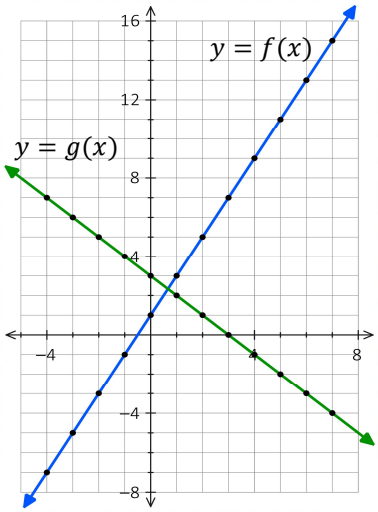
CHAPTER 5 REVIEW – Operations on Functions

1. The graphs of the linear function $y = f(x)$ and $y = g(x)$ are shown on the right.



(a) On the same grid, sketch the graph of $h(x) = (f + g)(x)$

(b) Determine a slope-intercept form equation for $f(x)$ and $g(x)$.

(c) Determine a simplified equation for $h(x)$.

(d) Determine an equation for the following functions. (Do not graph – include domain restriction for ii and iii)

i $k(x) = (f - g)(x)$

ii $y = (g - f)(x)$

iii $y = (f \cdot g)(x)$

iv $y = \left(\frac{f}{g}\right)(x)$

v $y = \left(\frac{g}{f}\right)(x)$

(e) Compare the simplified equations of $h(x) = (g - f)(x)$ and $k(x) = (f - g)(x)$.
Describe the relationship between these two functions using transformations terminology.

(f) Determine the range of $h(x) = (f + g)(x)$ if the domain of $g(x)$ is restricted to $[0, \infty)$.

(g) Determine and expression for each of the following composite functions:

i $y = (f \circ g)(x)$

ii $y = (g \circ f)(x)$

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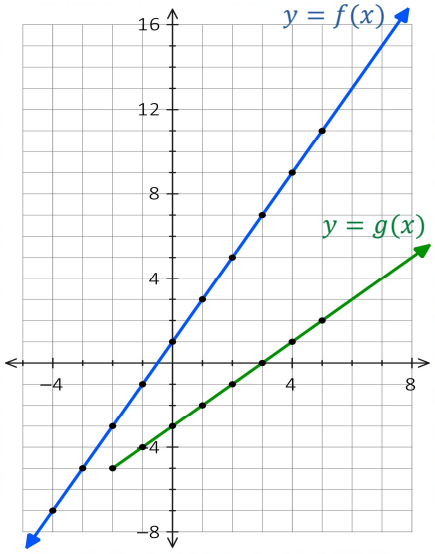
2. The graphs of $y = f(x)$ and $y = g(x)$ are shown on the right.



(a) On the same grid, sketch the graph of $h(x) = (f \cdot g)(x)$

(b) Determine a slope-intercept form equation for $f(x)$ and $g(x)$.
(As **always** – include any domain restrictions)

(c) Determine a simplified equation for $h(x)$, and state the domain and range.

(d) Determine an equation for the following combined functions. (Do not graph)

i $y = (f + g)(x)$ ii $y = (f - g)(x)$ iii $y = \left(\frac{f}{g}\right)(x)$

(e) Determine the range of $h(x) = (f \cdot g)(x)$ if the domain of $g(x)$ is restricted to $[-1, 5]$.

(f) Evaluate each of the following:

i $y = (f \circ g)(0)$ ii $y = (g \circ f)(1)$ iii $y = (f \circ g \circ g)(2)$

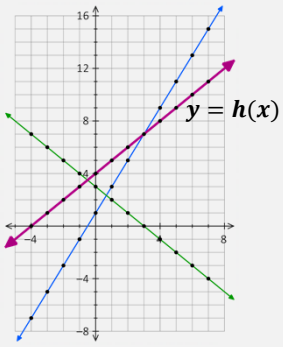
(g) State the domain and range of:

i $y = (f \circ g)(x)$ ii $y = (g \circ f)(x)$ iii $y = (g \circ g)(x)$

Answers from previous page

1. ADD all corresponding y-coordinates

(a)



(b) $f(x) = 2x + 1$ / $g(x) = -x + 3$ **(c)** $h(x) = x + 4$

(d) i $y = 3x - 2$
ii $y = -3x + 2$
iii $y = -2x^2 + 5x + 3$
iv $y = \frac{2x+1}{-x+3}$; $x \neq -3$
v $y = \frac{-x+3}{2x+1}$; $x \neq -\frac{1}{2}$

(e) Graphs are *vertical reflections* of one another

(f) $[4, \infty)$

(g) i $(f \circ g)(x) = -2x + 7$ ii $(g \circ f)(x) = -2x + 2$

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3. Two functions are defined $f(x) = x^2 - 4x - 5$ and $g(x) = x + 1$.

