Chapter 6 – Trigonometric Functions REVIEW

1. An angle that is co-terminal with an angle of $-\frac{17\pi}{6}$ is:

Exam Style

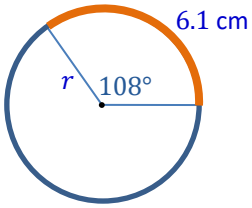
A. $\frac{\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

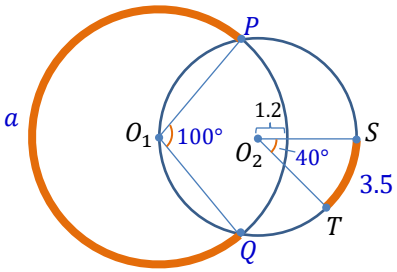
D. $\frac{11\pi}{6}$

2. Exam Style **NR** The radius of the circle shown, correct to the nearest tenth is _____ cm.



3. Exam Style **NR** In the diagram shown on the right, the centre of the larger circle, O_1 the centre of the smaller circle, O_2, and point S are colinear. The circles intersect at P and Q.

The length of the arc a, correct to the nearest tenth is ____.



4. The terminal arm of an angle in standard position θ passes through the point $P(-2,-3)$.

(a) **Sketch** to determine the exact value of all six trigonometric ratios of $\angle\theta$.
Be sure to label both the rotation angle θ and the reference angle α

(b) Determine the measure of θ, both in degrees *(round to the nearest whole number)* and radians *(Round to the nearest hundredth)*

Answers from previous page

8. B **9.** 92 **10.** C **11.** 4.9

Copyright © RTD Learning 2020 – all rights reserved

5. If $\boldsymbol{sec\,\theta = 3}$ and $\boldsymbol{cot < 0}$,

(a) **Sketch** a diagram of θ in standard position, to determine the exact value of **i** $csc\theta$ and **ii** $tan\theta$.

(b) Determine the measure of θ, both in degrees *(round to the nearest whole number)* and radians. *(Round to the nearest hundredth)*

6. Exam Style **NR** Given that $tan\,\theta = -3$ and $sec\,\theta < 0$, then the measure of θ, correct to the nearest hundredth of a radian is ____.

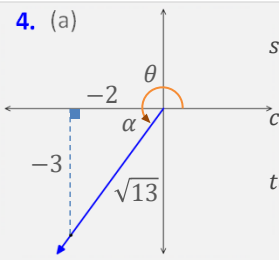
7. **i** For each of the following, determine the approximate measure of all angles that satisfy the equation on the given domain. ***Draw a diagram*** *for each, use the inverse trig functions to determine the reference angle. Provide solutions in the same angular unit as given in the question; for degrees, round to the nearest whole number - for radians, round to the nearest hundredth.*

ii State an expression(s) for all angles that satisfy the equation on $\{\theta \in \mathbb{R}\}$. *Use the same angular unit as given in the question*

(a) $sin\,\theta = -\frac{3}{4}$; $\{0° \le \theta < 360°\}$ (b) $cot\,\theta = \sqrt{5}$; $\{0 \le \theta < 2\pi\}$

Answers from previous page

1. C **2.** 3.2 **3.** 28.2 **4.** (a)



$sin\theta = \frac{-3\sqrt{13}}{13}$ $csc\theta = \frac{-\sqrt{13}}{3}$

$cos\theta = \frac{-2\sqrt{13}}{13}$ $sec\theta = \frac{-\sqrt{13}}{2}$

$tan\theta = \frac{3}{2}$ $cot\theta = \frac{2}{3}$

(b) $\theta \approx 236°$ or 4.12 rads

Copyright © RTD Learning 2020 – all rights reserved
EDGE math30-1edge.com

8. Exam Style

Given that $csc\ \theta = \frac{\sqrt{6}}{2}$ and $sec\ \theta < 0$, then the exact value of $tan\theta$ is:

A. $\frac{\sqrt{2}}{2}$

B. $\sqrt{2}$

C. $-\frac{\sqrt{2}}{2}$

D. $-\sqrt{2}$

9. Exam Style **NR**

Given that $log_3(\cot\theta) = 0.5$ and $\csc\theta < 0$, then measure of angle θ is _____ degrees.

