R:*** $[\boldsymbol{-7, 7}]$

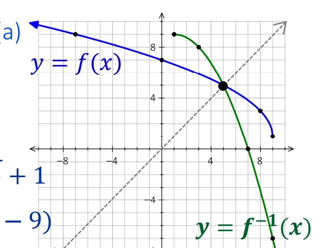
PQ 24 (a)



(b) Inv. pt is on line $y = x$, so $(\boldsymbol{2, 2})$

(c) $x = -2y + 6$ ➔ $x - 6 = -2y$ ➔ $\boldsymbol{f^{-1}(x) = -\frac{1}{2}x + 3}$

PQ 25 (a)



(b) Inv. pt is on line $y = x$, so $(\boldsymbol{5, 5})$

(c) $x = \sqrt{-4(y-9)} + 1$

$(x-1)^2 = -4(y-9)$

$\boldsymbol{f^{-1}(x) = -\frac{1}{4}(x-1)^2 + 9}$; $x \geq 1$

PQ 26

(a) $\{x \mid x \leq 11, x \in \mathbb{R}\}$

(b) $\{x \in \mathbb{R}\}$

Obtain the RANGE of $f(x)$

Must restrict domain to range of $f(x)$