(a) Determine a simplified expression for each of the following functions. (Obligatory "state any domain restriction" reminder)

i $y = \left(\frac{f}{g}\right)(x)$

ii $y = \left(\frac{g}{f}\right)(x)$

iii $y = (f - g)$

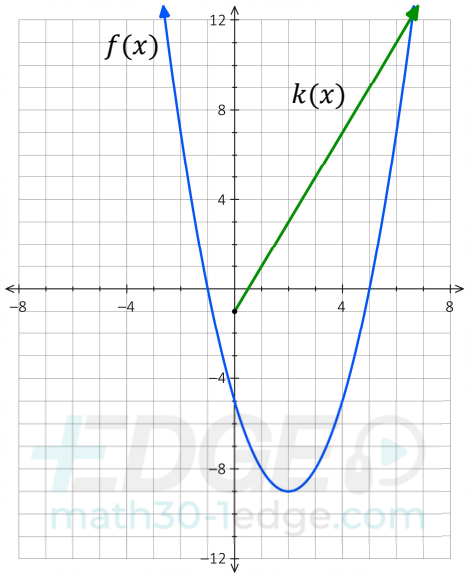
iv $y = (f \circ g)$

(b) A third function is given as $k(x) = 2x - 1\,;\, x \geq 0$. Given the graph of $f(x)$, determine the domain and range of each composite function:

i $y = (f - k)(x)$

ii $y = (f \circ k)(x)$

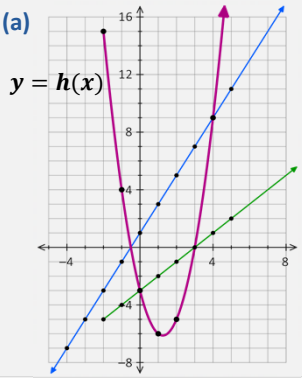
iii $y = (k \circ f)(x)$



Answers from previous page

2. MULT. all corresponding y-coordinates

(a)



(b) $f(x) = 2x + 1$ / $g(x) = x - 3\,;\, x \geq -2$

(c) $h(x) = 2x^2 - 5x - 3\,;\, x \geq -2$

(d) i $y = 3x - 2\,;\, x \geq -2$ iii $y = \frac{2x+1}{x-3}\,;\, x \geq -2, x \neq -3$

ii $y = x + 4\,;\, x \geq -2$

(e) $[-49/8, 22]$ *Use your graphing calculator to obtain minimum*

(f) i -5 ii 0 iii -7

(g) i D: $[-2, \infty)$ R: $[-9, \infty)$ ii D: $[-1.5, \infty)$ R: $[-5, \infty)$
iii D: $[1, \infty)$ R: $[-5, \infty)$

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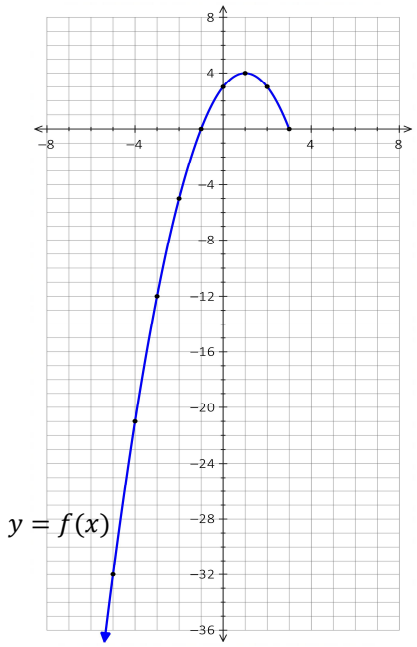
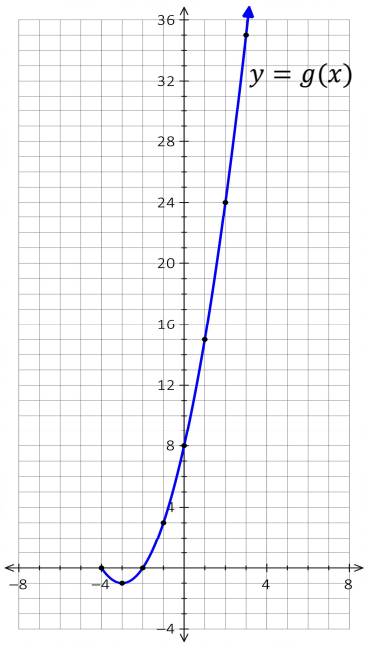
4. The graphs of $f(x)$ and $g(x)$ are shown below. Four new functions are defined as:





(a) Evaluate each of the following:

i $(f-g)(-1)$ **ii** $(f \cdot g)(0)$ **iii** $(f \circ g)(-3)$ **iv** $(g \circ f)(3)$

(b) State the domain of each of the following functions:

i $y = \left(\frac{f}{g}\right)(x)$ **ii** $y = \left(\frac{g}{f}\right)(x)$

iii $y = (f \circ g)(x)$ **iv** $y = (f \circ f)(x)$

(c) State the domain and range of each of the following functions:

i $y = (f+g)(x)$ **ii** $y = (f-g)(x)$

Answers from previous page

3. (a) **i** $y = x - 5$; $x \neq -1$ **ii** $y = \frac{1}{x-5}$; $x \neq -1, x \neq 5$

iii $y = x^2 - 5x - 6$ **iv** $y = x^2 - 2x - 4$

(b) **i** D: $[0, \infty)$ R: $[-12, \infty)$ **ii** D: $[0, \infty)$ R: $[-9, \infty)$ **ii** D: $\{x | x \leq -1, x \geq 5, x \in \mathbb{R}\}$ R: $\{y | y \geq -1, y \in \mathbb{R}\}$

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5. Four functions are defined $f(x) = log_3(x+1)$, $g(x) = \sqrt{16-2x}$, $h(x) = 2x-1$ and $k(x) = \frac{1}{x}$.

(a) Evaluate each of the following:

i $(f-g)(8)$ **ii** $\left(\frac{g}{h}\right)(0)$ **iii** $(f+h)(2)$ **iv** $(g \circ f)(80)$

v $(h \circ k)(0.25)$ **vi** $(k \circ g \circ h)(-0.5)$ **vii** $(g \circ h \circ h)\left(\frac{11}{8}\right)$ **viii** $(k \circ h \circ f)(0)$

ix $h(f+h)(8)$ **x** $g(g-h)(6)$ **xi** $k(f+g)(0)$ **xii** $h(f \cdot k)\left(-\frac{8}{9}\right)$

Edit note: *Add work space*

(b) Determine an expression for each of the following functions and state the domain.

i $y = (g \cdot k)(x)$ **ii** $y = \left(\frac{h}{k}\right)(x)$ **iii** $y = \left(\frac{k}{h}\right)(x)$

iv $y = (f \circ h)(x)$ **v** $y = (g \circ h)(x)$ **vi** $y = (h \circ g)(x)$

vii $y = (f \circ g)(x)$ **viii** $y = (k \circ g \circ h)(x)$ **ix** $y = (k \circ h \circ h)(x)$

Answers from previous page

4. (a) **i** -3 **ii** 24 **iii** 0 **iv** 8

(b) **i** $\{x| -4 < x \leq 3, x \neq -2, x \in \mathbb{R}\}$ **ii** $\{x| -4 \leq x < 3, x \neq -1, x \in \mathbb{R}\}$

iii $[-4,-1]$ **iv** $\{x|x \leq 0,\ 2 \leq x \leq 3, x \in \mathbb{R}\}$

(c) **i** D: $[-4,3]$ R: $[-21,35]$ **ii** D: $[-4,3]$ R: $[-35,-3]$

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6. Four functions are defined $f(x) = 2x + 1$, $g(x) = \sqrt{x-4}$, $h(x) = x^2$, and $k(x) = \frac{1}{x}$.

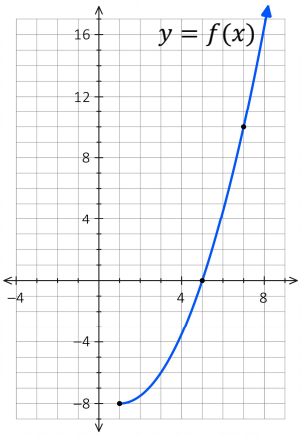
(a) Determine an expression for each of the following functions and state the domain.