10. Exam Style

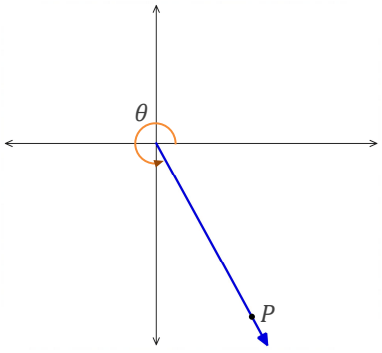
A point $P(n, -2n)$, shown below, lies on the terminal arm of an angle in standard position θ. The value of $sec\ \theta$ is:

A. $\sqrt{5}$

B. $\frac{\sqrt{5}}{5}$

C. $\frac{\sqrt{5}}{2}$

D. $\frac{2\sqrt{5}}{5}$



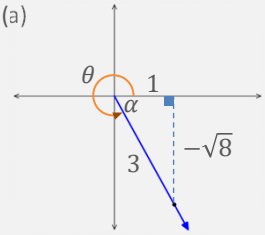
11. **NR**

The expression $sin^2(-\frac{5\pi}{6}) + \cos(-\frac{7\pi}{4}) + log_2(sec\frac{7\pi}{4})$ can be simplified to $\frac{2\sqrt{a}+b}{c}$.

The values of a, b, and c are, respectively, ____, ____, and ____.

Answers from previous page

5. (a)



i $csc\theta = \frac{-3\sqrt{8}}{8} = \frac{-3\sqrt{2}}{4}$

ii $tan\theta = -\sqrt{8} = -2\sqrt{2}$

(b) $\theta \approx 289°$ / 5.05 rads

6. 1.89

7. (a) i $\theta \approx 229°$ or $311°$

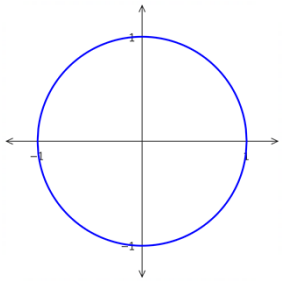
ii $\theta \approx 229° + 360°n$ or $311° + 360°n$ $n \in I$

(b) i $\theta \approx 0.42$ or 3.56

ii $\theta \approx 0.42 + 2\pi n$ or $3.56 + 2\pi n$; $n \in I$

Copyright © RTD Learning 2020 – all rights reserved

12. The terminal arm of an angle in standard position θ contains the point $P(x, y)$, where P is on the unit circle. If $sin\,\theta = -\frac{3}{10}$, then use the blank unit circle provided to:



(a) Determine the two possible values for x, expressed as exact values.

(b) Sketch and determine the two possible principal angle measures of θ, expressed to the nearest degree.

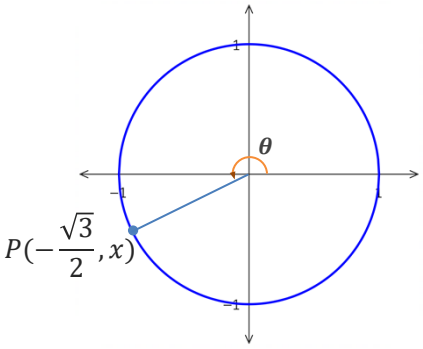
13. The terminal arm of an angle in standard position θ contains the point $P(\frac{\sqrt{5}}{3}, y)$, where P is on the unit circle.

(a) Determine the two possible values for $sin\,\theta$.

(b) Determine the largest possible principal angle value of θ. Round the nearest hundredth of a radian.

14. Point P intersects the unit circle and the terminal arm of angle θ in standard position, as shown on the right.

If the angle of rotation is changed to be become $\theta - \frac{11\pi}{6}$, then the coordinates of the point will be:



A. $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$ C. $(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$

B. $(\frac{1}{2}, -\frac{\sqrt{3}}{2})$ D. $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$

15. A point on the unit circle has coordinates $P(-\frac{5}{13}, m)$ and forms a principal angle in standard position, θ. If $tan\theta < 0$, then m is equal to:

A. $\frac{8}{13}$ C. $\frac{12}{13}$

B. $-\frac{8}{13}$ D. $-\frac{12}{13}$

Answers from previous page

8. *D* **9.** 210 **10.** *A* **11.** 234

Copyright © RTD Learning 2020 – all rights reserved

16. If the point $P(x, y)$ lies on the intersection of the unit circle and the terminal arm of $\theta = -\frac{49\pi}{5}$ in standard position, then the coordinates of P, to the nearest hundredth, are:

Exam Style

A. $(-0.81, 0.59)$
B. $(0.81, -0.59)$
C. $(-0.59, 0.81)$
D. $(0.59, -0.81)$

17. For each of the following, (i) Determine if the point could be on a unit circle, and if so (ii) determine the measure of the angle in standard position whose terminal arm passes through P. (round to the nearest degree)

(a) $P(-\frac{\sqrt{21}}{5}, \frac{2}{5})$

(b) $P(\frac{5}{6}, -\frac{\sqrt{10}}{6})$

18. For each of the following angles,
(i) State principal angle and the coordinates of the intersection point of the terminal arm and the unit circle
(ii) Determine the exact value of $cos\theta$, $sin\theta$, $tan\theta$, and $cot\theta$.

