CHAPTER 7 REVIEW – Trig Identities and Equations

1. For each of the following trigonometric equations

i Determine any **non-permissible values**, in degrees.

ii **Numerically verify** that that the equation could be an identify, using $\theta = 30°$.

iii **Graphically verify** that that the equation could be an identify, over the domain $-180° \leq \theta < 360°$.

iv Use algebraic process to **prove** that the equation is an identity.

(a) $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

(b) $\dfrac{\csc\theta}{\tan\theta+\cot\theta} = \cos\theta$

(c) $\dfrac{\cos\theta+\cot\theta}{1+\sin\theta} = \cot\theta$

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2. If the identity $\sec\theta + \tan\theta = \dfrac{\cos\theta}{1-\sin\theta}$ is verified using $\theta = \dfrac{5\pi}{6}$, then the exact value of each side is:

A. $-\dfrac{\sqrt{3}}{3}$

B. $-\dfrac{2\sqrt{3}}{3}$

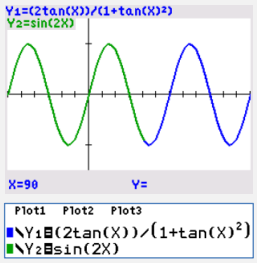
C. $-\sqrt{3}$

D. $-3\sqrt{3}$

Answers from previous page

1. (a) i N.P.V.s where $\cos\theta = 0$ ➧ $\boldsymbol{\theta \neq 90° + 180°n}$; $n \in I$

ii Substitute $30°$ into both sides, verify that each comes to $\sqrt{3}/2$.

iii 

Graphs look identical, so identity is **verified.**

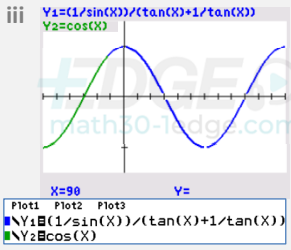
Note that at $x = 90°$, y_1 is not defined (Point Discontinuity), while y_2 is. So graphs are not actually identical!

iv

L.S.	R.S.
$\dfrac{2\tan\theta}{1+\tan^2\theta}$	$\sin 2\theta$
$= \dfrac{2\tan\theta}{\sec^2\theta}$	$= 2\sin\theta\cos\theta$ ✓
$= \dfrac{2\cdot\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$	
$= 2\cdot\dfrac{\sin\theta}{\cancel{\cos\theta}}\cdot\dfrac{\cancel{\cos^2}\theta}{1}$	L.S. = R.S. !
$= 2\sin\theta\cos\theta$ ✓	

(b) i N.P.V.s where $\cos\theta = 0, \sin\theta = 0$ ➧ $\boldsymbol{\theta \neq 90°n}$; $n \in I$

ii Substitute $30°$ into both sides, verify that each (again) comes to $\sqrt{3}/2$.

iii 

Graphs look identical, so identity is **verified.**

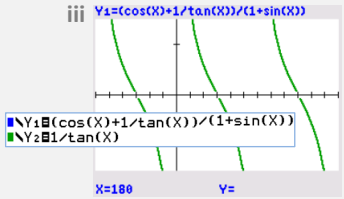
Note that at $x = 90°$ (and every $90°$ afterward) y_1 is not defined, whilst y_2 is)

iv

L.S.	R.S.
$\dfrac{\csc\theta}{\tan\theta + \cot\theta}$	$\cos\theta$
$= \dfrac{\frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}$	
$= \dfrac{\frac{1}{\sin\theta}}{\frac{\sin^2\theta}{\cos\theta\sin\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}}$	
$= \dfrac{1}{\sin\theta}\cdot\dfrac{\cos\theta\cancel{\sin\theta}}{\cancel{\sin^2\theta+\cos^2\theta}}$	L.S. = R.S. !
$= \cos\theta$ ⬅	

(c) i $\boldsymbol{\theta \neq 180°n, 270° + 360°n}$; $n \in I$

ii Substitute $30°$ into both sides, verify that each comes to $\sqrt{3}$.