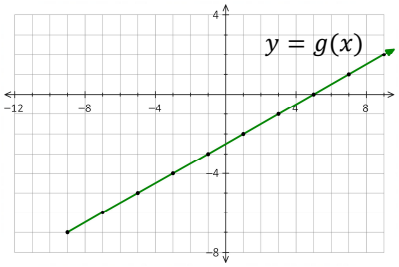
i $y = (g + k)(x)$ **ii** $y = \left(\frac{f}{k}\right)(x)$ **iii** $y = (k \circ g)(x)$ **iv** $y = (f \circ k)(x)$

(b) Determine an expression for each of the following functions, and state both the domain and range.

i $y = (f + h)(x)$ **ii** $y = (h \cdot k)(x)$ **iii** $y = (f \circ h)(x)$ **iv** $y = (h \circ g \circ f)(x)$

7. Use the graphs of $f(x)$ and $g(x)$ below to determine:





(a) The value of:

i $(f - g)(7)$ **ii** $(f \circ g)(7)$

(b) The domain and range of:

i $(f + g)(x)$

ii $\left(\frac{f}{g}\right)(x)$

iii $f(g(x))$

iv $g(f(x))$

Answers from previous page

5. **(a)** **i** 2 **ii** -4 **iii** 4 **iv** $2\sqrt{2}$ **v** 7 **vi** 0.25 **vii** $\sqrt{11}$ **viii** -1 **ix** 33 **x** $\sqrt{34}$ **xi** $1/4$ **xii** $-11/2$

(b) **i** $(g \cdot k)(x) = \frac{\sqrt{16-2x}}{x}$ D: $\{x|x \leq 8, x \neq 0, x \in \mathbb{R}\}$

ii $\left(\frac{h}{k}\right)(x) = 2x^2 - x$ D: $\{x|x \neq 0, x \in \mathbb{R}\}$

iii $\left(\frac{k}{h}\right)(x) = \frac{1}{x(2x-1)}$ D: $\{x|x \neq 0, x \neq 1/2, x \in \mathbb{R}\}$

iv $(f \circ h)(x) = log_3(2x)$ D: $\{x|x > 0, x \in \mathbb{R}\}$

v $(g \circ h)(x) = \sqrt{18-4x}$ D: $\{x|x \leq 9/2, x \in \mathbb{R}\}$

vi $(h \circ g)(x) = 2\sqrt{16-2x} - 1$ D: $\{x|x \leq 8, x \in \mathbb{R}\}$

vii $(f \circ g)(x) = log_3(\sqrt{16-2x} + 1)$ D: $\{x|x \leq 8, x \in \mathbb{R}\}$

viii $(k \circ g \circ h)(x) = \frac{1}{\sqrt{18-4x}}$ D: $\{x|x < 9/2, x \in \mathbb{R}\}$

ix $(k \circ h \circ h)(x) = \frac{1}{4x-3}$ D: $\{x|x \neq 3/4, x \in \mathbb{R}\}$

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Use the following information to answer questions 8, 9, and 10.

From the list of functions below, the functions $h(x) = \left(\frac{f}{g}\right)(x)$ and $k(x) = (f \circ g)(x)$ are formed.

1 $y = x^2 + 1$
2 $y = x^2 - 1$
3 $y = x^3 + 1$
4 $y = x^4 + 1$
5 $y = 0.5^x - 2$
6 $y = 0.5^x + 2$
7 $y = \sqrt{x} - 1$
8 $y = log_b(x + 2)$; $0 < b < 1$
9 $y = \frac{1}{x}$

For $h(x)$ function 1 is chosen for $f(x)$, while for $k(x)$ function 9 is chosen for $f(x)$.

8. Exam Style **NR** For the domain of $h(x)$ to be $\{x \in \mathbb{R}\}$, the function chosen for $g(x)$ must be function number ____, ____, or ____.

9. Exam Style **NR** For the domain of $k(x)$ to be $\{x | x \neq -1, x \in \mathbb{R}\}$, the function chosen for $g(x)$ must be function number ____ or _____.