(a) $\theta = \frac{11\pi}{6}$

(b) $\theta = -585°$

(c) $\theta = \frac{11\pi}{2}$

(d) $\theta = -600°$

Answers from previous page

12. (a) $\pm\frac{\sqrt{91}}{10}$ (b) 197° or 343° **13.** (a) $\pm\frac{2}{3}$ (b) 5.55 **14.** ***D*** **15.** ***C***

Copyright © RTD Learning 2020 – all rights reserved

19. For each of the following trigonometric equations, use the unit circle to (i) solve each equation on the interval $[0, 360°)$ and (ii) state a **general solution** (in degrees) for all solutions on $\{\theta \in \mathbb{R}\}$.

(a) $cos\,\theta = -\frac{1}{2}$

(b) $sin\,\theta = 0$

(c) $csc\,\theta = -1$

(d) $sin\,\theta = \frac{\sqrt{2}}{2}$

(e) $cos\,\theta = \frac{\sqrt{3}}{2}$

20. The point $P(-\frac{5}{7}, y)$ lies on the intersection of the unit circle and the terminal arm of angle θ in standard position, with θ on the interval $[0, 2\pi)$.

The largest possible measure of θ, measured to the nearest hundredth of a radian, is:

Exam Style

A. 0.78
B. 2.37
C. 3.92
D. 5.51

Answers from previous page

16. ***B*** **17.** (a) $(\frac{\sqrt{21}}{5})^2+(\frac{2}{5})^2=1$ ✓ ii: $156°$ (b) $(\frac{5}{6})^2+(-\frac{\sqrt{10}}{6})^2 \neq 1$ ✗ *Not on the unit circle*

18. (a) i $\frac{11\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ ii: $cos\theta = \frac{\sqrt{3}}{2}$ $sin\theta = -\frac{1}{2}$ $tan\theta = -\frac{\sqrt{3}}{3}$ $cot\theta = -\sqrt{3}$

(b) i $135°$ $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ ii: $cos\theta = -\frac{\sqrt{2}}{2}$ $sin\theta = \frac{\sqrt{2}}{2}$ $tan\theta = -1$ $cot\theta = -1$

(c) i $\frac{3\pi}{2}$ $(0, -1)$ ii: $cos\theta = 0$ $sin\theta = -1$ $tan\theta =$ *undefined* $cot\theta = 0$

(d) i $120°$ $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ ii: $cos\theta = -\frac{1}{2}$ $sin\theta = \frac{\sqrt{3}}{2}$ $tan\theta = -\sqrt{3}$ $cot\theta = -\frac{\sqrt{3}}{3}$

Copyright © RTD Learning 2020 – all rights reserved

21. Evaluate each of the following. Explain how each value is obtained, with respect to the unit circle.

(a) $tan\frac{3\pi}{2}$ (b) $cot\frac{3\pi}{2}$ (c) $csc\frac{3\pi}{2}$ (d) $sec\frac{3\pi}{2}$

(e) $tan\,\pi$ (f) $cot\,\pi$ (g) $csc\,\pi$ (h) $sec\,\pi$

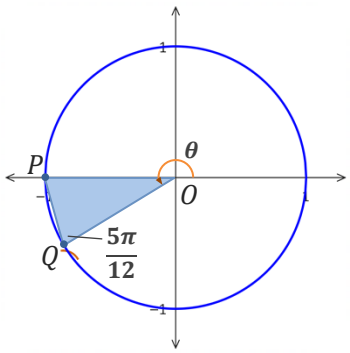
22. Exam Style **NR** If $sec\theta = 2.0245$, then the largest principal value of θ, to the nearest hundredth of a radian is ____.

23. Exam Style **NR** Point A $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$ and point B $(x, -\frac{\sqrt{2}}{2})$, where $x > 0$, lie on a unit circle. The length of the shortest arc that connects points A and B, correct to the nearest hundredth is _____.

24. In the diagram on the right, points P and Q are on a unit circle, with angle $OQP = \frac{5\pi}{12}$. Determine the:

(a) Measure of $\angle\theta$

(b) Length of the shortest **arc** connecting P and Q (*expressed as an exact value*).



