PART 1 – Machine Scored

Answers are on page 6
Full, step-by-step solutions are on rtdmath.com

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1. An exponential function is given by $f(x) = a(2)^x - 5$, where $a \in I,\ a < 0$. The domain of the inverse function is:

A. $x \in R$ B. $x > -5$ C. $x < -5$ D. $x < 5$

2. The equation $log_{b+1}(3m) = \frac{1}{2}$, can be written in terms of b as:

A. $9m^2 - 1$ B. $3m^2 - 1$ C. $(3m-1)^2$ D. $(9m-1)^2$

Use the following information to answer the next question.

The y-intercept and asymptote of $f(x) = ab^x + d$ (a, b and $\in I, b > 1$) can be expressed using the indicated codes.

Use the following codes (in bold) to complete the sentence below: www.rtdmath.com

1 a **2** d **3** $b+d$ **4** $a+d$ **5** $ab+d$ **6** x **7** y

NR #1

The y-intercept of $f(x)$ is $y =$ _____ .
first code

$f(x)$ has an asymptote that can be written in the form _____ = _____ .
second code *third code*

Use the following information to answer the next two questions.

A function is defined by $f(x) = log_b(ax + b)$, where b and a are integers, $a > 0$ and $b \geq 2$.

3. The domain of $f(x)$ can be expressed as:

A. $x > -\frac{b}{a}$ B. $x > -b$ C. $x > -\frac{a}{b}$ D. $x > -a$

4. The y-intercept of $f(x)$ can be expressed as:

A. $log_b(a+b)$ B. $log_b(a)$ C. 1 D. 0

5. A student used an algebraic process to solve the equation $\frac{3^{x^2+x}}{27^{3x-1}} = 3\left(\frac{1}{9}\right)^{x-2}$. He is able to simplify the equation to $x^2 + bx + c = 0$, where $b,c \in I$

The value of c is:

A. -8 B. -4 C. -2 D. -1

Use the following information to answer the next question.

6. If $2a^b = 6$ then an expression for b is:

A. $log_c\left(\frac{a}{2}\right)$ B. $log_{2a}c$ C. $log_c(2a)$ D. $log_a\left(\frac{c}{2}\right)$

The following statements are made of a function $f(x) = a(2^{x-1}) + d$, where a, and d are integers, $a > 0$:

Statement 1: The y-intercept of the function is $\frac{a}{2} + d$

Statement 2: There is an asymptote at $x = 1$

Statement 3: There is an asymptote at $y = d$

Statement 4: There can never be an x-intercept

Statement 5: There is an x-intercept when $d < 0$

Statement 6: The inverse function will have a domain $x \in R$

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7. According to the federal census, the population of Calgary in 1971 was 403 319, and by 2016 had grown to 1 239 220. The approximate average annual growth rate over that period is:

A. 1.03% B. 2.5% C. 3.1% D. 6.8%

8. A particular drug is administered to a patient so that the initial plasma level is 3600 mg/L. Exactly one day later the level was 1160 mg/L.

The approximate half-life for this drug is:

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A. 11.3 hours B. 14.7 hours C. 25.5 hours D. 39.2 hours

9. Exactly two years ago Harry invested $1000 into a GIC, which in that time has grown to $1127.84. Harry made his investment with the goal to double his money to $2000.

Assuming his rate of return stays the same, and he can withdraw at any point, the number of **additional** years Harry must wait, correct to the nearest tenth, is:

A. 5.8 years B. 9.2 years C. 11.5 years D. 9.5 years

NR #3

If $log_a(8) = 2b - 1$ and $log_2 a = b$, the largest positive value of b, correct to the nearest tenth is _____.

10. If $log_5 a = b + 2log_5 c$, then a is equal to :

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A. $\frac{25}{b^c}$ B. $5^b c^2$ C. bc^2 D. $\frac{5^c}{b^2}$

NR #4

11. An equation $log_{x+1}(2x + 10) = 2$ has ____ real solution(s), the largest of which is $x =$ _____.

If $log_8 m = \frac{2}{3}$ and $3(2^n) = 5$, determine the value of $log_m n^m$, correct to the nearest hundredth:

First digit of your answer *Second digit of your answer*

A. -0.31 B. 0.31 C. -0.88 D. 0.78

12. $log_3 5 = a$ and $log_3 2 = b$ then an expression for $log_3 360$ is:

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A. $6ab$ B. $2ab^3$ C. $a + b^3 + 2$ D. $a + 3b + 2$

NR #5 The equation $2^{3m-1} = 5^m$ has an exact solution that can be written in the form: $m = \frac{1}{a - log_2 c}$

The values of a and c are, respectively, _____.