10. State the domain of $y = f\big(g(x)\big)$, if function 7 is chosen for $f(x)$, and $g(x)$ is:

(a) Function 1 (b) Function 2 (c) Function 3 (d) Function 5

(e) Function 6 (f) Function 8

Answers from previous page

6. (a) i $(g + k)(x) = \sqrt{x - 4} + \frac{1}{x}$ D: $\{x | x \geq 4, x \in \mathbb{R}\}$ ii $\left(\frac{f}{k}\right)(x) = x(2x + 1)$ D: $\{x | x \neq 0, x \in \mathbb{R}\}$ iii $(k \circ g)(x) = \frac{1}{\sqrt{x - 4}}$ D: $\{x | x > 4, x \in \mathbb{R}\}$ iv $(f \circ k)(x) = \frac{2}{x} + 1$ D: $\{x | x \neq 0, x \in \mathbb{R}\}$

(b) i $(f \cdot h)(x) = (x + 1)^2$ D: $\{x \in \mathbb{R}\}$ R: $\{y | y \geq 0, y \in \mathbb{R}\}$ ii $(h \cdot k)(x) = x$ D: $\{x | x \neq 0, x \in \mathbb{R}\}$ R: $\{y | y \neq 0, y \in \mathbb{R}\}$ iii $(f \circ h)(x) = 2x^2 + 1$ D: $\{x \in \mathbb{R}\}$ R: $\{y | y \geq 1, y \in \mathbb{R}\}$ iv $(h \circ g \circ f)(x) = 2x - 3$ D: $\{x | x \geq 3/2, x \in \mathbb{R}\}$ R: $\{y | y \geq 0, y \in \mathbb{R}\}$

7. (a) i 9 ii -8

(b) i D: $\{x | x \geq 1, x \in \mathbb{R}\}$ R: $\{y | y \geq -10, y \in \mathbb{R}\}$ ii D: $\{x | x \geq 1, x \neq 5, x \in \mathbb{R}\}$ R: $\{y | y \geq 4, y \in \mathbb{R}\}$ iii D: $\{x | x \geq 7, x \in \mathbb{R}\}$ R: $\{y | y \geq -8, y \in \mathbb{R}\}$ iv D: $\{x | x \geq 1, x \in \mathbb{R}\}$ R: $\{y | y \geq -6.5, y \in \mathbb{R}\}$

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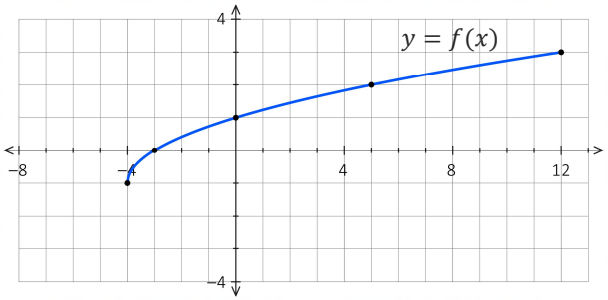
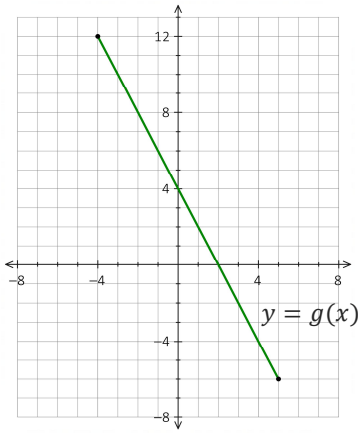
Use the following information to answer questions 8, 9, and 10.

The graphs of $f(x)$ and $g(x)$ are shown here.





11. The range of $y = (f - g)(x)$ is:

Exam Style

A. $[-13, 9]$
B. $[-13, 8]$
C. $[-8, 13]$
D. $[-9, 13]$

12. The domain of $y = (f \circ g)(x)$ is:

Exam Style

A. $[-4, 4]$
B. $[-4, 5]$
C. $[-4, 6]$
D. $[-4, 12]$

13. The range of $y = (f \circ g)(x)$ is

Exam Style

A. $[-6, 3]$
B. $[-1, 3]$
C. $[-1, 12]$
D. $[-6, 12]$

14. The range of $y = (g \circ f)(x)$ is

Exam Style

A. $[-1, 12]$
B. $[-1, 6]$
C. $[-2, 3]$
D. $[-2, 6]$

Answers from previous page ***and this page***

8. 146 **9.** 35 **10.** **(a)** $\{x \in \mathbb{R}\}$ **(b)** $\{x | x \neq \pm 1, x \in \mathbb{R}\}$ **(c)** $\{x | x \neq -1, x \in \mathbb{R}\}$ **(d)** $\{x | x \neq -1, x \in \mathbb{R}\}$ **(e)** $\{x \in \mathbb{R}\}$ **(f)** $\{x | x > 2, x \neq -1, x \in \mathbb{R}\}$

11. B **12.** A **13.** B **14.** D

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