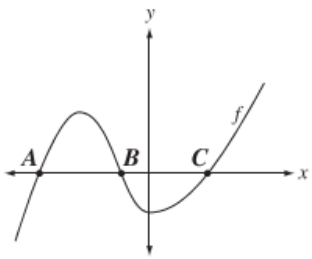
Unit 1 – Transformations of Functions

1 If the graph of $f(x) = (x-2)^2 + 1$ is transformed into the graph of $g(x) = f(x-4) + 3$, then the coordinates of the vertex on the graph of $y = g(x)$ will be

A. (2, 4)
B. (–2, 4)
C. (6, 4)
D. (–6, 4)

2 The partial graph of the function $y = f(x)$ is shown below.



Which of the following transformations will **always** create invariant points at *A*, *B*, and *C*?

A. $y = f(bx)$
B. $y = af(x)$
C. $y = f(x) + k$
D. $y = f(x-h)$

3 *When compared to the graph of* $y = f(x)$, *the graph of* $8y = f(-x)$ *has been reflected in the* ___**i**___ *and vertically stretched about the x-axis by a factor of* ___**ii**___.

The statement above is completed by the information in row

Row	*i*	*ii*
A.	*y*-axis	8
B.	*y*-axis	$\frac{1}{8}$

Row	*i*	*ii*
C.	*x*-axis	8
D.	*x*-axis	$\frac{1}{8}$

4 *The function* $f(x) = |x-2| + 3$ *is transformed into the function* $g(x) = |x+2| + 1$.
The transformations that will transform $y = f(x)$ *into* $y = g(x)$ *are a translation of*

_____ *units* _____ *and a translation of* _____ *units* _____.

Use the following code to complete the sentence above.

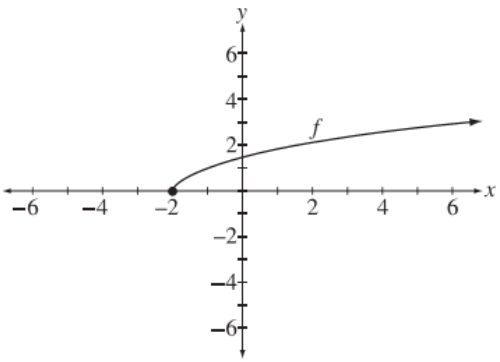
Reference Number	Numerical Value
1	1
2	2
3	3
4	4

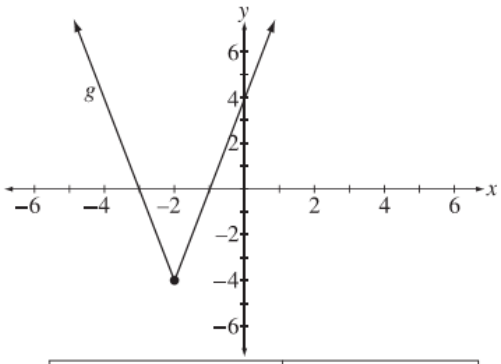
Reference Number	Translation Direction
5	left
6	right
7	up
8	down

Numerical Response

Use the following information to answer the next question

The graphs of the functions $y = f(x)$ and $y = g(x)$ are shown below. Each function is then horizontally stretched by a factor of 2 about the y-axis, resulting in the new functions $y = p(x)$ and $y = q(x)$ respectively. The domain and the range of each of the new functions are found in the table below.





Reference Number	Possible Domain
1	$x \geq -1$
2	$x \geq -2$
3	$x \geq -4$
4	$x \in R$

Reference Number	Possible Range
5	$y \geq 0$
6	$y \geq -2$
7	$y \geq -4$
8	$y \geq -8$
9	$y \in R$

5 Complete the statements below.

The domain and range of the new function $y = p(x)$ are numbered, respectively, _____ (Record in the **first** column) and _____ (Record in the **second** column).

The domain and range of the new function $y = q(x)$ are numbered, respectively, _____ (Record in the **third** column) and _____ (Record in the **fourth** column).

Numerical Response

Use the following information to answer the next question

The point $A(3, -5)$ lies on the graph of the function $y = f(x)$. The ordered pairs below represent possible coordinates of the new point corresponding to Point A after $f(x)$ undergoes a single transformation.

Point 1 (3, 5)
Point 2 (–3, 5)
Point 3 (–3, –5)
Point 4 (–5, –3)
Point 5 (–5, 3)
Point 6 (5, –3)

6 The corresponding point when the graph of $y = f(x)$ is reflected about the line

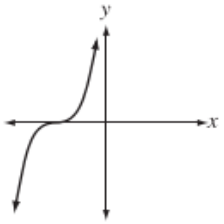
$y = x$ is Point _____ (Record in the **first** column)
$x = 0$ is Point _____ (Record in the **second** column)
$y = 0$ is Point _____ (Record in the **third** column)

Numerical Response

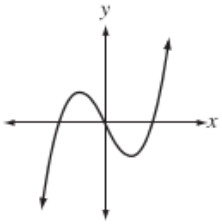
7

A student is asked to draw the inverse of each of the following graphs.

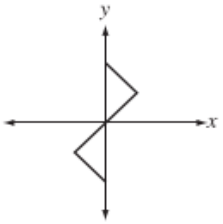
Graph 1



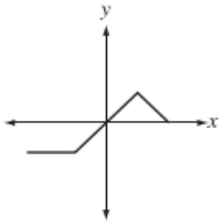
Graph 2



Graph 3



Graph 4



If no additional restrictions are given, which two graphs above will have an inverse that is a function?

A. Graph 1 and Graph 2

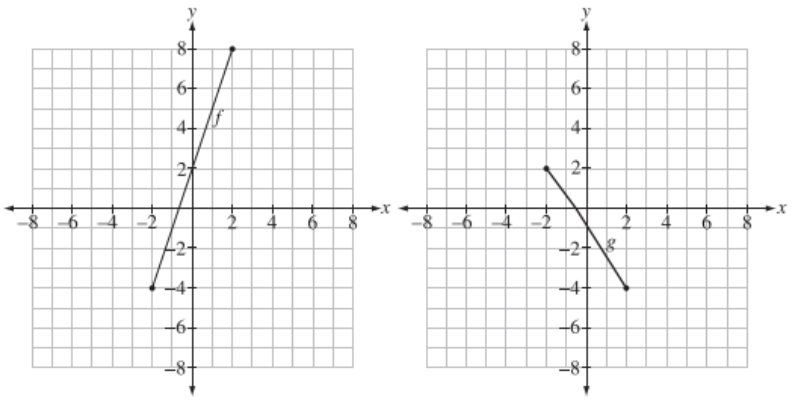
B. Graph 1 and Graph 3

C. Graph 2 and Graph 3

D. Graph 2 and Graph 4

8

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.



The range of the graph of $h(x) = (f - g)(x)$ would be

A. $\{y \mid 0 \le y \le 6, y \in R\}$

B. $\{y \mid -6 \le y \le 0, y \in R\}$

C. $\{y \mid -6 \le y \le 12, y \in R\}$

D. $\{y \mid -12 \le y \le 6, y \in R\}$

9

Use the following information to answer the next question.

The graph of $y = f(x)$ is reflected in the x-axis, stretched vertically about the x-axis by a factor of $\frac{1}{3}$, and stretched horizontally about the y-axis by a factor of 4 to create the graph of $y = g(x)$.

For Point $A(-3, 6)$ on the graph of $y = f(x)$, the corresponding image point, A', on the graph of $y = g(x)$ is

A. (9, 24)

B. (–12, –18)

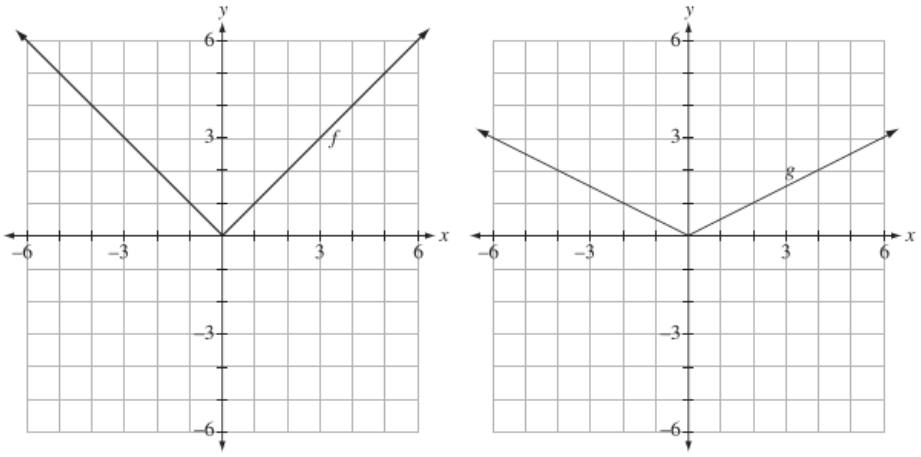
C. (1, 24)

D. (–12, –2)

10.

Use the following information to answer the next question.

The graphs of $f(x) = |x|$ and $y = g(x)$ are shown below. The graph of $f(x)$ undergoes a single transformation to become the graph of $g(x)$.



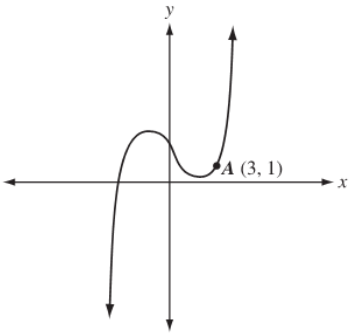
Determine both possible equations for the function $g(x)$.

Written Response

11.

Use the following information to answer the next question.

The graph of function f is shown below.
Function f is transformed to create function g.



If $g(x) - 2 = f(x + 5)$, then which quadrant contains Point A in the graph of function g?

A. Quadrant I
B. Quadrant II
C. Quadrant III
D. Quadrant IV

Use the following information to answer the next question.

12.

The graph of $y = \sqrt{x}$ is transformed into the graph of $y = \sqrt{-\frac{1}{2}x - 4}$.

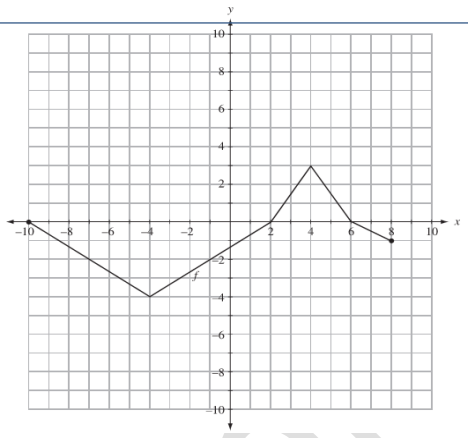
A list of possible transformations is shown below.

Reference Number	Transformation
1	Horizontal stretch by a factor of $\frac{1}{2}$
2	Horizontal stretch by a factor of 2
3	Reflection in the x-axis
4	Reflection in the y-axis
5	Horizontal translation of 4 left
6	Horizontal translation of 4 right
7	Horizontal translation of 8 left
8	Horizontal translation of 8 right

The transformations from the list above that describe the changes to the graph of $y = \sqrt{x}$ are _____, _____, and _____.

Numerical Response

13. The graph of $y = f(x)$ as shown is reflected in the y-axis, vertically stretched by a factor of 2 about the x-axis, horizontally stretched by a factor of ½ about the line $x = 0$, and then translated 3 units up.



SE Sketch the graph of the new function.

Written Response

14. *Use the following information to answer the next question.*

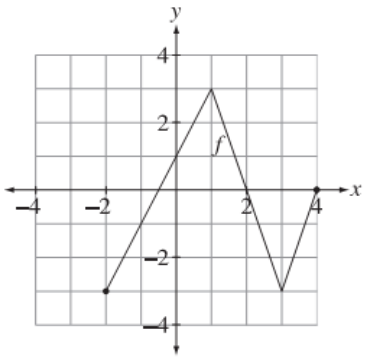
The inverse of $f(x) = 2x - 3$ can be written as $g(x) = \frac{a}{2}x + \frac{b}{2}$.

Numerical Response

Numerical Response

The values of a and b are, respectively, _____ and _____.