13. If $\frac{1}{2} log_2 4m = log_2 n + 2log_2 p$ then m can be expressed as:

A. $\frac{n^2 p^4}{4}$ B. $\frac{n^2 p^4}{2}$ C. $\frac{(n+2p)^2}{4}$ D. $\frac{(n+2p)^2}{2}$

NR #6 The equation $log_5 x - log_5 (x-1) = 3$ can be simplified to $ax - b = 0$, where a, b are positive integers. The value of a is _____.

14. The expression $2log4A - (3log2 - 6log\sqrt{A})$ can be simplified to:

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A. $log\left(\frac{A^5}{2}\right)$ B. $log(2A^5)$ C. $log\left(\frac{2}{A}\right)$ D. $log(2A^8)$

NR #7 In June 1946 an earthquake in Vancouver Island measured 7.3 on the Richter Scale. Later that year an earthquake measured on the Queen Charlotte fault had one-quarter the intensity. The Richter scale value of the Queen Charlotte fault earthquake, correct to the nearest tenth, was _____.

15. If $log_m n = 5$, then the value of $log_m(\sqrt[4]{n^3 m^2})$, correct to the nearest tenth is:

A. 3.3 B. 3.8 C. 5.3 D. 5.8

16. The equation $log_3(x-3) + log_3(x-2) = 2$ can be simplified to $x^2 + bx + c; b, c \in I$, where c is equal to:

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A. 3 B. 0 C. 4 D. 6

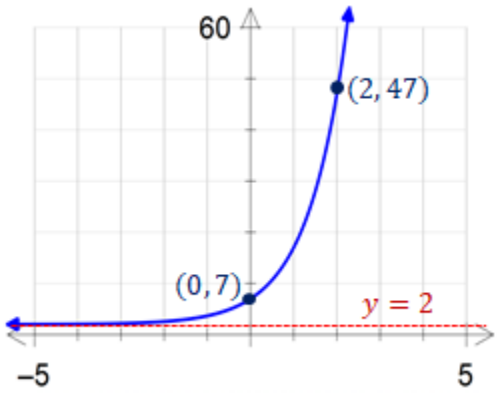
Answers are on the back page

PART 2 – Written Response

Use the following information to answer WR#1:

The graph on the right represents a function in the form $y = a(3)^x + k$.

The graph has a horizontal asymptote at $y = 2$.



❖ ***Written Response Question 1***

- Determine the values of a and k to derive the equation of the function. Show your reasoning. ***(2 marks)***

- Given a function $f(x) = 3(2)^x - 1$, determine the equation of the inverse function $g(x)$, and state the domain, range, and equation of the asymptote of $g(x)$. ***(3 marks)***

- BONUS *NOTE: An actual diploma exam question would never have a bonus component (sorry!)*
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Use an algebraic process to determine the exact value of any x or y intercepts for $g(x)$.

Use the following information to answer WR#2:

Strontium-90 is a radioactive isotope with applications in medicine and industry, and causes concern in fallout from nuclear weapons and accidents. Soil samples in particular area were tested for Strontium-90 over various years, and the results shown here:

Year	Millicuries (mCi) per square km

❖ ***Written Response Question 2***

- Assuming an exponential rate of decay, algebraically determine the half-life for Strontium-90. *(correct to the nearest tenth of a year)* Use your result to construct an equation that models the amount of Stontium-90 in the soil, in mCi per square km, as a function of time in years after the initial sample was taken. ***(3 marks)***

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- The amount of Iodine 131 in a sample after t days can be modeled by the equation $A = A_0(0.9172)^t$, where A_0 is the initial amount Iodine 131.

  Algebraically determine the minimum amount of time needed for a sample of Iodine 131 to decay to 10% of its initial amount. ***(2 marks)***

- BONUS *NOTE AGAIN: No actual bonus questions will be on your diploma exam!*

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  Determine the half-life for Iodine 131 from the second bullet, to construct an alternative equation in the form $A = A_0(b)^{\frac{t}{p}}$, where A is the percentage of Iodine 131 remaining after t days.

