Chapter 8 – Permutations and Combinations REVIEW

1. A grade 12 student is taking Biology, Math, Phys Ed, and Chemistry in her first semester. There are five periods in the day, meaning that she'll have a spare during one of them. One possible timetable is shown on the right.

Block	Course
1	Math 30-1
2	Biology 30
3	-Spare-
4	Phys-Ed 30
5	Chemistry 30

Each course if offered during every block.

Determine the number of ways the student can make her schedule if:

(a) She wishes to take either Math 30-1 or Chemistry during block 1.

(b) She wishes to take either Math 30-1 or Chemistry during block 1, and she wants to take her spare during block 3 or block 4. *Hint: Consider two cases.*

2. A couple is planning an evening at home and have three decisions to make. For their dinner they can order for delivery, order for takeout, or cook themselves. With their kids, they can either include them or send them to their grandparents for the night. And for their movie-watching, they can choose any two movies from a list of 7 still on their "must watch list".

Determine the number of ways the couple can make these three decisions.

3. In a particular jurisdiction the license plates consist of any three letters, excluding the vowels A, E, I, O, or U, followed by any three non-repeating digits, the first of which cannot be 0 or 1, followed by one of the letters X, Y, or Z.
One such plate is shown on the right.



(a) Determine the number of license plates that can be made that follow the criteria.

(b) Determine how many license plates subject to the given conditions do not begin with the letter B.

(c) Determine how many license plates subject to the given conditions do not include the letter C in any position.

4. A hockey team plays 7 games in a round-robin schedule. Each game can end in a win (W), loss (L), or overtime loss (O).

(a) Determine the number possible sets of results for these 7 games.

(b) In last year's tournament the team went 4-2-1. (4 wins, 2 losses, 1 overtime) Determine the number of ways that record could have played out.
For example, one way for this to have happened is they won the first four, lost the next two, and overtime lost the last game.

(c) In an earlier practice game, they team won by a final score of 3-2. Determine the number of possible scores after one period.

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5. Using all the letters in the word **MATHPOWER**, determine the number of arrangements that can be made: Reset for each scenario described below – *treat separately*

(a) That begin with the letters **MA**, in that order

(b) That begin with three vowels

(c) Where all of the vowels are together and in alphabetical order

(d) Where all of the vowels are together

(e) Where all the vowels are together, and all the consonants are together.

6. Using all the letters in the Italian word **PASSEGGIATA**, determine the number of arrangements that can be made: Reset for each scenario described below – *treat separately*

(a) That begin with PT, in that order

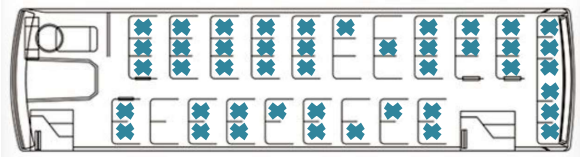
(b) That begin with three As

(c) That begin with at least one A

(d) Where all the vowels are together

(e) Where all the vowels are together, and all the consonants are together.

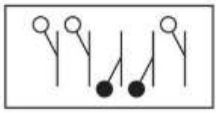
7. The diagram on the right shows the seating plan for a bus. Seats marked with an **x** are reserved, those with no **x** are open. There are 10 seats currently open. 6 passengers are still in line to book a seat.



(a) Determine the number of ways the remaining passengers can be assigned a seat.

(b) Determine the number of ways the remaining passengers can be assigned a seat if the first two passengers in line request to sit side-by-side.

8. A panel has five switches, each of which can be on (up) or off (down). On possible switch arrangement is shown on the right.



Determine the number of possible arrangement of the switches if:

(a) Three of the switches must be up, and two must be down.

(b) Any of the switches can be up or down.
Hint: There are multiple cases here – and many possible approaches!

Answers to Review Questions from Previous Page

1. (a) 48 (b) 24 **2.** 252 **3.** (a) 16 003 008 (b) 15 240 960 (c) 13 824 000
4. (a) 2187 (b) 105 (c) 12

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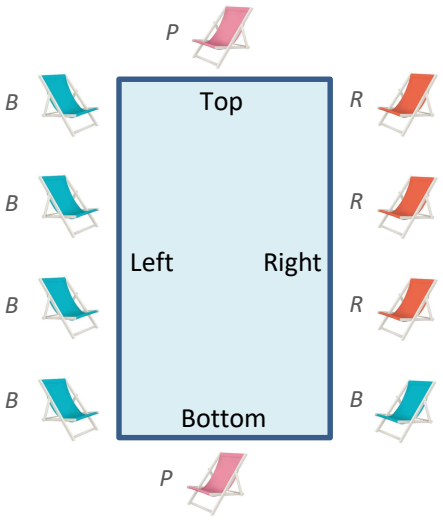
9. A pool has spots for 10 deck chairs that are identical except for colour. There are 5 blue chairs, 3 red chairs, and 2 pink chairs.

One possible arrangements of the chairs is shown on the right.



Determine the number of ways to arrange the deck chairs around the pool if: Reset for each scenario described below – *treat separately*

(a) There must be a pink chair at the top and bottom ends of the pool

(b) The left side must have only blue chairs

(c) The red chairs must be placed in any of the four spots on the right side of the pool.

10. A set of pool balls contains 9 balls that are identical except for colour; 4 red balls, 4 yellow balls, and 1 white ball. The balls come with a case with a spot for each ball, as shown below.