15. The graph of $y = f(x)$ is shown



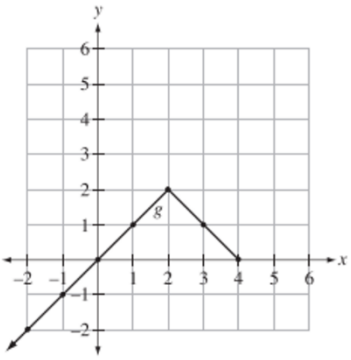
When the graph of $y = f(x)$ is reflected in the line $y = x$, the number of invariant points is

A. one
B. two
C. three
D. four

Unit 2 – Function Operations

The table shown below describes the function $y = f(x)$ and the graph shows the function $y = g(x)$.

x	$f(x)$
–1	undefined
0	4
1	5
2	6
3	6
4	7
5	8



16. If $h(x) = \frac{f(x)}{g(x)}$, then the value of $h(3)$ is _____

17.

$$h(x) = g(x) + (f \circ g)(x), \text{ where } f(x) = x^2 + 6x \text{ and } g(x) = \frac{1}{x-2}$$

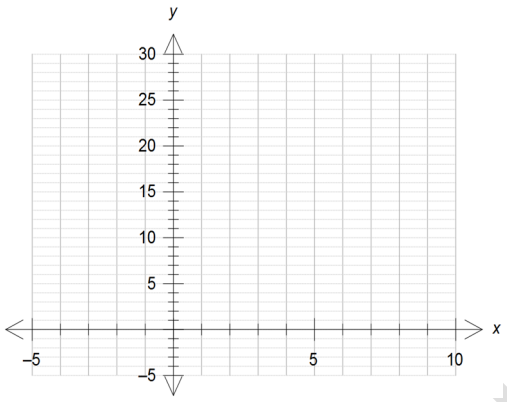
Numerical Response

The value of $h(5)$, to the nearest hundredth, is _____.

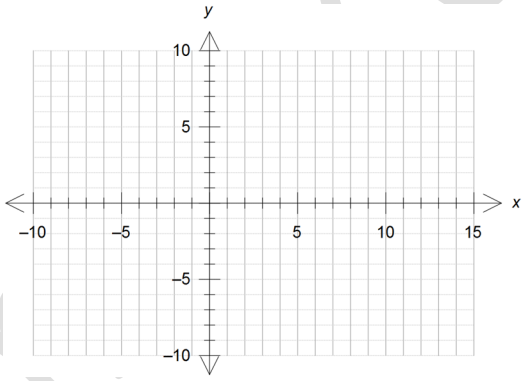
18. Given the functions $f(x) = 7 - x$ and $g(x) = 2x + 1$, sketch the graph of $h(x)$ for each question below **and** state the domain and range.

Written Response

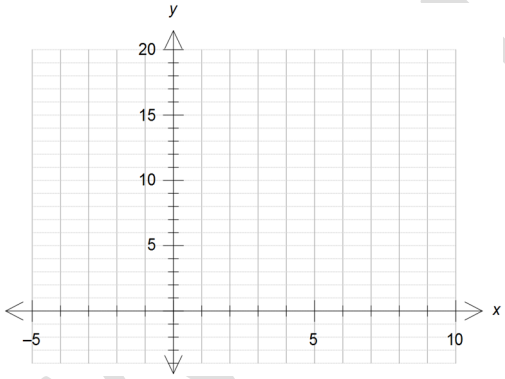
a) $h(x) = f(x)\,g(x)$



b) $h(x) = \left(\frac{g}{f}\right)(x)$



c) $h(x) = g(f(x))$



Use the following information to answer the next question.

For functions $f(x) = \sqrt{x-1}$, $g(x) = x^2 + 3$, $h(x) = 2x - 5$, and $k(x)$, where $k(x) = (h \circ g \circ f)(x)$, the simplified expression for $k(x)$ can be written in the form $k(x) = ax - b$.

19. The value of a is _____ (Record in the **first** box)

The value of b is _____ (Record in the **second** box)

Numerical Response

Use the following information to answer the next question.

Alex is given the following list of functions, where $b > 1$. She is asked to determine a new function, $h(x)$, which is the quotient of two different functions below and where the domain of $h(x)$ is $\{x \in R\}$.

Function 1 $y = x - b$

Function 2 $y = x^2 + b$

Function 3 $y = x^3 + b$

Function 4 $y = \log_b x$

Function 5 $y = b^x$

Function 6 $y = \sqrt{x + b}$

Numerical Response

20. If $h(x) = \frac{f(x)}{g(x)}$ and Alex selects Function 1 for $f(x)$, then the two functions that she could select for $g(x)$ are numbered _____ and _____.

Use the following information to answer the next question.

Two functions are shown below.

$f(x)$:{(1,4), (2,6), (3,5), (4,2)}

$g(x)$:{(1,5), (3,2), (5,8), (6,3)}

21. The value of $(g \circ f)(3)$ is

A. 6
B. 8
C. 9
D. 10

22. Given $f(x) = x^2 - 7x$, $g(x) = x - 2$, and $h(x) = \frac{2x^2 + x}{x - 2}$, determine a simplified equation for $j(x)$, given that $j(x) = \frac{f(x)}{g(x)} + h(x)$.

Written Response

23. Given that Point A (3, 4) lies on the graph of $g(x)$, and Point A' (3, 8) lies on the graph of $h(x)$, where $h(x) = (f \circ g)(x)$, the corresponding point that lies on the graph of $f(x)$ must be _____ .

Written Response

Use the following information to answer the next question.

Given that $f(x)=\sqrt{x-3}$ and $g(x)=\dfrac{x^2}{x^2-25}$, students are asked to select the domain of the graph of $y=h(x)$, where $h(x)=g(f(x))$, from the list provided in the table below.

Reference Number	Domain
2	$\{x \mid x \geq 3, x \in R\}$
3	$\{x \mid x \neq 28, x \in R\}$
4	$\{x \mid x \geq 3, x \neq 28, x \in R\}$
5	$\{x \mid x \geq 3, x \neq 5, x \in R\}$
6	$\{x \mid x \geq 3, x \neq 5, x \neq 28, x \in R\}$

Numerical Response

24. The reference number that corresponds to the domain of the graph of $h(x)=g(f(x))$ is _____ .

Unit 3 – Exponential Functions and Logarithms

25. The graph of $y=3^{x+2}$ is reflected in the line $y=x$. The equation of the new graph is

A. $y=\log_3 x-2$

B. $y=\log_3 x+2$

C. $y=\log_3(x-2)$

D. $y=\log_3(x+2)$

26. A student is asked to solve the equation $\dfrac{125^{x(x+1)}}{5^{(3x-4)}}=25^{(x-5)}$ using an algebraic process.

She is able to simplify the equation to the form $3x^2+bx+c=0$.

The value of c is

A. 6

B. 9

C. 14

D. 40

27. The solution to the equation $8^{(3x+4)} = 4^{(x-9)}$ is

A. $\frac{-13}{2}$

B. $\frac{-30}{7}$

C. $\frac{-17}{5}$

D. $\frac{-22}{7}$

28. The equation $m\log_p n + 5 = q$ can be written in exponential form as

A. $p^{(q-5)} = mn$

B. $p^{(q-5)} = n^m$

C. $p^{(q-5)} = \frac{m}{n}$

D. $p^{(q-5)} = m^n$

29. *Use the following information to answer the next question.*

A student sketched the graphs of $f(x) = \log_a(x+3) - 7$ and $g(x) = a^{(x-2)} + 5$, where $a > 1$, on a coordinate plane. She also drew the asymptotes of the two graphs using dotted lines.

The intersection point of the two dotted lines will be at

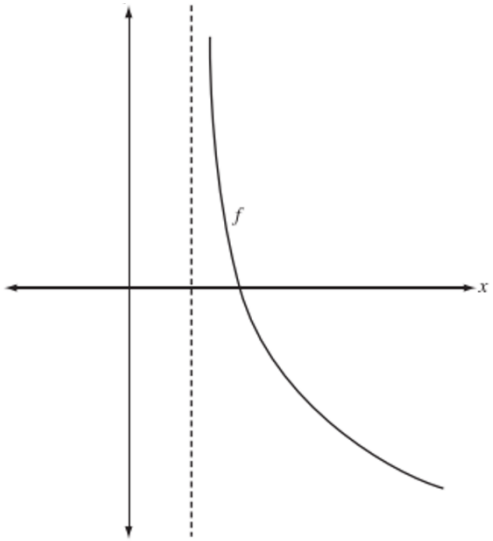
A. (3, 5)

B. (–3, 5)

C. (2, –7)

D. (–2, –7)

30. The partial graph of the logarithmic function $f(x) = a\log_b(x-h)$, where $b > 1$, is shown below.



To produce the graph of $y = f(x)$ above, the values of a and h in the function must satisfy

A. $a < 0$ and $h > 0$

B. $a < 0$ and $h < 0$

C. $a > 0$ and $h > 0$

D. $a > 0$ and $h < 0$

31.

The loudness of a sound is related to the logarithm of the ratio of the measured intensity, I, to a reference intensity, I_0. The loudness, L, of a sound is measured in decibels, dB, and can be determined using the following formula.

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

During an international soccer tournament in 2010, a noisemaker called the vuvuzela had a measured loudness of 127 dB at full volume.

If the intensity of the sound of the vuvuzela is 5 000 times greater than the intensity of the sound of a lawn mower, then the measured loudness of the lawn mower, to the nearest decibel, is

A. 3 dB
B. 37 dB
C. 90 dB
D. 123 dB

32. An expression that is equivalent to $\log\left(\frac{2\sin x}{\sin(2x)}\right)$, where $0° < x < 90°$, is

A. $\log 1$
B. $\log(\cos x)$
C. $-\log(\sin x)$
D. $-\log(\cos x)$

33. ***SE*** Solving the equation $3^{(2x+1)} = \left(\frac{1}{5}\right)^{(x-3)}$ algebraically, the solution, to the nearest hundredth, is _____.

Numerical Response

34. If $\log_a 8 = 3$ and $\log_4 b = \frac{3}{2}$, then the value of $\log_a b + \log_b a$, to the nearest hundredth, is _____.

Numerical Response

35. ***SE*** Solve algebraically $\log_7(x+1) + \log_7(x-5) = 1$.

Written Response

36. The population of a particular town on January 1, 2008 was 15 000. If the population of this town on January 1, 2016 was 32 450, what is the average annual rate of increase?

A. 0.1%
B. 1.1%
C. 10%
D. 110%

37. If $\log_a b = 1.26$, where $a, b > 0$, $a \neq 1$, then the value of $\log_a\left(\sqrt[3]{b}\right)$, to the nearest hundredth, is _____.

Numerical Response

38. Jordan needs $6 000 to take his family on a trip. He is able to make an investment which offers an interest rate of 8%/a compounded semi-annually. How much should Jordan invest now, to the nearest dollar, so that he has enough money to go on a family trip in 3 years?

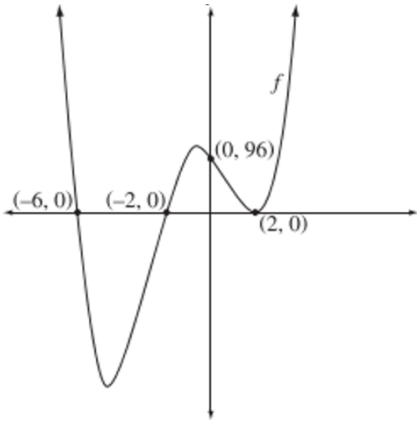
Written Response

Unit 4 – Polynomial Functions

39. For the polynomial function $P(x) = 2x^5 + 3x^4 - 10x^3 - 21x^2 + kx$, two of the zeros are –1 and –2. It can be determined that the largest zero of this function, to the nearest tenth, is _____.

Numerical Response

40. The graph of a particular function is shown below.



If the equation of the function graphed above is written in the form $f(x) = ax^4 + bx^3 + cx + d$, then the value of a is

A. 1
B. 2
C. 4
D. 96

www.tutormath.com

41. *If $P(x)$ is a polynomial function where $P\left(-\frac{2}{3}\right) = 0$ and $P(0) = 12$, then* ___i___ *is a factor of $P(x)$, and* ___ii___ *is a constant term in the equation of $P(x)$.*

The statement above is completed by the information in row

Row	*i*	*ii*
A.	$(3x - 2)$	12
B.	$(3x + 2)$	12
C.	$(3x - 2)$	-12
D.	$(3x + 2)$	-12

42. *Use the following information to answer the next question.*

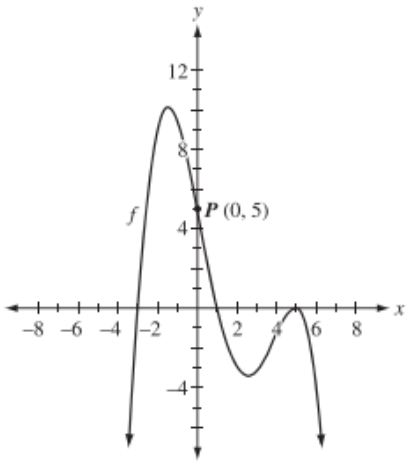
The roots of the equation $2x^3 - 3x^2 - 10x + 3 = 0$ can be written as $x = m$ and $x = \frac{n \pm \sqrt{p}}{4}$, where $m, n, p \in I$.