Answers

For full, worked-out solutions (as well as other practice materials) visit **www.rtdmath.com**

Multiple Choice

1. C **2.** A **3.** A **4.** C **5.** C **6.** D **7.** B **8.** B **9.** D **10.** B **11.** C **12.** D

13. A **14.** B **15.** D **16.** A

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Numerical Response

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1. 472 or 427 **2.** 135 *any order* **3.** 1.5 **4.** 13 **5.** 35 **6.** 124 **7.** 6.7

Written Response

1. *First bullet* $y = 5(3)^x + 2 \quad a = 5 \quad k = 2$ *Second bullet* $y = log_2[\frac{1}{3}(x+1)]$ Domain: $x \geq -1$ Range: $y \in R$

Asymptote: $x = 1$ *(vertical)* *Third bullet (bonus)* $x = 2,\ y = log_2(\frac{1}{3})$

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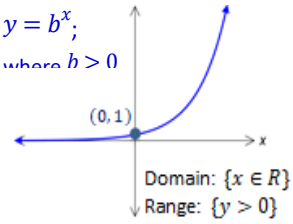
2. *First bullet* $y = 1.220(0.5)^{t/28.5}$ or $y = 1.220(0.976)^t$ NOTE: *Either form is ok!*

Second bullet 26.6 days *Third bullet (bonus)* $A = 100\left(\frac{1}{2}\right)^{t/8.02}$

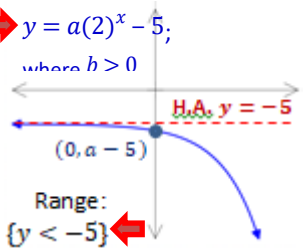
SOLUTIONS Math 30-1 Exponential Functions / LogsPractice

1. First consider graph of $f(x)$, since the domain of the inverse is the range of $f(x)$. (They switch!) So let's re-phrase this question … what's the RANGE of $f(x)$?

$y = b^x$; where $b > 0$



$y = a(2)^x - 5$; where $b > 0$



So the domain of the inverse is $x < -5$ ANSWER: **C**

2. $(b+1)^{\frac{1}{2}} = 3m$ First step – *convert to exp. form*

$\left((b+1)^{\frac{1}{2}}\right)^2 = (3m)^2$ To isolate b, square both sides (get that exp 1/'2 to be "1")

$\left((b+1)^{\frac{1}{2}}\right)^2 = 9m^2$

$b + 1 = 9m^2$

$-1 \quad -1$

$b = 9m^2 - 1$ ANSWER: **A**

NR#1 $f(0) = a(b^0) + d$ For y-intercept, set $x = 0$

$= a(1) + d$

$= a + d$ ← CODE 4

$y = 0$ ← Asymptote of basic exp. graph $y = b^x$, shown above in answer to #1

$y = d$ ← Horiz. Asymptote after being shifted vertically "d"units.

CODES 7 and 2 (either order) ANSWER: 472 or 427

3. $ax + b > 0$ $f(x) = log_b(ax + b)$

Isolate x

$ax > -b$ Whatever we're "logging" must be greater than 0. Can't log 0, can't log negatives!

$x > \frac{-b}{a}$ ANSWER: **A**

4. $f(0) = log_b(a(0) + b)$ For y-intercept, set $x = 0$

$= log_b(0 + b)$

$= log_b(b)$ ← $b^{\square} = b$? ANSWER: **C**

$= 1$

5. $\frac{3^{x^2+x}}{(3^3)^{3x-1}} = 3(3^{-2})^{x-2}$ *Re-write all three terms in base 3*

$\frac{3^{x^2+x}}{3^{9x-3}} = 3^1 * 3^{-2x+4}$ *Simplify using exponent rules*

$3^{x^2+x-(9x-3)} = 3^{-2x+4}$ *Once both sides are fully simplified – set the exponents equal*

$x^2 + x - 9x + 3 = -2x + 4$

$x^2 - 6x + 2 = 0$ ANSWER: **C**

c *value!*

6. $a^b = \frac{c}{2}$ ← First – isolate power term by dividing both sides by 2

$log_a\left(\frac{c}{2}\right) = b$ Then convert to log form to isolate "b"