Answers from previous page

19. (a) i $\theta = 120°, 240°$ ii $\theta = 120° + 360°n\,; n \in I$ or $240° + 360°n\,; n \in I$ (b) i $\theta = 0, 180°$ ii $\theta = 180°n\,; n \in I$

(c) i $\theta = 270°$ ii $\theta = 270° + 360°n\,; n \in I$ (d) i $\theta = 45°, 135°$ ii $\theta = 45° + 360°n\,; n \in I$ or $135° + 360°n\,; n \in I$

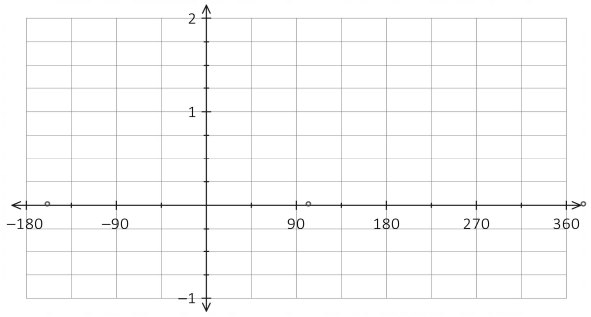
(e) i $\theta = 30°, 330°$ ii $\theta = 30° + 360°n\,; n \in I$ or $330° + 360°n\,; n \in I$

20. C

Copyright © RTD Learning 2020 – all rights reserved

25. Sketch each indicated function. State the amplitude, period, and range. *State the period using the same angular unit as shown in the graph.*

(a) $y = 1.25\sin x + 0.5$

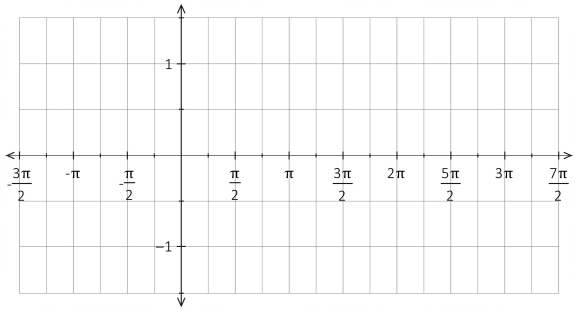


i *Amplitude:*

ii *Period:*

iii *Range:*

(b) $y = cos[\frac{2}{3}\left(x - \frac{\pi}{2}\right)]$



i *Amplitude:*

ii *Period:*

iii *Range:*

26. Without graphing, determine the indicated graph characteristics for each of the following sinusoidal functions. Provide the horizontal phase shift with respect to the basic graphs $y = \sin x$ or $y = \cos x$.

(a) $y = -3.25\sin(3x + 180°) + 1.55$

i *Range:*

ii *Period:*
in ***degrees***

iii *Horizontal*
Phase Shift:

(b) $y = 50\cos[\frac{5}{2}(x - \frac{7\pi}{2})] + 110$

i *Range:*

ii *Period:*
in ***radians***

iii *Horizontal*
Phase Shift:

Answers from previous page

21. (a) Coords on unit circle are $(0, -1)$
tan is $\frac{y}{x} \Rightarrow = \frac{-1}{0} \Rightarrow =$ ***undefined***

(b) cot is $\frac{x}{y} \Rightarrow = \frac{0}{-1} \Rightarrow = \mathbf{0}$

(c) csc is $\frac{1}{y} \Rightarrow = \frac{1}{-1} \Rightarrow = \mathbf{-1}$

(d) sec is $\frac{1}{x} \Rightarrow = \frac{1}{0} \Rightarrow =$ ***undefined***

(e) Coords on unit circle are $(-1, 0)$
tan is $\frac{y}{x} \Rightarrow = \frac{0}{-1} \Rightarrow = \mathbf{0}$

(f) cot is $\frac{x}{y} \Rightarrow = \frac{-1}{0} \Rightarrow =$ ***undefined***

(g) csc is $\frac{1}{y} \Rightarrow = \frac{1}{0} \Rightarrow =$ ***undefined***

(h) sec is $\frac{1}{x} \Rightarrow = \frac{1}{-1} \Rightarrow = \mathbf{-1}$

22. $\mathbf{5.23}$ **23.** $\mathbf{2.88}$

24. (a) Triangle POQ is isosceles, so $\angle QPO$ is also $\frac{5\pi}{12}$. So $\angle QOP = \pi - \frac{5\pi}{12} - \frac{5\pi}{12} = \frac{\pi}{6}$ So finally $\angle\theta = \pi + \frac{\pi}{6} = \mathbf{\frac{7\pi}{6}}$

(b) As we are on the unit circle, the arc length is the same as the angle, so $arc = \mathbf{\frac{\pi}{6}}$

Copyright © RTD Learning 2020 – all rights reserved
EDGE math30-1edge.com

27. Exam Style

The graph of a function $y = a\sin(bx) + d$ has a period of $225°$, a median line at $y = 2$, and the maximum value of the function is 5.