Numerical Response

The value of ***p*** is _____ .

Use the following information to answer the next question.

The graph of the polynomial function $y = f(x)$ is shown below. The equation of this graph can be written in the form $y = -\frac{1}{a}(x - 1)(x + b)(x - c)^2$.



Numerical Response

43. The value of ***a*** is _____ .

44. The polynomial function $P(x) = 4x^4 - x^3 - 8x^2 - 40$ has a linear factor of $x + 2$.

The remaining cubic factor is

A. $4x^3 + 7x^2 + 6x - 20$

B. $4x^3 + 7x^2 + 6x + 12$

C. $4x^3 - 9x^2 + 10x - 20$

D. $4x^3 - 9x^2 + 10x - 60$

45. *Use the following information to answer the next question.*

The binomial $x + 2$ is a factor of $f(x) = x^3 + 3x^2 + kx + 4$.

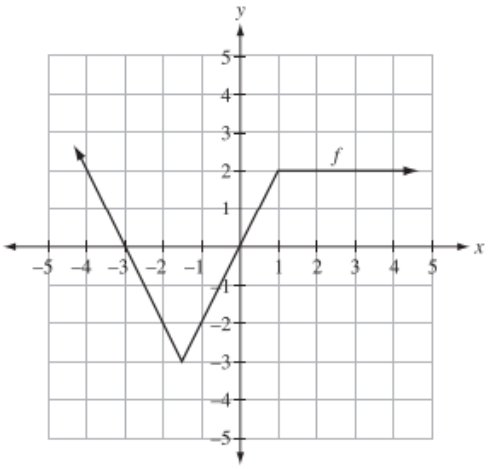
The value of k is _____. **Numerical Response** | | | | |

46. Fully factor: $P(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9$ **Written Response**

Unit 5 – Radical & Rational Functions

47. *Use the following information to answer the next question.* **Written Response**

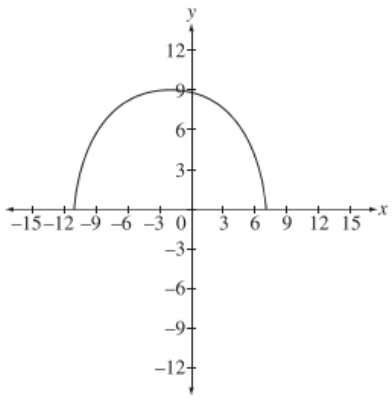
The graph of the function $y = f(x)$ is shown below.



Sketch the graph of $y = \sqrt{f(x)}$ and state the domain and range.

virtdmath.com

48. The graph of $y = \sqrt{f(x)}$ has a range of $[0, 9]$ as shown below.



If $f(x)$ is a quadratic function, then a point which **cannot** be on the graph of $y = f(x)$ is

A. $(-14, -63)$ **B.** $(-7, 56)$ **C.** $(4, 90)$ **D.** $(9, -40)$

Use the following information to answer the next question.

The graphs of the functions $y = f(x)$, $y = g(x)$, and $y = h(x)$ are shown below. Each function is transformed into $y = \sqrt{f(x)}$, $y = \sqrt{g(x)}$, and $y = \sqrt{h(x)}$, respectively.

Diagram 1

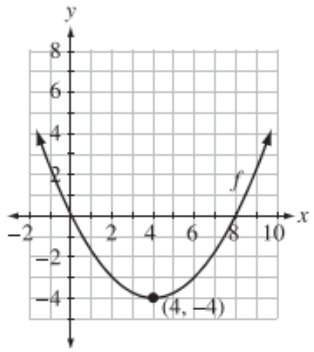


Diagram 2

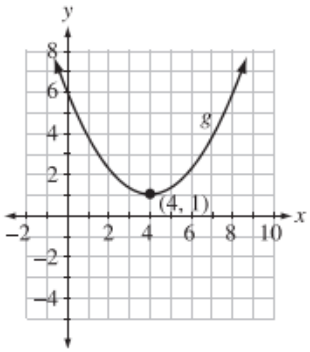
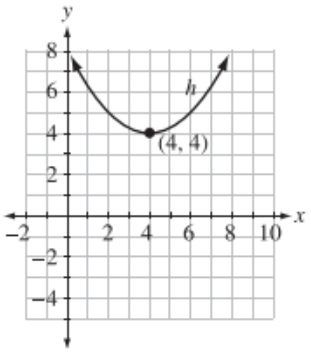


Diagram 3



49. The number of invariant points on the original graph and its transformed graph for

Diagram **1** is _____ **Numerical Response**

Diagram **2** is _____

Diagram **3** is _____

Use the following information to answer the next question.

The graph of the function $f(x) = \frac{1}{2}x - 3$ is transformed into $y = \sqrt{f(x)}$. The invariant points of the graph are located as (a, b) and (c, d).

50. The values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are, respectively, _____, _____, _____, and _____. **Numerical Response**

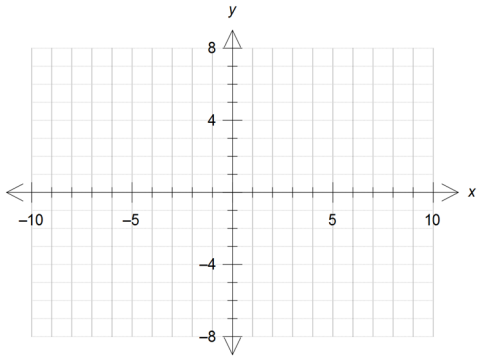
51. **Numerical Response**

For the function $f(x) = y = -2\sqrt{x+4} + 3$, the x-intercept is equal to $-k$. The value of k, to the nearest hundredth, is _____.

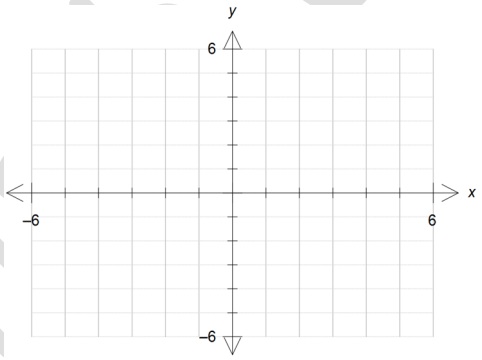
52. Sketch the graph of the following functions and determine the following characteristics for each function below: domain, x- and y-intercepts, and the equation of the vertical asymptote.

Written Response

a) $y = \dfrac{3x}{x^2 + 2x - 8}$



b) $y = \dfrac{x+3}{x^2 - 9}$



53. *If* $f(x) = \dfrac{1}{x+1} + 3$, *then the vertical asymptote is located at* _____ *and the horizontal asymptote is located at* _____. **Numerical Response**

Use the following code to complete the sentence above.

Possible Equations

1	$x = -3$	**3**	$x = 1$	**5**	$y = -3$	**7**	$y = 1$
2	$x = -1$	**4**	$x = 3$	**6**	$y = -1$	**8**	$y = 3$

54. The graph of the function $f(x) = \dfrac{3x^2 - 11x - 4}{(x-4)(x+4)}$ has a point of discontinuity at (m, n).

The value of n is

A. –11

B. 0

C. $\dfrac{11}{8}$

D. $\dfrac{13}{8}$

55. Determine the coordinates of the point of discontinuity on the graph of $f(x) = \frac{2x^2 - 15x + 7}{x - 7}$.

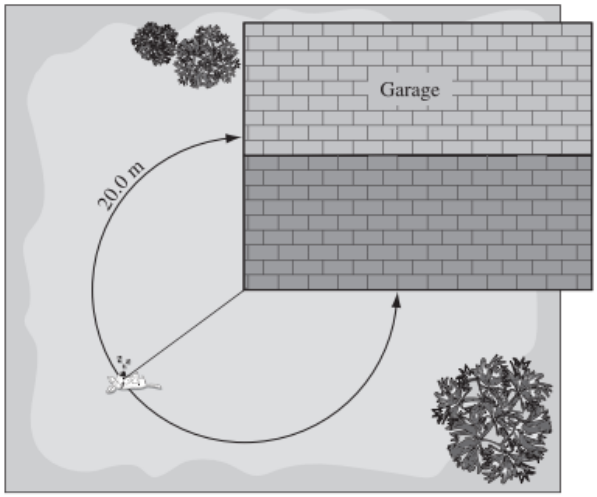
Written Response

56. For a function $f(x) = \frac{(3x - 2)(2x + 3)}{(1 - x)(3x - 2)}$ the vertical and horizontal asymptotes are given in row:

ROW	Vertical Asym.	Horizontal Asym.
A.	$x = 2/3$	$y = -2$
B.	$x = 1$	$y = -3$
C.	$x = 2/3$	$y = -3$
D.	$x = 1$	$y = -2$

Unit 6 – Trigonometry ◅●☑✔❖♥

A dog is tied to the corner of a rectangular garage. He is given enough leash to run along a 20.0 m circular path, completing $\frac{3}{4}$ of a circle, as shown in the diagram below.



Numerical Response

57. The length of the dog's leash, to the nearest tenth of a metre, is _____ m.

58. An angle, in radians, that is co-terminal with 30° is

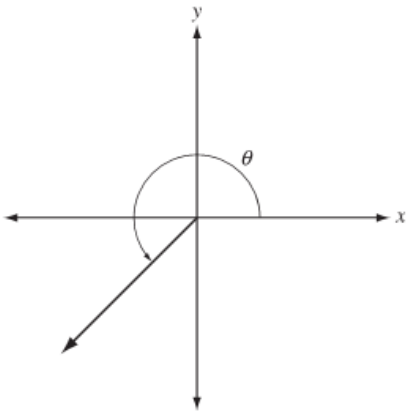
A. $-\frac{5\pi}{6}$

B. $-\frac{13\pi}{6}$

C. $\frac{7\pi}{6}$

D. $\frac{25\pi}{6}$

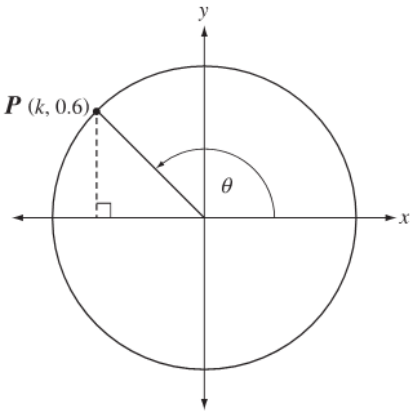
59. An angle, θ, in standard position, is shown below.



The best estimate of the rotation angle θ is

A. 1.25 radians
B. 3.12 radians
C. 4.01 radians
D. 5.38 radians

60. The unit circle is shown below with an angle, θ, in standard position. Point $P(k, 0.6)$ is on the terminal arm.



The value of k is ___**i**___ *, and the* ***exact*** *value of* $\sec\theta$ *is* ___**ii**___ .

The statement above is completed by the information in row

Row	*i*	*ii*
A.	−0.8	$-\frac{5}{4}$
B.	−0.8	$-\frac{5}{3}$
C.	0.8	$-\frac{5}{4}$
D.	0.8	$-\frac{5}{3}$

61. The terminal arm of an angle of 70° in standard position intersects the unit circle at the point $P(x, y)$. The coordinates of Point $P(x, y)$, rounded to the nearest hundredth, are $x = 0.ab$ and $y = 0.cd$.

Numerical Response

The value of x is 0._____ (Record in the **first and second** columns.)

The value of y is 0._____ (Record in the **third and fourth** columns.)

62. On a unit circle, Point $P\left(-\frac{5}{13}, \frac{12}{13}\right)$ lies on the terminal arm of angle θ in standard position.

What are the exact values of the 6 trigonometric ratios for angle θ?