iii 

Graphs look identical, so identity is **verified.**

Note that at $x = 180°$ (and afterward) y_1 is not defined, whilst y_2 is)

iv

L.S.	R.S.
$\dfrac{\cos\theta + \cot\theta}{1+\sin\theta}$	$\cot\theta$
$= \dfrac{\cos\theta + \frac{\cos\theta}{\sin\theta}}{1+\sin\theta}$	
$= \dfrac{\cos\theta\sin\theta + \cos\theta}{\sin\theta}\cdot\dfrac{1}{1+\sin\theta}$	
$= \dfrac{\cos\theta\cancel{(\sin\theta+1)}}{\sin\theta\cancel{(1+\sin\theta)}}$	L.S. = R.S. !
$= \cot\theta$ ⬅	

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Use the following information to answer the next question

Each of the following trig expressions can be simplified to a numerical value, where $cos\theta \neq 0$ and $sin\theta \neq 0$.

(a) $\dfrac{\sec x \sin x \cot x}{\tan x \csc x}$ (b) $\csc x - \cot x \cos x$ (c) $\dfrac{\sin 2x}{1 + \cos 2x}$

3. Exam Style **NR** Use the codes below to indicate the simplified form of each trigonometric expression:

The first expression, **(a)**, simplifies to code ____ (first digit of answer),

the second expression, **(b)** simplifies to code ____, (second digit)

and the third expression, **(c)**, simplifies to code ____. (third digit)

code	simplified form
1	$\cos x$
2	$\sin x$
3	$\sec x$
4	$\tan x$

4. The trigonometric expression $\dfrac{\sec^2 x}{\sec^2 x - 1}$, where $\sec x \neq \pm 1$, is equivalent to

Exam Style

A. $\sec^2 x$

B. $\csc^2 x$

C. $\cot^2 x$

D. $\tan^2 x$

5. The identity $\sec\theta + \tan\theta = \dfrac{\cos\theta}{1 - \sin\theta}$ has non-permissible values of

A. $\dfrac{\pi}{2} + \pi n\,; n \in I$

B. $\dfrac{\pi}{2} n\,; n \in I$

C. πn and $\dfrac{\pi}{2} + 2\pi n\,; n \in I$

D. πn and $\dfrac{\pi}{2} + 2\pi n\,; n \in I$

Answers from previous page

2. C

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6. Use an algebraic process to prove that each of the following trigonometric equations are identities.

(a) $\dfrac{\tan x + \sin x}{\cot x + \csc x} = \sin^2 x \sec x$

(b) $\dfrac{\cot x}{\csc x - 1} = \dfrac{\csc x + 1}{\cot x}$

(c) $\cot x - \cot x \cos 2x = \sin 2x$

(d) $\dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x} = 2 \tan x$

Answers from previous page

3. 124 **4.** B **5.** A

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7. **For each:** i - Use an algebraic process to prove that each of the following trigonometric equations are identities.
ii – Determine any non-permissible values, in radians.

(a) $\dfrac{1+\cot x}{\cot x} = \tan x + \csc^2 x - \cot^2 x$

(b) $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$

8. Use an applicable sum or difference formula to simplify $\sin(x+\frac{\pi}{2}) - \cos(x+\pi)$.

Answers from previous page

6. **(a)** Proceed on the L.S., a few steps in arrive at: $\dfrac{\sin x + \sin x \cos x}{\cos x} \cdot \dfrac{\sin x}{\cos x + 1}$ Factor / cancel from there to get to: $\dfrac{\sin^2 x}{\cos x}$ *Which is one step away from R.S. !*

(b) Option 1: Proceed on *both* the L.S. and R.S;, so that both become simplified expressions in terms of SIN and COS … then at that point multiply the top / bottom of either side by the conjugate

Option 2: (Shorter!) Proceed on either the L.S. or R.S., multiplying the top/ bottom by the conjugate as a first step. *One possibility, proceed on L.S.:* $\dfrac{\cot x}{\csc x - 1} \cdot \dfrac{\csc x + 1}{\csc x + 1}$

(c) Proceed on the L.S., **factor** and at the same time apply most suitable COS double-angle formula, to get to: $\dfrac{\cos x}{\sin x} \cdot 2\sin^2 x$ *Now cancel / simplify to equate to R.S.!*

(d) Proceed on the L.S., obtain a common denominator and then simplify to get to : $\dfrac{\cos x + \cos x \sin x - \cos x + \cos x \sin x}{\cos^2 x}$ *Then cancel / simplify to equate to R.S.!*

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9. Use an applicable sum or difference formula to determine the simplified exact value of each of the following.