The value of b can be written as $\frac{m}{n}$.

NR The values of a, m, n and d are, respectively, ____, ____, ____, and ____.

28. Without graphing, state a general equation, in **radians**, for all x-intercepts ($x \in R$) of each function.

(a) $y = -3\sin(2x)$ (b) $y = \cos(\frac{3}{4}x)$ (c) $y = \sin[\frac{3}{2}(x - 30°)]$

29. Without graphing, determine the indicated graph characteristics for each of the following functions.
Express domain restrictions and period in radians.

(a) $y = \tan(x)$ (b) $y = 3\tan(2x)$ (c) $y = -\tan(\frac{2}{3}x)$

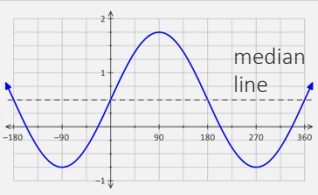
i *Domain:* i *Domain:* i *Domain:*

ii *Range:* ii *Range:* ii *Range:*

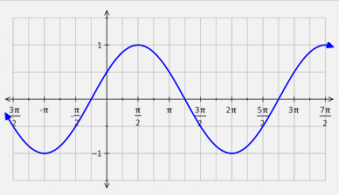
iii *x-intercepts:* iii *x-intercepts:* iii *x-intercepts:*

Answers from previous page

25. (a) i 1.25
ii $360°$
iii $[-0.75, 1.75]$



(b) i 1
ii 3π
iii $[1, 1]$



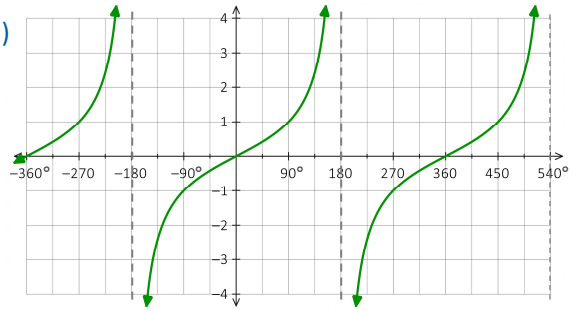
26. (a) i $[-1.7, 4.8]$ ii $120°$ iii $60°$ left (b) i $[60, 160]$ ii $\frac{4\pi}{5}$ iii $\frac{7\pi}{2}$ right

Copyright © RTD Learning 2020 – all rights reserved

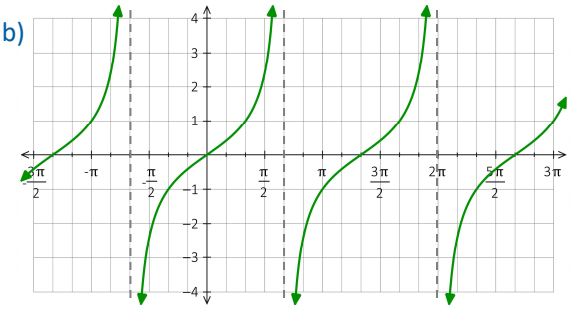
30. Describe where the graph of $y = tanx$ is undefined, using radian measure. Explain why the function is undefined where it is, with respect to the unit circle.

31. Given each of the following graphs, determine an equation in the form $y = \tan(bx)$.

(a)

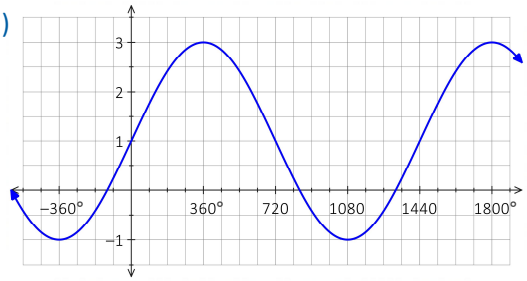


(b)