63. If $\sin\theta = -2\cos\theta$ and $\frac{\pi}{2} \le \theta \le \pi$, then the **exact** value of $\tan(2\theta)$ is

A. $\frac{4}{3}$ **C.** $-\frac{4}{3}$

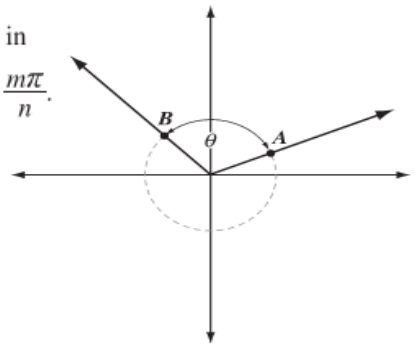
B. $\frac{4}{5}$ **D.** $-\frac{4}{5}$

64. The terminal arm of θ, when drawn in standard position, contains the point $P(x, y)$ where P is on the unit circle. If $\sin\theta = \frac{7}{10}$ and $\tan\theta < 0$, then what is the value of x?

A. $-\frac{\sqrt{51}}{10}$ **C.** $\frac{3}{10}$

B. $\frac{\sqrt{51}}{10}$ **D.** $-\frac{3}{10}$

65. Point $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and Point $B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lie on the terminal arm of two different angles in standard position on the circle. The angle, θ, where $0 < \theta < \pi$, can be expressed in the form $\frac{m\pi}{n}$.



The values of m and n are, respectively, _____ and _____.

Numerical Response

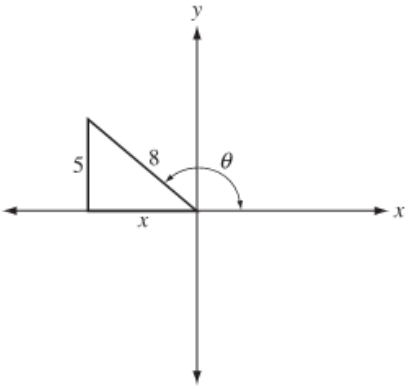
66. The **exact** value of $\sin\left(-\frac{\pi}{6}\right) + \cos\left(\frac{7\pi}{4}\right)$ is

A. $\frac{\sqrt{2}-1}{2}$ **C.** $\frac{\sqrt{3}+\sqrt{2}}{2}$

B. $\frac{\sqrt{2}+1}{2}$ **D.** $\frac{\sqrt{3}-\sqrt{2}}{2}$

67. Given that $\csc\theta = \frac{8}{5}$, where $\frac{\pi}{2} < \theta < \pi$, determine the **exact** value of $\tan\theta$.

Written Response



68. If $\cos(x - 20°) = \frac{1}{2}$, where $0° < x < 90°$, then the value of x, to the nearest degree, is _____°.

Use the following information to answer the next question.

69.

For the angles $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$, the following statements are given.

Statement 1 These angles in degrees are, respectively, 30°, 150°, 210°, and 300°.

Statement 2 They are all part of the solution set $\theta = \frac{\pi}{6} + 2n\pi, n \in I$.

Statement 3 The values of $\sin\left(\frac{7\pi}{6}\right)$ and $\cos\left(\frac{5\pi}{6}\right)$ are both negative.

Numerical Response

The statement that is true from the list above is number _____.

70. *If* $\csc\theta = \frac{2}{\sqrt{3}}$, *where* $0 \le \theta < 2\pi$, *then* θ *lies in Quadrant* ___***i***___, *and* $\cot\theta$ *is equal to* ___***ii***___.

ROW	*i*	*ii*
A.	I or II	$\pm\sqrt{3}$
B.	I or IV	$\pm\sqrt{3}$
C.	I or II	$\pm 1/\sqrt{3}$
D.	I or IV	$\pm 1/\sqrt{3}$

71. The partial graph of $f(x) = a\sin[b(x - c)] + d$, where $a > 0$ and x is in radians, is shown below. Two of its maximum points and two of its minimum points are labelled.

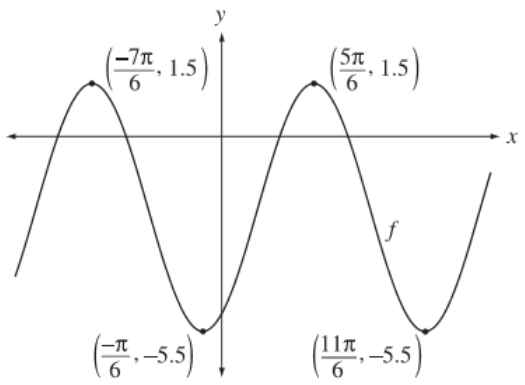
The minimum positive value of ***c***, to the nearest hundredth of a radian, is

A. 1.05

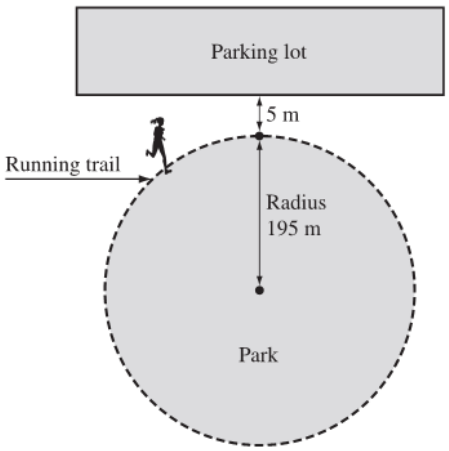
B. 1.57

C. 2.09

D. 2.62



72. A local park with a radius of 195 m has a circular running trail surrounding it, as shown below. The shortest distance from the running trail to the parking lot is 5 m. At a constant speed, Ellie can complete 4 full laps around the park in 32 minutes. Ellie's distance from the parking lot as she runs, s, in metres, as a function time, t, in minutes, can be represented by the function $s(t) = a\cos\left[b(t-c)\right] + d$.



The values of ***b*** and ***d*** in the function are, respectively,

A. $\frac{\pi}{16}$ and 195

B. $\frac{\pi}{16}$ and 200

C. $\frac{\pi}{4}$ and 195

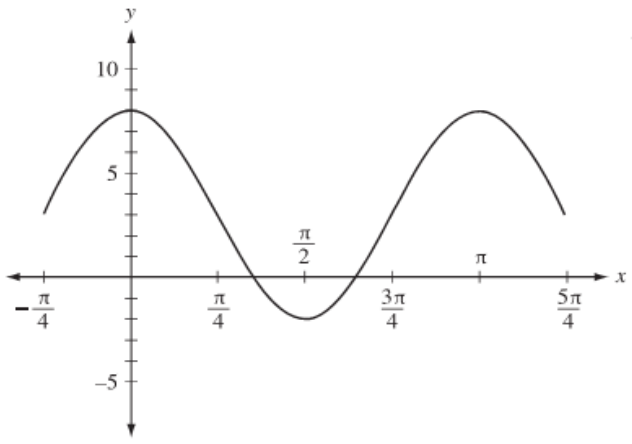
D. $\frac{\pi}{4}$ and 200

73. *A function is represented by the equation* $y = \sin(3x + \pi) + 7$. *The value of the phase shift is* ___***i***___ *and the period of the corresponding graph is* ___***ii***___ .

The statement above is completed by the information in row

Row	*i*	*ii*
A.	π	3
B.	π	$\frac{2\pi}{3}$
C.	$\frac{\pi}{3}$	$\frac{2\pi}{3}$
D.	$\frac{\pi}{3}$	3

74. The partial graph of the cosine function below has a minimum point at $\left(\frac{\pi}{2}, -2\right)$ and a maximum point at $(\pi, 8)$ as shown below. The equation of the function can be expressed in the form $y = a\cos\left(b(x-c)\right) + d,\ a, b, c, d \in W$.



Numerical Response

With a minimum possible phase shift, the values of ***a***, ***b***, ***c***, and ***d*** are, respectively, _____, _____, _____, and _____.

75. An incorrect solution to the equation $\cos\theta = \frac{\cos\theta}{\csc\theta}$, where $0° \le \theta < 360°$, is shown below.

The step that records the **first** mistake in the solution is

A. Step 2
B. Step 3
C. Step 4
D. Step 5

Step 1 $\cos\theta = \cos\theta\left(\frac{1}{\csc\theta}\right)$

Step 2 $1 = \frac{1}{\csc\theta}$

Step 3 $\csc\theta = 1$

Step 4 $\cos\theta = 1$

Step 5 $\theta = 0°, 180°$

76. The non-permissible values of θ for the identity $\frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$ are

A. $n\pi,\ n \in I$
B. $2n\pi,\ n \in I$
C. $\frac{\pi}{2} + n\pi,\ n \in I$
D. $\pi + 2n\pi,\ n \in I$

77. If the identity $\frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}$ is verified using $x = \frac{2\pi}{3}$, then the **exact** value of each side is

A. $-\sqrt{3}$
B. $\sqrt{3}$
C. $-\frac{\sqrt{3}}{3}$
D. $\frac{\sqrt{3}}{3}$

78. The steps shown below were used to determine the complete solution set to the equation $3\sin^2 x + \cos^2 x + 5\sin x - 4 = 0$, where $0 \le x < 2\pi$.

The **first** recorded error is in Step

A. 1
B. 2
C. 4
D. 6

Step 1 $2\sin^2 x + \sin^2 x + \cos^2 x + 5\sin x - 4 = 0$

Step 2 $2\sin^2 x + 1 + 5\sin x - 4 = 0$

Step 3 $2\sin^2 x + 5\sin x - 3 = 0$

Step 4 $(2\sin x - 1)(\sin x + 3) = 0$

Step 5 $\sin x = \frac{1}{2}$ or $\sin x = -3$

Step 6 $x = \left\{\frac{\pi}{6}\right\}$

79. The values of θ, where $180° \le \theta < 360°$, in the equation $2\cos^2\theta + \cos\theta = 0$, are

A. $\theta = 90°, 120°$

B. $\theta = 240°, 270°$

C. $\theta = 60°, 90°, 270°, 300°$

D. $\theta = 90°, 120°, 240°, 270°$

80. Determine a general solution of $\cot^2\theta - 1 = 0$, expressed in radians. **Written Response**

81. Determine the solution set for the equation $2\cos^2 x + \sin x - 1 = 0$, where $-\pi \le x \le \pi$. **Written Response**

82. *Use the following information to answer the next question.*

A Mathematics 30–1 class was asked to determine a general solution to the equation $2\sin\theta\cos\theta - \cos\theta = 0$, in degrees. The answers provided by four different students are shown below.

Student 1 $\theta = 60° + n(120°),\ n \in I$

Student 2 $\theta = 90° + n(360°),\ n \in I$, and $\theta = 30° + n(120°),\ n \in I$

Student 3 $\theta = n(180°),\ \theta = 60° + n(360°)$, and $\theta = 300° + n(360°),\ n \in I$

Student 4 $\theta = 90° + n(180°),\ \theta = 30° + n(360°)$, and $\theta = 150° + n(360°),\ n \in I$

The two students who provided a correct general solution are numbered

A. 1 and 3

B. 1 and 4

C. 2 and 3

D. 2 and 4

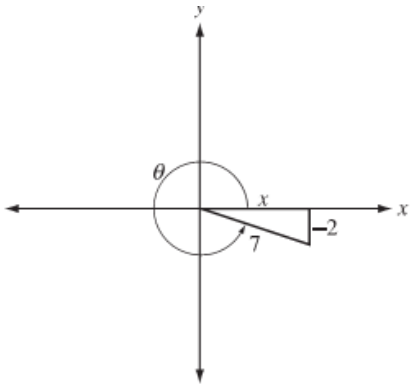
rtdmath.com

83. Graphically solve for θ, where $-180° \le \theta \le 0°$, given $(2 - \sqrt{3}\sec\theta)(\sec\theta + 3) = 0$. State answers to the nearest degree.

Written Response

84. Given that $\sin\theta = -\frac{2}{7}$ and $\cot\theta < 0$, determine the **exact** value of $\cos\left(\theta - \frac{2\pi}{3}\right)$.

Written Response



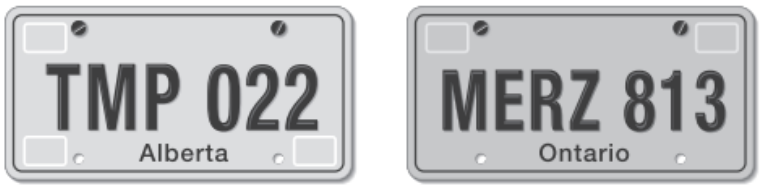
Unit 7 – Perms, Combs, and Binomial Theorem

85. A soccer team plays six games in one month. Each game results in a win, loss, or tie.

For this soccer team, how many different sets of results are possible for the six games?