$b = log_a\left(\frac{c}{2}\right)$ ANSWER: **D**

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NR#2

Check **Statement 1:**

$f(0) = a(2^{(0)-1}) + d$ ← *For y-int, set* $x = 0$

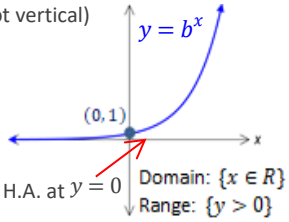
$= a(2^{-1}) + d$

$= a\left(\frac{1}{2}\right) + d$

$= \frac{a}{2} + d$ **Statement 1 is TRUE**

Check **Statement 2:**

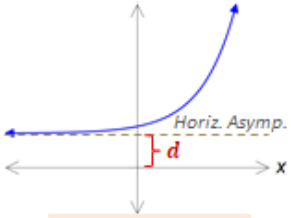
Exponential functions have HORIZONTAL asymptotes (not vertical)



Statement 2 is FALSE

Check **Statement 3:**

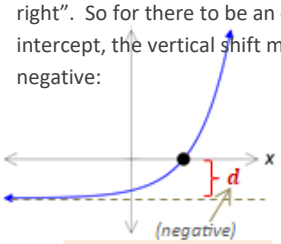
Consider the graph on the left shifted vertically by "d":



Statement 3 is TRUE

Check **Statement 4:**

Since $a > 0$ the graph must "rise right". So for there to be an x-intercept, the vertical shift must be negative:



Statement 3 is FALSE

There ***can be*** *an* x*-intercept*

Check **Statement 5:**

See rationale for statement 4

Statement 5 is TRUE

Check **Statement 6:**

The domain of the inverse is the RANGE of $y = f(x)$ ← *Which is NOT* $y \in R$

Statement 6 is FALSE

ANSWER: 135

7. $y = ab^{\frac{t}{p}}$ ← *Use this formula, with $p = 1$ since we want a growth rate per 1 year*

$1239220 = 403\,319(b)^{45}$

end amount; Initial amount; # of years from 1971 to 2016

$\frac{1239220}{403\,319} = b^{45}$ *Isolate the power term*

$b^{45} \approx 3.072555$ *Take the "45th root" of both sides to solve for b*

$\sqrt[45]{b^{45}} \approx \sqrt[45]{3.072555}$

$b \approx 1.025$ ($b = 1 +$ *growth rate*)

growth rate $= 2.5\%$

ANSWER: B

8. $y = ab^{\frac{t}{p}}$ ← *Use this formula, **find** p*

$1160 = 3600\left(\frac{1}{2}\right)^{\frac{24}{p}}$

We were given amount after 1 day (answers are all in hours); FIND p; end amount; Initial amount; Half-life problem

$\frac{1160}{3600} = \left(\frac{1}{2}\right)^{\frac{24}{p}}$ *Isolate the power term*

$log_{1/2}\left(\frac{1160}{3600}\right) = \frac{24}{p}$ *Convert to log form (or, if more to your liking, "log both sides")*

$p = \frac{24}{log_{1/2}\left(\frac{1160}{3600}\right)}$ ➡ $p \approx 14.7$ (***hours***, *since t was in hrs.*)

ANSWER: B

9. $y = ab^{\frac{t}{p}}$ ← *Use this formula, where $p = 1$*

$1127.84 = 1000(b)^2$

end amount; Initial amount; After two years; FIND b

$\frac{1127.84}{1000} = b^2$ *Isolate the power term*

$b^2 = 1.12784$

$b = \sqrt{1.12784}$ *Square root both sides*

$b \approx 1.062$ *Use this in equation*

Now, use $b = 1.062$ to solve for "t" when $y = 2000$.

$y = 1127.84(1.062)^t$

$2000 = 1127.84(1.062)^t$

$\frac{2000}{1127.84} = (1.062)^t$

Convert to log form to solve for t:

$t = log_{1.062}\left(\frac{2000}{1127.84}\right)$

$t \approx 9.5$ ANSWER: D

NR#3 Convert each to exp. form:

$a^{2b-1} = 8$ $\quad 2^b = a$

Substitute "2^b" for a in first equation

$(2^b)^{2b-1} = 8$

Re-write using base 2:

$2^{2b^2-b} = 2^3$

Set exponents equal and solve:

$2b^2 - b = 3$ ➡ $2b^2 - b - 3 = 0$

$(2b - 3)(b + 1) = 0$ $\quad b = 3/2$ or ~~-1~~ *extraneous*

ANSWER: 1.5

10. Get the log terms on the same side:

$log_5 a - 2log_5 c = b$

Apply log laws on left side:

$log_5 a - log_5 c^2 = b$

$log_5 \frac{a}{c^2} = b$

Convert to exp. form to isolate "a":