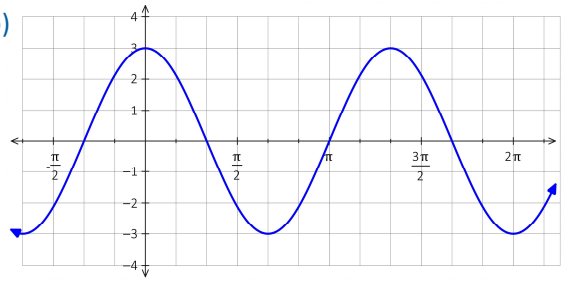


32. Given each of the following, determine an equation in the form $y = a\sin(bx) + d$ or $y = a\cos(bx) + d$.

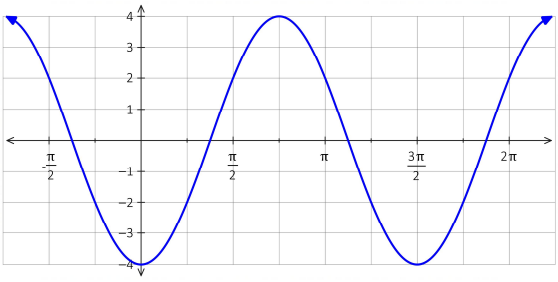
(a)



(b)



(c)



Answers from previous page

27. 3852 **28.** (a) $x = \frac{\pi}{2}n\,;n \in I$ (b) $x = \frac{2\pi}{3} + \frac{4\pi}{3}\,;n \in I$ (c) $x = \frac{\pi}{6} + \frac{2\pi}{3}n\,;n \in I$

29. (a) i $\{x \neq 90° + 180°n\,;n \in I\,, x \in \mathbb{R}\}$ ii $\{y \in \mathbb{R}\}$ iii $x = 180°n\,;n \in I$

(b) i $\{x \neq 45° + 90°n\,;n \in I\,, x \in \mathbb{R}\}$ ii $\{y \in \mathbb{R}\}$ iii $x = 90°n\,;n \in I$

(c) i $\{x \neq 135° + 270°n\,;n \in I\,, x \in \mathbb{R}\}$ ii $\{y \in \mathbb{R}\}$ iii $x = 270°n\,;n \in I$

Copyright © RTD Learning 2020 – all rights reserved
EDGE math30-1edge.com

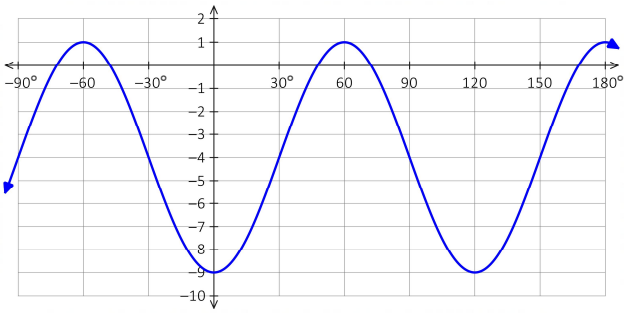
33. Determine an equation for each of the following sinusoidal functions, in the form $y = a\sin[b(x-c)] + d$ and $y = a\cos[b(x-c)] + d$ as described below:

(i) As sine equation of minimum phase shift, with $a > 0$

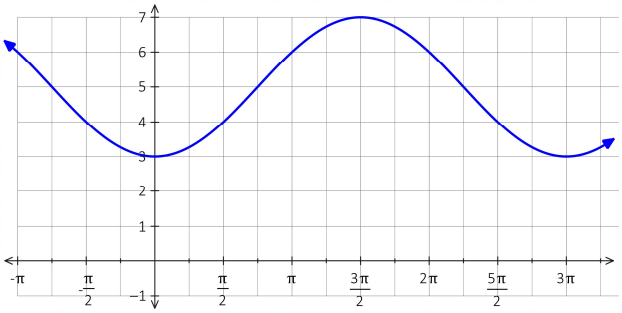
(ii) As cosine equation of minimum phase shift, with $a > 0$

(iii) As either a sine or cosine equation with $a < 0$, where the phase shift is made smaller by applying a vertical reflection.

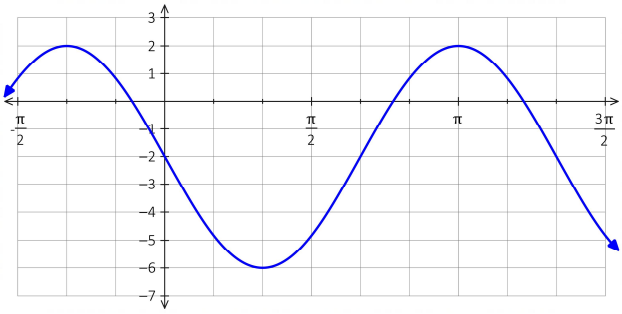
(a)



(b)



(b)



Answers from previous page

30. *tanx is equal to* $\frac{sinx}{cosx}$ *, or* $\frac{y}{x}$*, so tan is undefined where cosx ("x" on the unit circle) is equal to 0. That is, at* $x = \frac{\pi}{2} + n\pi\,;n \in I$

31. (a) $y = \tan(\frac{1}{2}x)$ (b) $y = \tan(\frac{3}{4}x)$

32. (a) $y = 2sin(\frac{1}{4}x) + 1$ (b) $y = 3\cos(\frac{3}{2}x)$ (c) $y = -4\cos(\frac{4}{3}x)$

EDGE math30-1edge.com

Copyright © RTD Learning 2020 – all rights reserved

34. Exam Style

The graph of a function $y = a\cos(bx) + d$ has its first minimum point at $(\frac{2\pi}{3}, -1)$, and the maximum value of the function is 9.