A. 729 **C.** 216
B. 720 **D.** 64

86. Prior to 2010, standard licence plates in Alberta consisted of 3 letters followed by 3 digits, and standard Ontario licence plates consisted of 4 letters followed by 3 digits. Letters and digits can be repeated on licence plates. There are 23 letters in the alphabet that can be used, as the letters I, O, and Q are not allowed. All of the digits from 0 to 9 may be used. An example of each licence plate is shown below.



Numerical Response

In 2009, the number of standard licence plates possible in Ontario was ***r*** times greater than the number of standard licence plates possible in Alberta. The value of ***r*** as a whole number is _____.

87. The identification code on a bank card consists of 1 digit followed by 2 letters. The code must meet the following conditions:

- The digit must be odd.
- The letters A, E, I, O and U cannot be used.
- Letters cannot be used more than once.

The number of possible identification codes that can be created with these conditions is _____.

Numerical Response

88. A 6 person health council must be created from a group of 8 doctors and 40 nurses. The expression that represents the number of different councils that contain at most 2 doctors is

A. $({}_{40}P_6)+({}_8P_1)({}_{40}P_5)+({}_8P_2)({}_{40}P_4)$

B. $({}_{40}C_6)+({}_8C_1)({}_{40}C_5)+({}_8C_2)({}_{40}C_4)$

C. $({}_{40}P_5)({}_8P_1)+({}_{40}P_4)({}_8P_2)$

D. $({}_{40}C_5)({}_8C_1)+({}_{40}C_4)({}_8C_2)$

89. In a local high school, from a group of 16 students comprised of 7 singers and 9 actors, a 6-person executive council is selected.

How many different executive councils are possible if the council must include **at least** 4 actors?

A. 336

B. 2 646

C. 3 528

D. 3 612

90. The binomial $(2x+3)^{(a-5)}$ has 7 terms. The value of ***a*** is ___*i*___ and the coefficient on the first term is ___*ii*___ .

The statement above is completed by the information in row

ROW	*i*	*ii*
A.	11	64
B.	11	128
C.	12	64
D.	12	128

91. In the expansion of $\left(x^3+\frac{1}{2x^2}\right)^8$ written in descending powers of x, the sixth term is

A. $\frac{28}{x}$

B. $\frac{7}{4x}$

C. $\frac{1}{64x^6}$

D. $\frac{7}{16x^6}$

92. In the expansion of $(3a - b^2)^{10}$, what is the coefficient of the term containing a^4b^{12}?

A. −17 010 **C.** 630

B. −630 **D.** 17 010

93. In the expansion of the binomial $\left(2a + \frac{1}{a}\right)^8$, the constant term is _____.

94.

There are 6 terms in the expansion of $(x + 4)^n$.

The value of ***n*** *is* ___***i***___ *, and the value of the constant term in the expansion is* ___***ii***___ .

The statement above is completed by the information in row

Row	***i***	***ii***
A.	5	1 024
B.	5	4 096
C.	6	1 024
D.	6	4 096

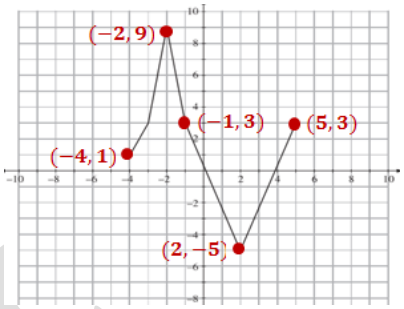
rtdmath.com

ANSWERS

1. C **2.** B **3.** B **4.** 4528 *or* 2845 **5.** 3547 **6.** 531 **7.** B

8. C **9.** D **10.** $g(x)=\left|\frac{1}{2}x\right|$ *or* $g(x)=\frac{1}{2}|x|$ **11.** B **12.** 247 **13.** ↓

14. 13 **15.** B **16.** 6 **17.** 2.44 **18.** *Go to website*



19. 21 **20.** 25 **21.** B **22.** $h=3x$ **23.** (4, 8)

24. 4 **25.** A **26.** C **27.** B **28.** B

29. B **30.** A **31.** C **32.** D **33.** 0.98 **34.** 3.33 **35.** $x=6$

36. C **37.** 0.42 **38.** $4742 **39.** 2.5 **40.** A **41.** B **42.** 17

43. 15 **44.** C **45.** 4 **46.** $P(x)=(x+1)(x-3)(2x+1)(x+3)$ **47.** **D:** $x\le -3$, or $x\ge 0$ **R:** $y\ge 0$

48. C **49.** 410 **50.** 6081 *or* 8160 **51.** 1.75 **52.** *See on website* **53.** 28 **54.** D

55. (7, 13) **56.** D **57.** 4.2 **58.** D **59.** C **60.** A **61.** 3494

62. $\sin\theta=\frac{12}{13};\cos\theta=-\frac{5}{13};\tan\theta=-\frac{12}{5};\csc\theta=\frac{13}{12};\sec\theta=-\frac{13}{5};\cot\theta=-\frac{5}{12}$ **63.** A **64.** A

65. $m=7, n=12$ **66.** A **67.** Since θ is a second quadrant angle, $x=-\sqrt{39}$.

Therefore, $\tan\theta=-\frac{5}{\sqrt{39}}$ or $\tan\theta=-\frac{5\sqrt{39}}{39}$. **68.** 80

69. 3 **70.** C **71.** A **72.** D

73. C **74.** 5203 **75.** A **76.** A **77.** D **78.** D **79.** B

80. $x=\frac{\pi}{4}+\frac{n\pi}{2};n\epsilon I$ **81.** $\frac{-5\pi}{6}, \frac{-\pi}{6}, \frac{\pi}{2}$ **82.** D **83.** $-109°$ and $-30°$ **84.** $\frac{-3\sqrt{5}-2\sqrt{3}}{14}$

85. A **86.** 23 **87.** 2100 **88.** B **89.** D **90.** A **91.** B

92 D **93.** 1120 **94.** A

www.rtomath.c

Math 30-1 Exemplars - SOLUTIONS

Question 1

$g(x)$ shifts $f(x)$ **4 right** and **3 up**

All pts $(x,y)\rightarrow(x+4,y+3)$

So...$(2,1)\rightarrow(6,4)$

Vertex of $f(x)$: $2+4$, $1+4$

Answer: **C**

Question 3

$y=f(x)$ is transformed to $y=\frac{1}{8}f(-x)$

Isolate y to see vert. stretch

$(-)$ is inside, so horiz. reflection

Answer: **B**

Question 5

The domain of $y=f(x)$ is $x\geq -2$

After a horiz. str. of 2, the domain of $y=p(x)$ is $\boldsymbol{x\geq -4}$ (All points move twice as far from the y-axis.)

The range of $y=f(x)$ is $y\geq 0$

A horiz.str. does not affect the range *(only x-coords get multiplied by 2)* So the range of $y=p(x)$ stays $\boldsymbol{y\geq 0}$.

The domain of $y=g(x)$ is $x\in R$

A horiz. str. will not change this, so, the domain of $y=q(x)$ is $\boldsymbol{x\in R}$.

The range of $y=g(x)$ is $y\geq -4$

A horiz.str. does not affect the range *(only x-coords get multiplied by 2)* So the range of $y=q(x)$ stays $\boldsymbol{y\geq -4}$.

Answer

3	5	4	7

Question 8

For the range of a graph we need to consider the lowest and highest points. For example $f(x)$ has a lowest point where $y=-4$, and a highest point where $y=8$, so its range is $[-4,8]$.

The lowest point on $(f-g)(x)$ occurs when the lowest value of f $(y=-4)$ is subtracted by the highest value on g, $(y=2)$ ***That is, the lowest pt is $y=-6$.***

The highest point occurs when the highest value of f $(y=8)$ is subtracted by the lowest value on g, $(y=-4)$ ***That is, the highest pt is $y=12$.***

Answer: **C**

Question 2

A, B, and C are all on the x-axis

So, transformation must be VERTICAL (stretch or reflection)

A is a horiz stretch – pts move $1/b$ times as far from y-axis. *Not invariant.*

B is a vertical stretch – all pts move a times as far from x-axis. (And A, B, C are all 0 units from it, so they don't move!) ***invariant.***

C is a vertical shift – all pts move k units up, including A, B, and C. *Not invariant.*

D is a horizontal shift – all pts move h units right (or left if h is negative), including A, B, and C. *Not invariant.*

Answer: **B**

Question 4

$f(x)$ has a vertex at $(2,3)$

$g(x)$ has a vertex at $(-2,1)$

$g(x)$ is 4 units left of $f(x)$, and 2 units down

$x+2$ inside gives horiz. shift

$+1$ outside gives vert. shift

From x-coord 2 to -2

From y-coord 3 to 1

Answer

4	5	2	8

Or...

2	8	4	5

Question 6

Reflection about $y=x$ gives the INVERSE function, where all pts $(x,y)\rightarrow(y,x)$

So, the point $A(3,-5)$ becomes $\boldsymbol{(-5,3)}$.

Reflection about $x=0$ (the y-axis) gives a horiz. refl., where $(x,y)\rightarrow(-x,y)$

So, the point $A(3,-5)$ becomes $\boldsymbol{(-3,-5)}$.

Reflection about $y=0$ (the x-axis) gives a vert. refl., where $(x,y)\rightarrow(x,-y)$

So, the point $A(3,-5)$ becomes $\boldsymbol{(3,5)}$.

Answer

5	3	1	

Question 7

If a graph is of a FUNCTION there will be 1 (and only 1) y value assigned for each x in the domain. (So graph 3 is NOT a function since when $x=0$ it can be seen that there are three different y values. Look up "*Vertical Line Test*" for more info.)

When graphing an inverse function, all pts $(x,y)\rightarrow(y,x)$. So for its inverse to be a function we are looking for graphs where each y in the range maps from 1 (and only 1) x. (Look up "*Horizontal Line Test for inverses*" for more info)

For graph 2 we can see that when $y=0$ there are 3 different x values, so the inverse of graph 2 will NOT be a function.

However for graphs 1 and 3 there are no scenarios where a particular y has more than one associated x. So the inverses of these graphs ARE FUNCTIONS! (even though graph 3 is not a function itself!)

Answer: **B**

Question 9

For the described transformations, all pts $(x,y)\rightarrow(4x,-\frac{1}{3}y)$ Note, we do NOT use the reciprocal of the horiz. str. factor unless we are writing an equation, in this case it would be $g(x)=\frac{1}{3}f(\frac{1}{4}x)$.

So, the point $(-3,6)$ becomes $\boldsymbol{(-12,2)}$

$4*(-3)$

$-\frac{1}{3}*6$

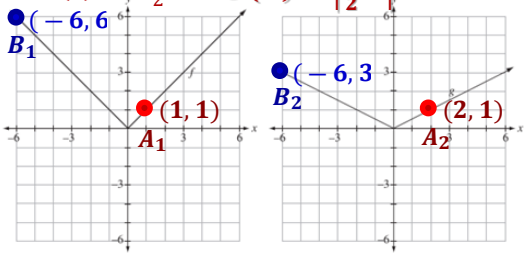
Answer: **D**

RD EXAM PREP

Question 10

Option 1 – Horizontal Stretch

Consider two points that line up horizontally – such as A_1 and A_2 as shown below. Point A_2 is two times as far from the y-axis as A_1. For a horiz. str. by factor of 2, the equation is $g(x) = f(\frac{1}{2}x)$ or $g(x) = |\frac{1}{2}x|$



Option 2 – Vertical Stretch

Consider two points that line up vertically – such as B_1 and B_2 as shown above. Point B_2 is half as far from the x-axis as B_1. For a vert. str. by factor of 1/2, the equation is $g(x) = \frac{1}{2}f(x)$ or $g(x) = \frac{1}{2}|x|$

Question 14

To determine an inverse function, we:

Re-write using y instead of $f(x)$

$y = 2x - 3$

Interchange x and y

$x = 2y - 3$

Isolate y

$x + 3 = 2y$ ➡ $\frac{2y}{2} = \frac{x}{2} + \frac{3}{2}$

➡ $g(x) = \frac{1}{2}x + \frac{3}{2}$ *Answer* 1 3

Question 16

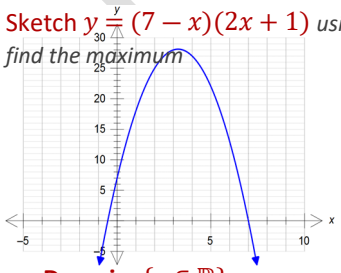
At $x = 3$, the value of f is 6, and the value of g is 1.