$5^b = \frac{a}{c^2}$ ➡ $a = 5^b c^2$ ANSWER: B

NR#4 Convert to exp. form to solve:

$(x + 1)^2 = 2x + 10$

Expand left side, move right side terms over

$x^2 + 2x + 1 - 2x - 10 = 0$

Simplify and solve resulting quadratic:

$x^2 - 9 = 0$ ➡ $x^2 = 9$

$x = 3$ or ~~-3~~ ***Extraneous*** *– subbing into the original equation would lead to logging a negative (illegal)*

ANSWER: 13

11. Convert each to exp. form:
(first isolate power term on 2nd one)

$8^{2/3} = m$ $\quad 2^n = 5/3$ ➡ $n = log_2\left(\frac{5}{3}\right)$

$m = 4$

Now, substitute into: $log_m n^m$

$= 4log_4(log_2\left(\frac{5}{3}\right))$ *Put into your calc*

```
4log₄(log₂(5/3))
                -0.8806616548
```

≈ -0.88 ANSWER: C

12.

Split up 360 into multiples of **5** and **2** *(since we're given those logs)*

$= log_3(5 * 8 * 9)$

Split up using log laws:

$= log_3 5 + log_3 8 + log_3 9$

$= log_3 5 + log_3 2^3 + 2$ ← $3^{\square} = 2$?

$= log_3 5 + 3log_3 2 + 2$

$= a + 3b + 2$ ANSWER: D

NR#5 Convert to exp. form (using base 2, as indicated) to solve:

$log_2(5^m) = 3m - 1$

Use log laws on left side:

$mlog_2 5 = 3m - 1$

Re-arrange get "m" terms on same side:

$1 = 3m - mlog_2 5$

Factor out "m" to isolate

$1 = m(3 - log_2 5)$

$\frac{1}{(3 - log_2 5)} = m$

ANSWER: 35

13. **Step 1** #'s in front become exponents:

$log_2(4m)^{\frac{1}{2}} = log_2 n + log_2 p^2$

Step 2 Combine LS to single log:

$log_2\sqrt{4m} = log_2(np^2)$

Step 3 Since same base – drop logs:

$\sqrt{4m} = np^2$

➡ $2\sqrt{m} = np^2$ ➡ $\sqrt{m} = \frac{np^2}{2}$

Step 4 Square both sides to isolate m

$(\sqrt{m})^2 = \left(\frac{np^2}{2}\right)^2$ ➡ $m = \frac{n^2p^4}{4}$

ANSWER: A

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NR #6 Log laws / combine to single log:

$log_5\left(\frac{x}{x-1}\right) = 3$

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Convert to exp. form:

$5^3 = \frac{x}{x-1}$

Cross-multiply to solve:

$125(x-1) = x$

$125x - 125 = x$

$125x - x - 125 = 0$

$124x - 125 = 0$ ANSWER: 124

14. Log laws / #'s *in front* become exponents:

$= log(4A)^2 - log2^3 + log(A^{1/2})^6$

(Also distribute – sign through brackets)

$= \log(4^2A^2) - log8 + logA^3$

Log laws / combine to single log:

log ——— *Pos (+) logs go up top*; *Neg (−) logs down here*

$= log\left(\frac{16A^2 * A^3}{8}\right)$

Simplify inside brackets:

$= log(2A^5)$ ANSWER: B

NR #7 Express Richter values as exponents of 10:

$\frac{10^{higher\ Richter}}{10^{lower}} =$ *"times as intense"*

$\frac{10^{7.3}}{10^{x}} = 4$ — *Van. Island Richter is higher / goes on top*; *If Q. Charlotte had 1/4th intensity, then Van. Island was 4 times as intense*

Unknown / Richter for Q.Charlotte earthquake was smaller (goes on bottom)

$10^{7.3-x} = 4$

Convert to log form to solve:

$log_{10}4 = 7.3 - x$ ➡ $x = 7.3 - log10$

$x \approx 6.7$ ANSWER: 6.7

15. Convert first log to exp. form:

$m^5 = n$ *We can now substitute into second log*

$log_m(n^{\frac{3}{4}}m^2)$

Becomes....

$log_m((m^5)^{\frac{3}{4}}m^2)$

Now simplify:

Multiply exponents: $5 * \frac{3}{4}$

$= log_m(m^{\frac{15}{4}}m^2)$

Add exponents: $\frac{15}{4} + 2 = \frac{15}{4} + \frac{8}{4}$

$= log_m(m^{\frac{23}{4}})$

$= \frac{23}{4}$ ANSWER: D

16. Combine L.S. to single log:

$log_3[(x-3)(x-2)] = 2$

Then convert to exp. form:

$3^2 = (x-3)(x-2)$

Solve resulting quadratic equation.