The value of b can be written as $\frac{m}{n}$.

NR The values of a, m, n and d are, respectively, ____, ____, ____, and ____.

35. Exam Style

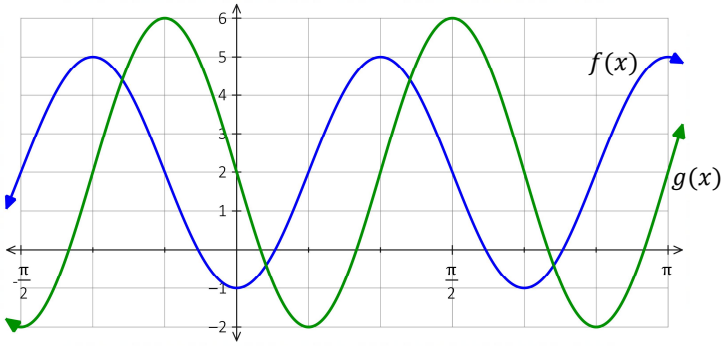
The graph of $f(x) = a\sin[b(x-c)] + d$ below is transformed to become the graph of $g(x)$. The parameters in $f(x)$ that are changed to produce the graph of $g(x)$ are:

A. a and b

B. a and c

C. d and b

D. d and c



36. Determine a sine equation, with $a > 0$, for $y = g(x)$ from question 35 above.

37. Exam Style

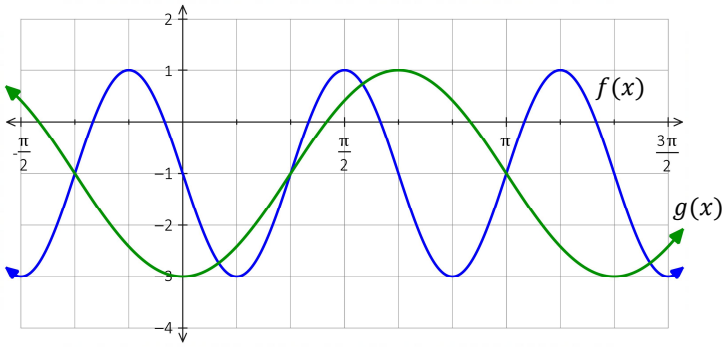
The graph of $f(x) = a\cos[b(x-c)] + d$ below is transformed to become the graph of $g(x)$. The parameter in $f(x)$ that is changed to produce the graph of $g(x)$ is:

A. c, which is $\frac{\pi}{6}$ greater in $g(x)$

B. c, which is $\frac{\pi}{6}$ less in $g(x)$

C. b ; the value of b in $g(x)$ is $\frac{4}{3}$ times the value of b in $f(x)$

D. b ; the value of b in $g(x)$ is $\frac{1}{2}$ the value of b in $f(x)$



Answers from previous page

33. (a) i $y = 5sin[3(x-30°)] - 4$ ii $y = 5cos[3(x-60°)] - 4$ iii $y = -5cos(3x) - 4$

(b) i $y = 2sin[\frac{2}{3}(x-\frac{3\pi}{4})] + 5$ ii $y = 2cos[\frac{2}{3}(x-\frac{3\pi}{2})] + 5$ iii $y = -2cos(\frac{2}{3}x) + 5$

(c) i $y = 4sin[\frac{3}{2}(x-\frac{2\pi}{3})] - 2$ ii $y = 4cos[\frac{3}{2}(x+\frac{\pi}{3})] - 2$ iii $y = -4sin(\frac{3}{2}x) - 2$

Copyright © RTD Learning 2020 – all rights reserved

Use the following information to answer the next two questions

The average daily temperature in Wainright, Alberta can be modeled by a sinusoidal function in the form $T(n) = a\cos[b(n-c)] + d$, where T is the temperature in °C, and n is the day # of the year, through to 365.