So, the value of $f(3)/g(3)$ is $\frac{6}{1} = 6$

Answer 6

Question 18

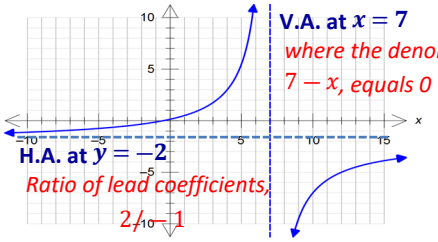
(a) Sketch $y = (7 - x)(2x + 1)$ *using calc, find the maximum*



Domain: $\{x \in \mathbb{R}\}$

Range $\{y \leq 28.125\}$

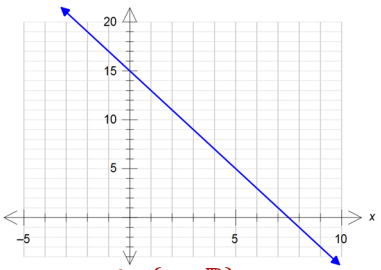
(b) Sketch $y = (2x + 1)/(7 - x)$ *using calc, recall that horiz. asymp. occurs at the ratio of the lead coefficients*



Domain: $\{x \neq 7\}$

Range $\{y \neq -2\}$

(c) Sketch $y = 2(7 - x) + 1$ *which simplifies to* $y = -2x + 15$



Domain: $\{x \in \mathbb{R}\}$

Range $\{y \in \mathbb{R}\}$

Question 11

Re-arrange: $g(x) = f(x + 5) + 2$, which shows a horiz. translation 5 units left and a vertical translation 2 units up.

Answer B

➡ All pts $(x, y) \rightarrow (x - 5, y + 2)$ So, $(3, 1) \rightarrow (-2, 3)$

Question 12

Re-arrange: $y = \sqrt{-\frac{1}{2}(x + 8)}$, which shows a horiz. reflection, a horiz. str. factor of 2, and a horiz. translation 8 units left.

You must factor the inside to get the hor. shift right!

Answer 2 4 7

**numbers can be in any order*

Question 13

➡ All pts $(x, y) \rightarrow (-\frac{1}{2}x, 2y + 3)$

So, $(-10, 0) \rightarrow (5, 3)$

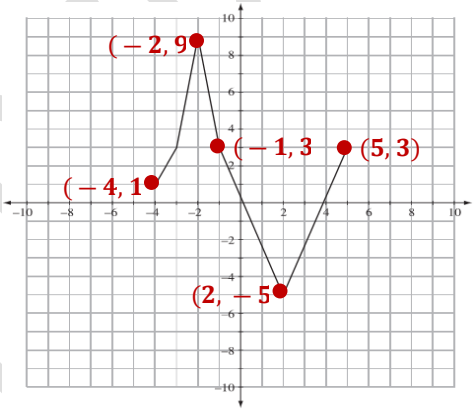
$-\frac{1}{2} * (-10)$ $\quad 2 * (0) + 3$

$(-4, -4) \rightarrow$

$(2, 0) \rightarrow (-1, 3)$

$(4, 3) \rightarrow (-2, 9)$

$(8, -1) \rightarrow (-4, 1)$

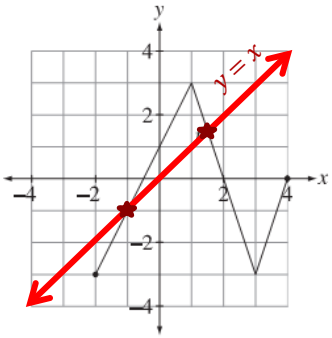


Question 15

When graphing an inverse function, all pts $(x,y) \rightarrow (y,x)$.

So, any invariant point would occur where the x and y coordinates are the same. *That is, on the line $y = x$*

To identify invariant pts, draw the line $y = x$ on top of the graph of f. *Answer* B



Question 17

At $x = 5$, the value of f is 55, and the value of g is $1/3$. ($(5)^2 + 6(5)$)

So, the value of $h(5) = g(5) + f(g(5))$

$= \frac{1}{3} + (\frac{1}{3})^2 + 6(\frac{1}{3})$

Answer 2 . 4 4

RTD EXAM PREP

Question 19

$k(x) = 2[(\sqrt{x-1})^2 + 3] - 5$ *simplifies to…*

$= 2(x - 1 + 3) - 5$

$= 2(x + 2) - 5$

$= 2x + 4 - 5$

$= \mathbf{2x - 1}$

Answer **2 1**

Question 21

From the pt $(3, 5)$

First note that $f(3) = 5$, then note that $g(5) = 8$

Answer **B**

Question 22

Simplify: $h(x) = \dfrac{x^2 - 7x}{x - 2} + \dfrac{2x^2 + x}{x - 2}$

$= \dfrac{x^2 - 7x + 2x^2 + x}{x - 2}$

$= \dfrac{3x^2 - 6x}{x - 2} \Rightarrow = \dfrac{3x(x - 2)}{x - 2} \Rightarrow = \mathbf{3x}$

Question 25

Reflection in the line $y = x$ means find the **inverse**.

$x = 3^{y+2}$ Switch x and y

$y + 2 = log_3(x)$ Convert to log form, and solve for y

$y = log_3(x) - 2$

Answer **A**

Question 26

$\dfrac{(5^3)^{x(x+1)}}{5^{3x-4}} = (5^2)^{x-5}$ Re-write all terms using a *common base*

$\dfrac{5^{3x^2+3x}}{5^{3x-4}} = 5^{2x-10}$ Apply exponent rules to simplify

$5^{(3x^2+3x)-(3x-4)} = 5^{2x-10}$

Set exponents equal

$3x^2 + 3x - 3x + 4 = 2x - 10$

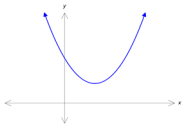
$3x^2 - 2x + \mathbf{14} = 0$

Answer **C**

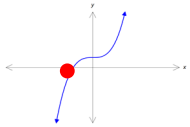
Question 20

The domain of a rational function will be $x \in \mathbb{R}$ if the "bottom" (denominator) can never be 0. Given that $b > 1$…

❷ $x^2 + b$ can never be 0



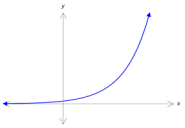
❸ $x^3 + b$ **CAN** be 0



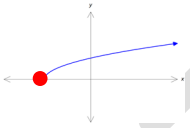
❹ $log_b x$ **CAN** be 0



❺ b^x can never be 0



❸ $\sqrt{x + b}$ **CAN** be 0



Answer **2 5**

Question 23

Given: $g(3) = 4$, and $f(g(3)) = 8$ ➡ So, $f(4) = 8$

But hey, this is, 4

So, the corresponding point is $(4, 8)$

Question 24

Simplify: $h(x) = \dfrac{(\sqrt{x-3})^2}{(\sqrt{x-3})^2 - 25}$

Can't √ negatives!

$= \dfrac{x - 3}{x - 3 - 25}$

$= \dfrac{x - 3}{x - 28}$

For the domain of $h(x)$, we must consider:

- Before simplifying, $h(x)$ had a radical component, so $x \geq 3$
- The domain of the simplified func. / can't divide by 0 **So, $x \neq 28$**

Combined, we get $x \geq 3, x \neq 28$

Answer **4**

Note, we do not need to consider the domain of $g(x)$ on its own, that is $x \neq \pm 5$, because the inputs for $g(x)$ are values of f, not "x".

Question 27

$(2^3)^{3x+4)} = (2^2)^{x-9}$ Re-write all terms using a *common base*

$2^{9x+12} = 2^{2x-18}$ Apply exponent rules to simplify

$9x + 12 = 2x - 18$ Set exponents equal

$7x = -30 \Rightarrow x = -\dfrac{30}{7}$

Answer **B**

Question 28

$m log_p n = q - 5 \Rightarrow log_p n^m = q - 5 \Rightarrow p^{q-5} = n^m$

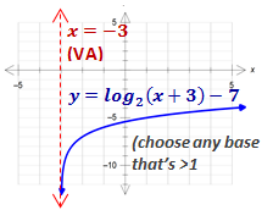
Isolate log term — Apply log law

Answer **B**

Question 29

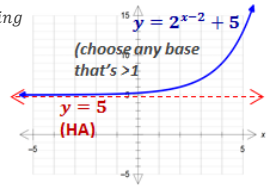
$f(x)$ has a V.A. at $x = -3$

Think: You can't log 0 or negatives, so the domain of f is $x + 3 > 0 \Rightarrow x > -3$



$g(x)$ has a H.A. at $y = 5$

Think: The power term $2^{anything}$ can never be 0 or a negative (must be +), so the +5 after means the range of g is $y > 5$

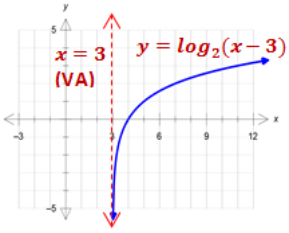


Answer **B**

Question 30

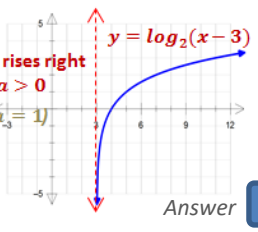
$h > 0$

Since the V.A. is positive (for example, it could be something like $x = 3$, we know that $h > 0$



$a < 0$

Since the graph "falls right", we know that $a < 0$.



Answer **A**

RTD EXAM PREP

Question 31

$L_{noise\ maker} = 127db$ (Given)

$L_{difference} = 10log5000$ (Difference between loudness of noise maker and lawn mower)

$\approx 164\ db$

So, loudness of the lawn mower is:

$\approx 164\ db - 127\ db$

$\approx$ **90 db**

Answer **C**

Question 32

$log\left(\frac{2sinx}{sin2x}\right)$

$= log\left(\frac{2sinx}{2sinxcosx}\right)$ Apply double angle identity for sin

$= log\left(\frac{1}{cosx}\right)$ Simplify / cancel

$= log1 - log\ (cosx)$ Apply log law

$= 0 - log\ (cosx)$ Answer **D**

Question 33

$log(3^{2x+1}) = log\left(\frac{1}{5}\right)^{x-3}$ "log both sides" to solve exp. eqn. where no common base is possible

$(2x+1)log(3) = (x-3)log\left(\frac{1}{5}\right)$

$2x(log3) + log3 = x(log0.2) - 3log\ (0.2)$

$log3 + 3log0.2 = x(log0.2) - 2x(log3)$

$x(log0.2 - 2log3) = log3 + 3log0.2$

$x = \frac{(log3 + 3log0.2)}{(log0.2 - 2log3)}$ $x \approx 0.98$

Question 34

$a^3 = 8$ $\quad 4^{3/2} = b$

Convert each to exp. form

$a = 2, b = 8$ Evaluate

$a = \sqrt[3]{8}$ $\quad b = \sqrt[2]{4^3}$

Now find:

$log_2 8 + log_8 2$

$= 3 + 1/3$

Since $2^3 = 8$ Since $8^{1/3} = 2$

Answer **3 . 3 3**

Question 35

$log_7[(x+1)(x-5)] = 1$ Combine to a single log

$7^1 = x^2 - 5x + 1x - 5$ Convert to exp. Form (and expand)

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$ Solve resulting quadratic equation

$x = 6$ or $x = -2$

Check each equation by subst. into the original equation

$log_7(6+1) + log_7(6-5) = 1$

$log_7(7) + log_7(1) = 1$

$1 + 0 = 1$ ✓

$log_7(-2+1) + log_7(-2-5) = 1$

$log_7(-1) + log_7(-7) = 1$ ✗ can't log negatives – sol. is **EXTRANEOUS**

➡ $x = 6$

Question 36

Use $y = ab^{\frac{t}{b}}$ (end amount; initial amount; mult. growth factor; ...in how long; period of time for growth factor (here "1"))

Solve for "b":

$32\ 450 = 15000b^8$

$2.16333 = b^8$ *Isolate the power term*

$b = (2.16333)^{\frac{1}{8}}$ *take the eighth root of both sides*

$b = 1.1012$ *THINK:* $b = 1 + growth\ rate$

growth rate is right here!