(a) $\sec 105°$

(b) $\tan \frac{25\pi}{12}$

10. Given that $\cos\theta = -\frac{4}{5}$ and $\cot\theta < 0$, determine the exact value of:

(a) $\sin(\theta + \frac{5\pi}{6})$

(b) $\cos(\theta - \frac{3\pi}{4})$

(c) $\sin(2\theta)$

(d) $\cos(2\theta)$

Answers from previous page

7. **(a)** Work on both sides, a step working on the LS is: $\frac{\sin x + \cos x}{\sin x} \cdot \frac{\sin x}{\cos x}$ On RS, use an identity for "$\csc^2 x$", to get to: $\tan x + 1$ *L.S. is a few steps away from this!*

(b) Work on RS, to get to: $\frac{\cos x}{\sin x} \cdot \frac{\cos x \sin x}{\sin^2 x + \cos^2 x}$ Then apply Pyth. Identity, cancel out to get L.S.

7. ii $x \neq \frac{\pi}{2}n; n \in I$

8. $\cos x + \sin x$

ii $x \neq \frac{\pi}{2}n; n \in I$

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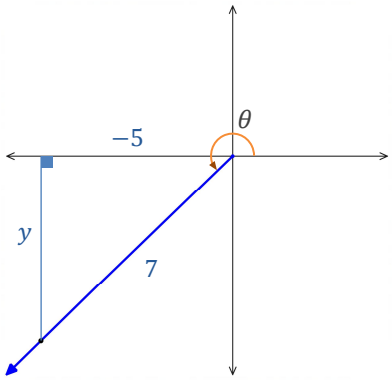
11. Given the diagram of an angle in standard position θ, determine the simplified exact value of:



(a) $\tan(\theta - \frac{3\pi}{4})$ (b) $\tan(2\theta)$

12. If $\cos\theta = -3\sin\theta$ and $\sec\theta > 0$ then the exact value of $\tan(2\theta)$ is

Exam Style

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $-\frac{4}{3}$

D. $-\frac{3}{4}$

13. Exam Style **NR** Given $\sin(\theta - \frac{\pi}{5}) = -\frac{\sqrt{3}}{2}$, where $\frac{3\pi}{2} < \theta < 2\pi$, the value of θ, correct to the nearest hundredth, is _____.

Answers from previous page

9. (a) $-\sqrt{2} - \sqrt{6}$ (b) $2 - \sqrt{3}$ Note: To arrive at this beauty answer you must continue simplifying / *rationalize* (use conjugate) when you get to: $\frac{3-\sqrt{3}}{3+\sqrt{3}}$

10. (a) $\frac{-3\sqrt{3}-4}{10}$ (b) $\frac{7\sqrt{2}}{10}$ (c) $-\frac{24}{25}$ (d) $\frac{7}{25}$

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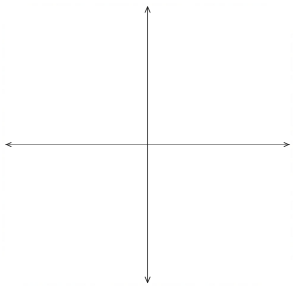
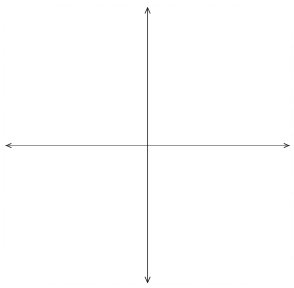
14. Given $\cos A = -\frac{7}{25}$, where $\cot A > 0$, and $\sin B = \frac{5}{13}$, where $\cot B < 0$, determine the exact value of:

(a) $\sin(A - B)$ **(b)** $\cos(2B)$ **(b)** $\tan(A + B)$





15. An expression equivalent to $\frac{1 - \cos(2x)}{1 + \cot^2 x}$, where $x \neq \pi n\,; n \in I$ is equivalent to:

Exam Style

A. $2\sin^2 x$

B. $2\sin^4 x$

C. $2\cot^2 x$

D. $2\cot^2 x$

16. If $\cos(2\theta) = \frac{4}{5}$ where $0 \leq \theta < \frac{\pi}{2}$, use an appropriate identity to determine the exact value of $\sin\theta$.

Answers from previous page

11. **(a)** $20\sqrt{6} + 49$ **(b)** $20\sqrt{6}$ **12.** *D* **13.** 5.86

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17. Given that $\cos\theta = -\frac{2}{3}$ where $\tan\theta > 0$, the exact value of $\sin(2\theta)$ is:

Exam Style

A. $\frac{2\sqrt{5}}{9}$

B. $-\frac{2\sqrt{5}}{9}$

C. $\frac{4\sqrt{5}}{9}$

D. $-\frac{4\sqrt{5}}{9}$

18. Given the equation $2\cos^2 x + \sin x - 1 = 0$ use an algebraic process to determine solutions;

(a) On the interval $[0, 2\pi)$

(b) on $[-\pi, \pi)$

(c) On $(-\infty, \infty)$

*That is, a **general solution***

19. Given the equation $(\tan x + \sqrt{3})(\sqrt{3}\sec x + 2) = 0$, use an algebraic process to determine;

(a) The solutions on the interval $[-\pi, 2\pi)$

(b) The general solution

Verify Graphically

Answers from previous page

14. **(a)** $\frac{323}{325}$ **(b)** $\frac{119}{169}$ **(c)** $\frac{253}{204}$ **15.** *B*

16. Ans: $\frac{\sqrt{10}}{10}$ *Solution:* Since $\cos 2\theta = 1 - 2\sin^2\theta$, solve $\frac{4}{5} = 1 - 2\sin^2\theta \rightarrow 2\sin^2\theta = \frac{1}{5} \rightarrow \sin^2\theta = \frac{1}{10}$

(square root both sides, rationalize denominator)

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20. Given the equation $\tan^2 x - \sec x - 5 = 0$ use an algebraic process to determine;
For any solutions that cannot be expressed as exact values in terms of π, round to the nearest hundredth

(a) Solutions on the interval $[0, 2\pi)$

(b) Solutions on $[-\pi, \pi)$

(c) The general solution

21. Given the equation $\cos x + 1 = \sin x$;

(a) Square both sides as the first step in a process to express the equation in terms of a single trigonometric function.

(b) Algebraically determine a general solution, in degrees.

(c) Verify your solutions graphically.
Are there any extraneous solutions?

Answers from previous page ***and this page***

17. C **18.** (a) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (b) $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}$ (c) $\frac{\pi}{2} + \frac{2\pi n}{3}; n \in I$

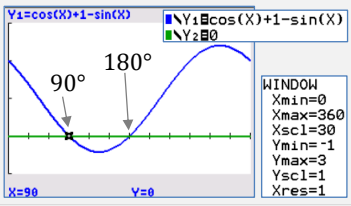
19. (a) $-\frac{5\pi}{6}, -\frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{3}$ (b) $\frac{2\pi}{3} + \pi n, \frac{5\pi}{6} + 2\pi n, \frac{7\pi}{6} + 2\pi n; n \in I$

20. (a) $1.23, \frac{2\pi}{3}, \frac{4\pi}{3}, 5.05$ (b) $-\frac{2\pi}{3}, -1.23, 1.23, \frac{2\pi}{3}$

(c) $1.23 + 2\pi n, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, 5.05 + 2\pi n; n \in I$

21. (a) $2\cos^2 x + 2\cos x = 0$ (b) $90° + 360°n, 180° + 360°n; n \in I$

21. (c) Graphing shows there are only two solutions, 270° is *extraneous*.



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