Determine the number of ways the balls can be arranged in the case if:

Reset for each scenario described below – *treat separately*

(a) There top row (spots 1, 2, and 3) must only contain red balls.

(b) The top row must contain balls of the same colour.

(c) The white ball must go in the front row. (In spot 7, 8, or 9)

Now consider a second set of balls where each ball is different. There are four red balls numbered 1 through 4, four yellow balls numbered 5 through 8, and a white ball that is not numbered.

How many arrangements of these new numbered balls can be made in the same case if:

(d) There top row must only contain red balls. (That is, only balls numbered 1 through 4)

(e) The top row must contain balls of the same colour.

Finally, consider a scenario where only the front row (spots 7, 8, and 9) is being used to arrange three of the balls from the second, numbered set.

(f) Determine the number of ways the front row can be arranged using one red ball, one yellow ball, and the white ball.

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5. (a) 5040 (b) 4320 (c) 5040 (d) 30 240 (e) 8640

6. (a) 15 120 (b) 10 080 (c) 453 600 (d) 25 200 (e) 7200 **7.** (a) 151 200 (b) 6720 **8.** (a) 10 (b) 32

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11. Three friends are going to a movie together with each friend brining a date. The friends each wish to sit next to their date. When they arrive at the theatre, they find a row of 7 consecutive empty seats.

The number of ways the friends can be arranged in these empty seats such that everyone sits side-by-side (there are no gaps), is given by the expression:

Exam Style

A. $2 \times 3!\,2!$

B. ${}_7C_6 \times 3!\,2!$

C. ${}_7P_6 \times 2!$

D. $\frac{6!}{3!} \times 2!$

12. A student is asked to consider the method that would be used to solve the following problems:

Exam Style

1 The number of ways the starting line of a hockey team can be set from 4 centres, 3 left wing, and 3 right wings.

2 The number of ways 2 boys and 3 girls can be chosen from a group of 10 boys and 6 girls.

3 The number triangles that can be drawn using any of the 5 points marked on the circumference of a circle.

4 The number of four-letter "words" that can be made using the letters in the word NUMERICAL

5 The number of ways to buy 3 cakes, from a selection of 7 available cakes.

6 The number of possible 3-person subcommittees that can be formed from a group of 6 people, if the subcommittee consists of a chair, vice-chair, and secretary.

NR The three problems whose solution involves using $\binom{n}{r}$ are numbered ___, ___, and ___.

13. A five-person intramurals dodgeball team is to be chosen from a group of 5 teachers and 6 students.

Determine the number of teams that can be made if:

Reset for each scenario described below – *treat separately*

(a) There must be exactly 2 teachers on the team

(b) There must be at least one teacher on the team

(c) The star student player, Rieker, must be on the team along with exactly one teacher.

(d) The star student player, Rieker, must be on the team along with at least one teacher.

(e) There can be any number of students or teachers, but the students Parvis and Sajedul cannot both be on the team.

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9. (a) 56 (b) 60 (c) 84 **10.** (a) 30 (b) 60 (c) 210 (d) 17 280 (e) 34 560 (f) 96

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13. For a team photo, 7 friends stand in a line. The photographer will place one of the friends, Jessica, at one of the two ends of the line and asks another one of the friends, Ramon, to stand in the middle.

The number of ways the friends can be placed for the photo is:

Exam Style

A. 120
B. 240
C. 360
D. 480

14. Students in a class must interview every other student and submit their interview results in a report. At the conclusion of the activity, 132 interviews took place.

Algebraically determine the number of students in the class.

15. Rebecca is preparing ingredients for a stew. She wishes to include 6 different vegetables from 9 available vegetable types.

Explain why the number of types of vegetables that can be selected for her stew can be found using $\binom{9}{3}$.

16. Exam Style

A teacher is making an intramural schedule for 18 floor-hockey teams. Teams are to be divided equally into 3 division, where each team will play every other team in its division twice. Teams will not play teams in any other division than their own.

NR The total number of games the teacher must schedule is _____.

17. In the expansion of $(3x + 2y^2)^6$, the coefficient of the third term is:

Exam Style

A. 324
B. 2430
C. 4320
D. 4860

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11. A **12.** 235 **13.** (a) 200 (b) 456 (c) 50 (d) 205 (e) 378

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18. One term in the expansion of $(4x^2 - 3)^6$ is $\boldsymbol{ax^4}$; where $a \in I$. Determine the value of a.

19. In the expansion of $(2x - \frac{1}{x^2})^{4a+2}$ there are 7 terms.

(a) Determine the value of a.

(b) Fully expand the binomial.

(c) State the value of the constant term.

20. One term in the expansion of $(2x + ay^2)^6$ is $-20\,000x^by^6$.
Determine the values of a and b.

Answers to Review Questions from Previous Page ***and this page***

13. B **14.** $n = 12$ **15.** $_9\boldsymbol{C_3}$ *(choosing 3 not to use)* ... is the same as $_9\boldsymbol{C_6}$ *(choosing 6 to use)*

16. $\boldsymbol{90}$ **17.** **D**

18. $a = 19\,440$ **19.** (a) $a = 1$ (b) $64x^6 - 192x^3 + 240 - \frac{160}{x^3} + \frac{60}{x^6} - \frac{12}{x^9} + \frac{1}{x^{12}}$ (c) 240 **20.** $a = -5, b = 3$

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