ANSWER: **10.1%** Answer **C**

Question 37

$log_a b^{\frac{1}{3}}$ *Simplify to isolate* $log_a b$

$= \frac{1}{3} log_a b$

Given: This is 1.26

$= \frac{1}{3}(1.26)$ Answer

≈ 0.42 **0 . 4 2**

Question 38

$y = ab^{\frac{t}{p}}$

$6000 = a(1.04)^{\frac{3}{0.5}}$

$a = \frac{6000}{(1.04)^6}$ $\quad a = \$4742$ *answer*

$y = 6000$ *(end amount)*

$a =$ ***wanted*** *(start amount)*

$b = 1 + \frac{0.08}{2}$ *(mult. growth factor)*

$t = 3$ *(how long)*

$p = 0.5$ *(since compounded semi-annually)*

Question 40

Since $P\left(-\frac{2}{3}\right) = 0$, one of the factors would be $(x + \frac{2}{3})$ OR $(3x+2)$

Mult both terms by 3

Since $P(0) = 12$ the constant term is 12. (For example, think of the function $y = x^3 - 3x + 12$)

Answer **B**

Question 39

Since -1 and -2 are zeros, $P(-1) = 0$ and $P(-2) =$

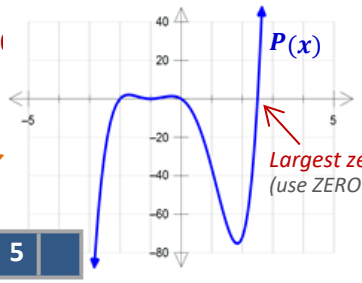
$2(-1)^5 + 3(-1)^4 - 10(-1)^3 - 21(-1)^2 + k(-1) = 0$ *Use either to solve for k*

$-2 + 3 + 10 - 21 - k = 0$

$-10 = k$

So, $P(x) = 2x^5 + 3x^4 - 10x^3 - 21x^2 - 10x$

GRAPH to see largest zero



Largest zero is 2.5 (use ZERO on calc!)

Answer **2 . 5**

Question 41

Since $P\left(-\frac{2}{3}\right) = 0$, one of the factors would be $(x + \frac{2}{3})$ OR $(3x+2)$

Mult both terms by 3

Since $P(0) = 12$ the constant term is 12. (For example, think of the function $y = x^3 - 3x + 12$)

Answer **B**

Question 42

Fully factor: **Step ❶** Potential zeros: $\pm 1, \pm 3$ **Step ❷** Test: $P(1) = 2(1)^3 - 3(1)^2 - 10(1) + 3$, which $\neq 0$ Similarly $P(-1) \neq 0$, but $P(3) = 2(1)^3 - 3(1)^2 - 10(1) + 3$, which $= 0$ So, $(x-3)$ is a factor. **Step ❸** Synthetic division to find $(2x^3 - 3x^2 - 10x + 3) \div (x-3)$

3	2	−3	−10	3
	↓	−6	−9	3
	2	3	−1	0

Step ❹ Synth division gives remaining factor of : $3x^2 - 10x + 3$, which doesn't factor. So use quad formula to find remaining roots.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ➡ $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$ ➡ $x = \frac{-3 \pm \sqrt{17}}{4}$

Answer **1 7**

RTD EXAM PREP

Question 43

Since the zeros of the function are –3, 1, and 5 (multiplicity 2), the equation that represents the function is $f(x)=\frac{-1}{a}(x+3)(x-1)(x-5)^2$. Use the y-intercept (0, 5) to find the leading coefficient.

$5=\frac{-1}{a}(0+3)(0-1)(0-5)^2$

$5=\frac{-1}{a}(-75)$

$-\frac{1}{15}=\frac{-1}{a}$

$\therefore f(x)=-\frac{1}{15}(x+3)(x-1)(x-5)^2$

Question 44

Since $x+2$ is a factor, the remaining factor can be found by dividing $P(x) \div (x+2)$

-2	4	−1	−8	0	−40
		−8	18	−20	40
	4	−9	10	−20	0

Remainder

Answer C

$4x^3-9x^2+10x-20$

Question 45

Since $x+2$ is a factor, $P(-2)=0$

$(-2)^3+3(-2)^2+k(-2)+4=0$

$-8+12-2k+4=0$

$8=2k$

$k=4$

Answer

4			

Question 46

$P(-1)=2(-1)^4+3(-1)^3-17(-1)^2-27(-1)-9=0 \rightarrow (x+1)$ is a factor

1	2	3	−17	−27	−9
	↓	2	1	−18	−9
	2	1	−18	−9	0

$Q(x)=2x^3+x^2-18x-9$

$Q(3)=2(3)^3+3^2-18(3)-9=0 \rightarrow (x-3)$ is a factor

−3	2	1	−18	−9
	↓	−6	−21	−9
	2	7	3	0

$2x^2+7x+3=(2x+1)(x+3)$

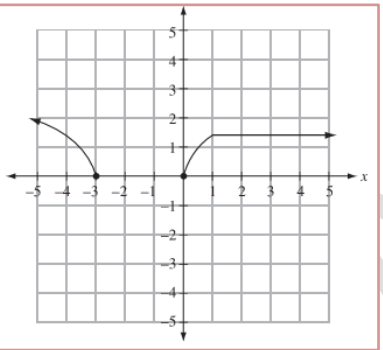
$\therefore P(x)=(x+1)(x-3)(2x+1)(x+3)$

www.rtdmath.com

Question 47

D: $(-\infty, -3]$ or $[0, \infty)$

R: $[0, \infty)$



Question 48

Point C$(4,\ 90)$ transforms to $(4,\sqrt{90})$

But $\sqrt{90} > 9$ (Max value of range)

So point C is not possible on $f(x)$

Answer C

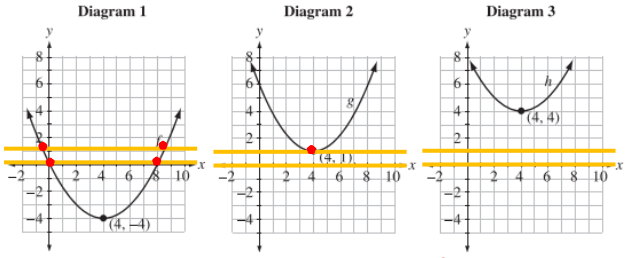
Question 49

Draw horizontal lines at $y=0$ and $y=1$

(Invariant points where $f(x)=0$ or 1)

Answer

4	1	0	



4 invariant points | 1 invariant point | 0 invariant points

Question 50

Invariant points where $f(x)=0$ or 1

$\frac{1}{2}x-3=0$

$\frac{1}{2}x=3$

$x=6$

Pt $(6, 0)$

$\frac{1}{2}x-3=1$

$\frac{1}{2}x=4$

$x=8$

Pt $(8, 1)$

Answer

6	0	8	1

OR

8	1	6	0

Question 51

For x-intercept set $y=0$

$0=-2\sqrt{x+4}+3$

$2\sqrt{x+4}=3$

$\sqrt{x+4}=3/2$ *Sq root both sides*

$x+4=9/4$

$x=-7/4$

Answer

1	.	7	5

Question 52

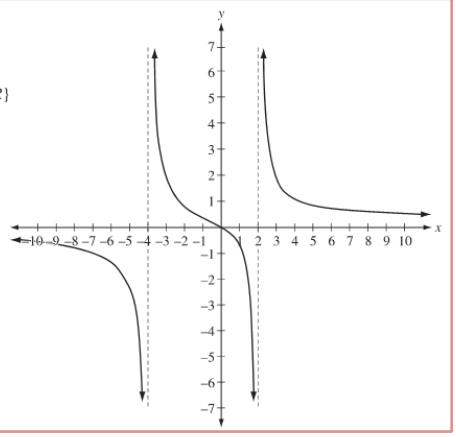
$y = \frac{3x}{(x+4)(x-2)}$

D:$\{x \mid x \neq -4, 2, x \in R\}$

x-intercept: 0

y-intercept: 0

Equation of vertical asymptotes: $x = -4$ and $x = 2$



$y = \frac{x+3}{x^2-9}$

Possible solution:

$y = \frac{x+3}{(x+3)(x-3)}$

$y = \frac{1}{x-3}$

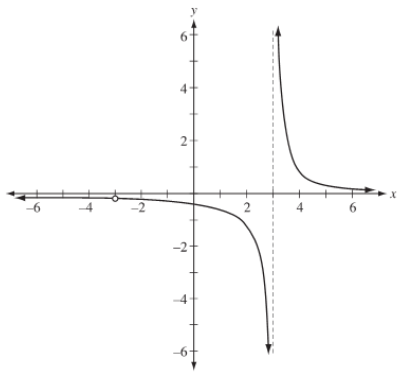
D:$\{x \mid x \neq \pm 3, x \in R\}$

No x-intercept

y-intercept: $-\frac{1}{3}$

Equation of vertical asymptote: $x = 3$

There is a point of discontinuity at $x = -3$.



Question 53

Consider graph of $y = \frac{1}{x}$ *V.A. at* $x = 0$ *H.A. at* $y = 0$

Shifted 1 unit left & 3 units up

V.A. at $x = -1$ H.A. at $y = 3$

Answer **2 8**

Question 54

$f(x) = \frac{(3x+1)(x-4)}{(x-4)(x+4)}$ *Factor*

$f(x) = \frac{(3x+1)}{(x+4)}$ P.D. at $x = 4$ (from factor that cancels)

$f(4) = \frac{3(4)+1}{(4)+4}$ subst.x-coord of PD into simplified form of $f(x)$

$= \frac{13}{8}$ *Answer* **D**

Question 55

There is a point of discontinuity when $x = 7$.

$\therefore f(x) = 2x - 1$, for $x \neq 7$

$y = 2(7) - 1$

$y = 13$

So the point of discontinuity is (7, 13).

Question 56

$f(x) = \frac{(3x-2)(2x+3)}{(1-x)(3x-2)}$ *Simplify*

$f(x) = \frac{2x+3}{1-x}$

V.A. at $x = 1$ For H.A., refer back to original function.

$f(x) = \frac{6x^2 \ldots}{-3x^2 \ldots}$ H.A. at ratio of lead coefficients. (Same degree top / bottom)

H.A. at $y = -2$ *Answer* **D**

Question 57

$\frac{3\pi}{2} = \frac{20.0m}{r}$ $\theta = \frac{3}{4} * 2\pi$

$\frac{3\pi}{2} r = 20.0m$

$r = \frac{20}{\frac{3\pi}{2}}$

$r \approx 4.2$

Answer **4 . 2**

Question 58

www.rtdmath.com

$\frac{\pi}{6} + 2\pi$ Add or subtract 1 rotation (2π) from 30°, that is $\frac{\pi}{6}$.

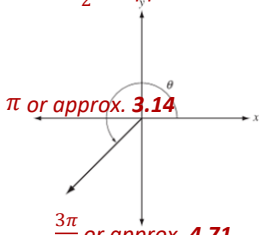
$= \frac{\pi}{6} + \frac{12\pi}{6}$

$= \frac{13\pi}{6}$ Add another rotation...