But watch for extraneous solutions!

$9 = x^2 - 5x + 6$

$0 = x^2 - 5x - 3$

ANSWER: A

Written #1

First bullet The horiz. asymptote represents the vert. shift, "k"

So.... $k = 2$

Sub into equation and solve for "a"

$y = a(3)^x + 2$

Use any given point to solve for "a"

$7 = a(3)^0 + 2$ ➡ $7 = a(1) + 2$ ➡ $a = 5$

So equation is:

$y = 5(3)^x + 2$

Second bullet For inverse, switch x and y

$f(x)$: $y = 3(2)^x - 1$

$g(x)$: $x = 3(2)^y - 1$

Then solve for y: Isolate power term, 2^y, to convert to log form

$x + 1 = 3(2)^y$ ➡ $\frac{x+1}{3} = 2^y$ ➡ $log_2\left(\frac{x+1}{3}\right) = y$ ➡

$y = log_2\left(\frac{x+1}{3}\right)$ or $y = log_2\left[\frac{1}{3}(x+1)\right]$

Third bullet $g(x)$ has an x-int when $y = 0$

Using $x = 3(2)^y - 1$ *equation:*

$x = 3(2)^0 - 1$

$x = 3(1) - 1$ ➡ $x = 2$ x-intercept

Using $y = log_2\left(\frac{x+1}{3}\right)$ *equation:*

$0 = log_2\left(\frac{x+1}{3}\right)$

➡ $2^0 = \frac{x+1}{3}$ *Same result!* $3 = x + 1$

$g(x)$ has a y-int when $x = 0$

Using $y = log_2\left(\frac{x+1}{3}\right)$ *equation:*

$y = log_2\left(\frac{0+1}{3}\right)$ ➡ $y = log_2\left(\frac{1}{3}\right)$

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Written #2

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First bullet

Use $y = ab^{t/p}$ to solve for p:

$$0.959 = 1.220\left(\frac{1}{2}\right)^{\frac{10}{p}}$$ ← *Use values from table, after 10 years amount went from 1.220 to 0.959*

Isolate power term:

$$\frac{0.959}{1.220} = \left(\frac{1}{2}\right)^{\frac{10}{p}}$$

Then convert to exp. form to solve:

$$\frac{10}{p} = log_{1/2}\left(\frac{0.959}{1.220}\right) \Rightarrow p = \frac{10}{log_{0.5}\left(\frac{0.959}{1.220}\right)} \Rightarrow p \approx 28.8 \text{ years}$$

Half-life

Alternatively you could "log both sides"

$$log\frac{0.959}{1.220} = log\left(\frac{1}{2}\right)^{\frac{10}{p}} \Rightarrow log\left(\frac{0.959}{1.220}\right) = \frac{10}{p}log\left(\frac{1}{2}\right) \Rightarrow p = \frac{10log\left(\frac{1}{2}\right)}{log\left(\frac{0.959}{1.220}\right)}$$

Same result, $p \approx 28.8$

Equation that models amount of Strontium-90:

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$$y = 1.220\left(\frac{1}{2}\right)^{\frac{t}{28.8}}$$

Second bullet

Initial percentage is 100, so use $A_0 = 100$

$$10 = 100(0.9172)^t$$

Isolate power term:

$$\frac{10}{100} = (0.9172)^t$$

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$$0.1 = (0.9172)^t$$

Then convert to exp. form to solve:

$$t = log_{0.9172}(0.1) \Rightarrow t \approx 26.6 \text{ days}$$

Alternatively you could "log both sides"

$$log0.1 = log0.9172^t$$

$$log0.1 = tlog0.9172$$

$$\frac{log0.1}{log0.9172} = t$$

Third bullet

"bonus"!

The half-life is the period of time ("p" in the equation) necessary for the initial amount to drop to half.

$$0.5 = 1(0.9172)^t$$ ← *Make the initial amount* 1*, and the end amount* 0.5

Then convert to exp. form to solve:

$$t = log_{0.9172}(0.5)$$

$$t \approx 8.02 \text{ days}$$

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$$A = 100\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$$

Alternate equation using half-life of 8.02 days

And with that – you're done!



(Or try another practice exam)

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