The town experiences its first minimum on January 18th (18 days from the start of the year), which is -11.2°C. The first maximum is on July 20th (201 days from the start of the year), which is 21.6°C

38. Exam Style

The values of a and b in the equation $T(n)$ are, respectively,

A. 16.4 and 0.0172

B. 16.4 and 58.09

C. 5.2 and 0.0172

D. 5.2 and 58.09

39. Exam Style **NR**

From the temperature function $T(n)$, the number of days in the year the average temperature will be at least 18°C is _____.

40. Suppose that a second temperature function was developed for Sao Paulo, Brazil, which is in the southern hemisphere, where the warmest month is January. The temperature is also much warmer year- round and fluctuates less. Which parameters a, b, c, or d would we expect to change, and how?

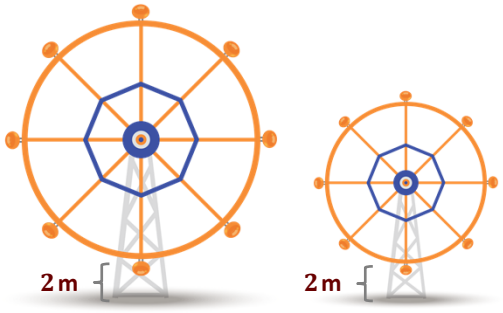
Use the following information to answer the next question

A carnival features two Ferris wheels, one larger and one smaller. On both wheels the distance from the lowest carriage to the ground is 2m, and both take one minute to complete a revolution.

The height of a passenger, $h(t)$ in metres, on each wheel can be modeled as sinusoidal function in the form

$$h(t) = a\sin[b(x-c)] + d$$

Where t is the number of seconds after the ride begins.



41. Exam Style

The parameters that **must** be different in the two functions are,

A. A and C

B. A and D

C. C and D

D. A only

Answers from previous page

34. 5324 **35.** B **36.** $y = 4sin[3(x - \frac{\pi}{3})] + 2$ **37.** D

Copyright © RTD Learning 2020 – all rights reserved

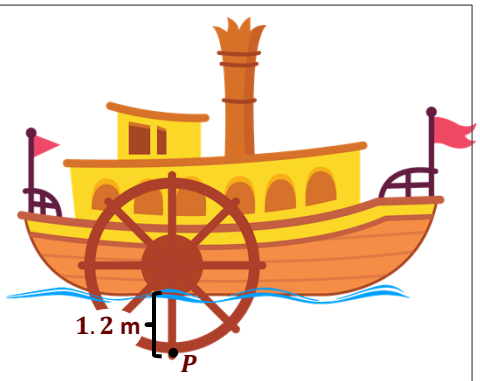
Use the following information to answer the next question

A paddlewheel of a ferry boat has a radius of 4.4 metres and rotates at a constant rate of one rotation every 6.2 seconds. At its lowest, a point P marked on the wheel is 1.2 metres below the water.

The height, h metres, of the point P above the surface of the water on the edge of the wheel at any time t, in seconds, can be modeled by the function

$$h(t) = a\sin[b(x-c)] + d.$$

P is at the lowest point on the wheel when a stopwatch is started, and its height modeled.



42. (a) Sketch a graph that shows the height of the point P from the time the stopwatch is started, over two complete rotations of the wheel. Provide a scale for each axis and label fully.



(b) Determine an equation for $h(t)$, as a:

i Minimum phase shift sine function, where $a > 0$ ii Minimum phase shift cosine function, where $a < 0$

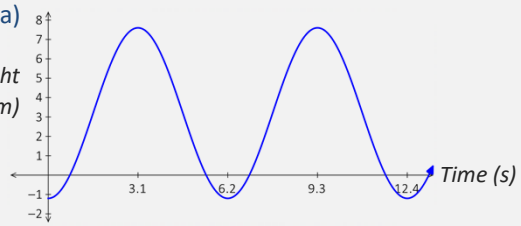
(b) Determine the number of seconds it would take for the height of P to go from **water level** to 6 metres above the water. *Round to the nearest hundredth.*

Answers from previous page ***and this page***

38. A **39.** 78 **40.** The median temperature would be higher, so d would increase. The seasonal temperature differences would be less, so a would decrease. The maximum would occur earlier in the year so c would increase. (alternatively, "c" could remain the same and a vertical reflection applied)

41. B

42. (a)



(b) i $h(t) = 4.4\sin[\frac{\pi}{3.1}(x-1.55)] + 3.2$

ii $h(t) = -4.4\cos(\frac{\pi}{3.1}x) + 3.2$

(c) i **1.48** seconds

Copyright © RTD Learning 2020 – all rights reserved
EDGE math30-1edge.com