$\frac{13\pi}{6} + \frac{12\pi}{6} = \frac{25\pi}{6}$ *Answer* **D**

Question 59

$\frac{\pi}{2}$ *or approx.* ***1.57***



π *or approx.* ***3.14***

$\frac{3\pi}{2}$ *or approx.* ***4.71***

So, θ must be between 3.14 and 4.71

Answer **C**

Question 60

$x^2 + y^2 = 1$ Equation of Unit Circle

$(k)^2 + (0.6)^2 = 1$

$k^2 = 1 - 0.36$

$k^2 = 0.64$ Sq root both sides

$k = 0 \pm 0.8$ k is (-) since P is in quad II

$\boldsymbol{k = -0.8}$

$sec\theta = \frac{hyp}{adj}$

(since unit circle)

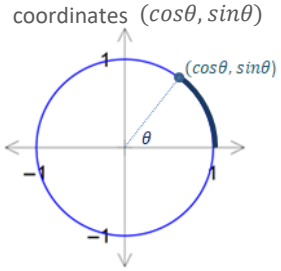
$sec\theta = \frac{1}{-0.8}$

$\boldsymbol{sec\theta = -\frac{5}{4}}$

Answer **A**

Question 61

Any pt on unit circle has coordinates $(cos\theta, sin\theta)$



$P(cos70°, sin70°)$

$P(0.34, 0.94)$

Answer

3 4 9 4

RTD EXAM PREP

Question 62

Any pt on unit circle has coordinates $(cos\theta, sin\theta)$

$\frac{12}{13} \div \frac{-5}{13}$ $\quad 1 \div \frac{12}{13}$ $\quad 1 \div \frac{-5}{13}$ $\quad 1 \div \frac{-12}{5}$

$$\sin\theta = \frac{12}{13};\ \cos\theta = -\frac{5}{13};\ \tan\theta = -\frac{12}{5};\ \csc\theta = \frac{13}{12};\ \sec\theta = -\frac{13}{5};\ \cot\theta = -\frac{5}{12}$$

Question 63

In Quad II, so tan is (-)

$$tan\theta = \frac{sin\theta}{cos\theta}$$

$$= \frac{-2\cancel{cos\theta}}{\cancel{cos\theta}}$$

$$= -2$$

Now,

$$tan2\theta = \frac{2tan\theta}{1 - tan^2\theta}$$

$$= \frac{2(-2)}{1-(-2)^2}$$

$$= \frac{-4}{-3}$$

Answer **A**

Question 64

sin is (+) and **tan** is (-), so we're in Quad II. *And x is (-)*

$x^2 + (7/10)^2 = 1$ Equation of unit circle

$x^2 = 1 - \frac{49}{100}$ $\quad x^2 = \sqrt{51/100}$

$x^2 = 51/100$ $\quad$ Answer **A**

Question 65

$\theta = \frac{3\pi}{4} - \frac{\pi}{6}$

$= \frac{9\pi}{12} - \frac{2\pi}{12}$

$= \frac{7\pi}{12}$

A *is at* $\frac{\pi}{6}$

B *is at* $\frac{3\pi}{4}$

Answer

7	1	2	

Question 66

$= sin\left(\frac{11\pi}{6}\right) + cos\left(\frac{7\pi}{4}\right)$ Answer **A**

coterminal

$= -\frac{1}{2} + \frac{\sqrt{2}}{2}$ $\quad = \frac{-1+\sqrt{2}}{2}$

Question 67

Since θ is a second quadrant angle, $x = -\sqrt{39}$.

Therefore, $\tan\theta = -\frac{5}{\sqrt{39}}$ or $\tan\theta = -\frac{5\sqrt{39}}{39}$.

Question 68

$x - 20° = cos^{-1}(1/2)$

$x - 20° = 60°$

$x° = 80°$ Answer

8	0		

Question 69

$\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$

30° $\quad$ 150° $\quad$ 210° $\quad$ 330°

*Statement **1** is false*

*Statement **2** is false*

(adding 2π to $\frac{\pi}{6}$ does not give the next angle, $\frac{5\pi}{6}$)

*Statement **3** is **true***

$sin\frac{7\pi}{6} = \frac{-1}{2}$ $\quad cos\frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$

Answer

3			

Question 70

If $csc\theta = 2/\sqrt{3}$, then $sin\theta = \sqrt{3}/2$

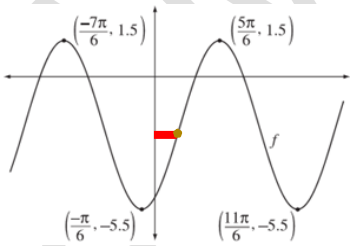
sin is (+) in I or II

$cot\theta = \frac{cos\theta}{sin\theta}$ From unit circle, when y-coord is $\sqrt{3}/2$

$= \frac{\pm 1/2}{\sqrt{3}/2}$ $\quad = \pm\frac{1}{2} \div \frac{2}{\sqrt{3}}$ $\quad = \pm\frac{1}{\sqrt{3}}$

Answer **C**

Question 71



c is midway between the min ($\frac{-\pi}{6}$) and the max ($\frac{5\pi}{6}$)

$c = \frac{-\frac{\pi}{6} + \frac{5\pi}{6}}{2}$ $\quad c = \approx 1.05$

Answer **A**

Question 72

$b = \frac{2\pi}{period}$

$d =$ *median dist from P lot*

$b = \frac{2\pi}{8}$ *4 laps in 32 mins, so 1 lap in 8mins*

$d = 5m + 195m$

$d = 200m$

$b = \frac{\pi}{4}$ $\quad$ Answer **D**

Question 73

$y = sin\left[3\left(x + \frac{\pi}{3}\right)\right] + 7$

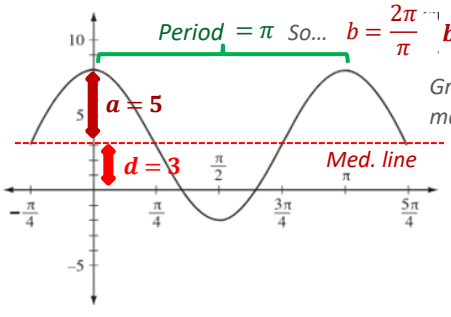
Factor out the b value to see the phase shift

phase shift $= \frac{\pi}{3}$

$period = \frac{2\pi}{b}$

$\boldsymbol{period = \frac{2\pi}{3}}$ Answer **C**

Question 74



Period $= \pi$ *So...* $b = \frac{2\pi}{\pi}$ $\boldsymbol{b = 2}$

Graph "starts" at its max, so $\boldsymbol{c = 0}$

Answer

5	2	0	3

Question 75

*In **step 2** both sides are divided by "$cos\theta$"*

*This is **not allowed** as it's deletes solutions*

Answer **A**

Question 76

NPVs where we'd divide by 0

$\underbrace{1 - cos^2\theta}_{} \neq 0$ $\quad sin^2\theta \neq 0$

Identity: This is also $sin^2\theta$

$sin^2\theta \neq 0$ *Sq root both sides*

$sin\theta \neq 0$

$\theta \neq 0, \pi, 2\pi,$ *etc*

Answer **A**

RTD EXAM PREP

www.rtdmath.com

Question 77

Left Side:

$$= \frac{\sin\left(\frac{2\pi}{3}\right)}{1-\cos\left(\frac{2\pi}{3}\right)}$$

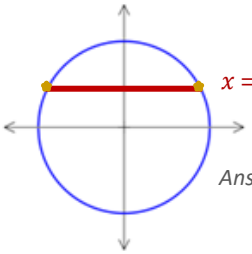
$$= \frac{\frac{\sqrt{3}}{2}}{1-\left(-\frac{1}{2}\right)}$$

Answer **D**

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} \Rightarrow = \frac{\sqrt{3}}{2} * \frac{2}{3} \Rightarrow = \frac{\sqrt{3}}{3}$$

Question 78

*All is good to step 6. There are no solutions for $\sin x = -3$, but $\sin = \frac{1}{2}$ has **two** solutions*



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Answer **D**

Question 79

$$2\cos\theta(\cos\theta + 1) = 0$$

$\cos\theta = 0$ $\quad$ $\cos\theta = -1$

$\theta = 90°, 270°$ $\quad$ $\theta = 180°$

Note: $90°$ is not within given solution domain

$\theta = 180°$ *or* $270°$

Question 80

www.rtdmath.com

$\cot^2\theta = 1$

$\cot\theta = \pm 1$

$\tan\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, etc.

Therefore, the general solution is $\theta = \frac{\pi}{4} + \frac{n\pi}{2}, n \in I$.

Question 81

$2\cos^2 x + \sin x - 1 = 0$

$2(1 - \sin^2 x) + \sin x - 1 = 0$

$2 - 2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = 1$

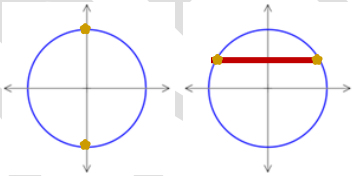
$\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}\right\}$

Question 82

Factor to solve:

$\cos\theta(2\sin\theta - 1) = 0$

$\cos\theta = 0$ $\quad$ $\sin\theta = 1/2$



$\theta = 90°, 270°$ $\quad$ $\theta = 30°, 150°$

General Solution options:

$90°$ *then every* $180°$ $\quad$ $90°$ *then every* $360°$

$30°$ *then every* $360°$ $\quad$ $30°$ *then every* $120°$

$150°$ *then every* $360°$

Answer **D**

Question 83

$\theta = -109°$ and $-30°$

Enter the following function into the calculator.

$$y_1 = \left(2 - \frac{\sqrt{3}}{\cos x}\right)\left(\frac{1}{\cos x} + 3\right)$$

A window that could be used is x: [–180, 0, 30], y: [–5, 5, 1].

The x-intercepts are the solutions to the original equation.

Question 84

www.rtdmath.com

$x^2 + (-2)^2 = 7^2$

$x^2 = \sqrt{45}$

$x = \pm 3\sqrt{5}$

Since θ is in Quadrant IV, $x = 3\sqrt{5}$ and $\cos\theta = \frac{3\sqrt{5}}{7}$.

$$\cos\left(\theta - \frac{2\pi}{3}\right) = \cos\theta\cos\frac{2\pi}{3} + \sin\theta\sin\frac{2\pi}{3}$$

$$= \left(\frac{3\sqrt{5}}{7}\right)\left(-\frac{1}{2}\right) + \left(-\frac{2}{7}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-3\sqrt{5}}{14} - \frac{2\sqrt{3}}{14}$$

$$= \frac{-3\sqrt{5} - 2\sqrt{3}}{14}$$

Question 85

$= \underbrace{3 * 3 * 3 * 3 * 3 * 3}_{\text{Six games}}$

$= 3^6$ Answer **A**

Question 86

Alberta:

$\underbrace{23 * 23 * 23}_{\text{Letters}} * \underbrace{10 * 10 * 10}_{\text{Digits}}$

Ontario:

$\underbrace{23 * 23 * 23 * 23}_{\text{Letters}} * \underbrace{10 * 10 * 10}_{\text{Digits}}$

Answer

2	3		

(Ontario has one extra "23")

Question 87

$\underbrace{5}_{\text{1, 3, 5, 7, or 9}} * \underbrace{21}_{\text{26 letters – 5 (vowels)}} * \underbrace{20}_{\text{26 letters – 5 (vowels) –1 (no repetition)}}$

Answer

2	1	0	0

Question 88

*No arranging / designating positions, so it's a **COMB***

*"At most" two doctors means **0** or **1** or **2** doctors. (Three cases)*

Answer **B**

Question 89

4 actors or 5 actors or 6 actors… (3 cases)

$= {}_9C_4 * {}_7C_2 + {}_9C_5 * {}_7C_1 + {}_9C_6 \quad = 3612$

Answer **D**

RTD EXAMPREP

Question 90

# of terms = 1 + degree

$a - 5 = 6$ $\quad$ $\boldsymbol{a = 11}$

$(2x + 3)^6$ *First term has coefficient:*

${}_6C_0 * 2^6$

coeff = 64

Answer **A**

Question 91

$$t_{5+1} = {}_8C_5 * (x^3)^{8-5}\left(\frac{1}{2x^2}\right)^5$$

$$t_6 = 56(x^3)^3\left(\frac{1}{2x^2}\right)^5$$

$$= 56 * x^9 * \frac{1}{32x^{10}}$$

$$= \frac{56x^9}{32x^{10}}$$

$$= \frac{7}{4x}$$

Answer **B**

Question 92

On formula, coeff. of "x" (first term) is $n - k$. Since we need power of first term to be 4, and $n = 10$, that means $k = 6$.

$$t_{6+1} = {}_{10}C_6(3a)^{10-6}(-b^2)^6$$

$$= 210 * 81a^4 * (b^{12})$$

$$= 17010a^4b^{12}$$

Answer **D**

Question 93

The exponent of the variable for a constant term must be zero; i.e., a^0.

${}_8C_4(2a)^4\left(\frac{1}{a}\right)^4$

$70(16a^4)\left(\frac{1}{a^4}\right)$

1 120

Therefore, the constant term is 1 120.

Question 94

"n" is one less than the # of terms

OR: $\#\ of\ terms = n + 1$

$\boldsymbol{n = 5}$

Constant term is LAST TERM in expansion:

$$t_{5+1} = {}_5C_5 * x^0 * 4^5$$

$$t_6 = 1024$$

Answer **A**

RTD EXAM PREP