# Chapter 3 Exponential Functions and Logarithms REVIEW

**1.** Use an algebraic process to solve each of the following equations. Verify your answers.

(a) $4^{2x-3} = 1$

(b) $(16^{2x-1})(8^{x-2}) = \left(\frac{1}{4}\right)^{x+2}$

(c) $\left(\frac{3}{2}\right)^{\frac{x}{2}} = \left(\frac{8}{27}\right)^{2x+2}$

**2.** For each of the following exponential functions, state the (i) domain, (ii) range, and coordinates of any (iii) $x$-intercepts and (iv) $y$-intercepts. Sketch each, be sure to include the asymptote. *Try doing as much as you can without using your graphing calculator! Use an algebraic process to determine any intercepts.*

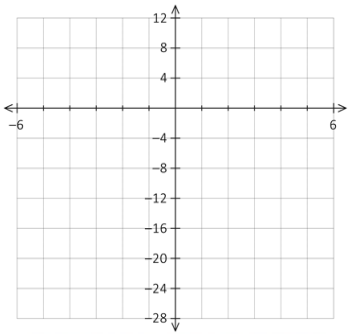
(a) $f(x) = 3\left(\frac{1}{2}\right)^{x-1} - 24$

i – Domain: _____

ii – Range: _____

iii – $x$-int: _____

iv – $y$-int: _____



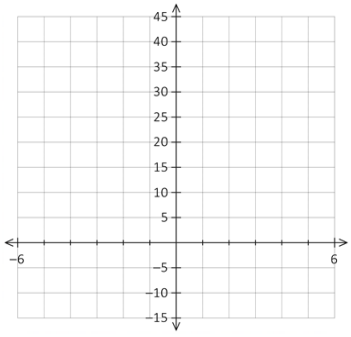

(b) $g(x) = -5(2)^x + 40$

i – Domain: _____

ii – Range: _____

iii – $x$-int: _____

iv – $y$-int: _____





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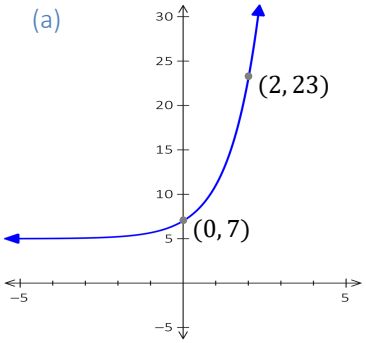
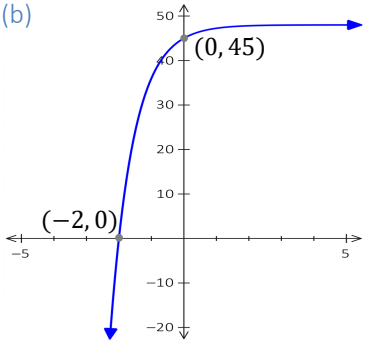
**3.** The range of the exponential functions given by the graphs below are $\{y|y > 5, y \in \mathbb{R}\}$ for $f(x)$ and $\{y|y < 48, y \in \mathbb{R}\}$ for $g(x)$. Determine an equation for each function, in the form $y = a(b)^x + d$.

(a)




(b)




**4.** Evaluate each of the following logarithms. *Try without using a log button on your calculator.*

(a) $log_3 81$ (b) $log_{16} 32$ (c) $log_4 0.125$ (d) $log_{\sqrt{3}} 27$

(e) $log \sqrt[3]{100}$ (f) $log_5 \left(\frac{1}{\sqrt{125}}\right)$

*Answers to Review Questions from Previous Page*

**1.** (a) 3/2

(b) 6/13

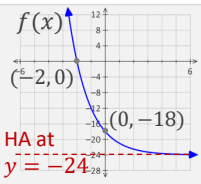
(c) $-13/12$

**2.** (a) i - $\{x \in \mathbb{R}\}$

ii - $\{y|y > -24, y \in \mathbb{R}\}$

iii - $(-2, 0)$

iv - $(0, -18)$

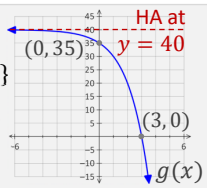



(b) i - $\{x \in \mathbb{R}\}$

ii - $\{y|y < 40, y \in \mathbb{R}\}$

iii - $(3, 0)$

iv - $(0, 35)$





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**5.** Refer to #3 above. If $a$ and $d$ are both elements of the real numbers, where $a \neq 0$, $d \neq 0$, identify which of the following scenarios provides an $x$-intercept. Justify by providing a graph.

(a) $a > 0,\ d > 0$ (b) $a > 0,\ d < 0$ (c) $a < 0,\ d > 0$ (d) $a < 0,\ d < 0$

**6.** Solve each of the following for $x$, without using a log button on your calculator.

(a) $log_4 512 = x$ (b) $log_4 x = -3$ (c) $log_x\left(\frac{1}{16}\right) = -\frac{4}{3}$

(d) $3^{2x} = 729$ (e) $4(2)^{x+1} = \frac{1}{\sqrt{8}}$ (f) $log_{\frac{27}{8}} x = -\frac{2}{3}$

(g) $log_4 32\sqrt{8} = x$ (h) $log_9\left(\frac{\sqrt{27}}{3}\right) = x$

*Answers to Review Questions from Previous Page*

**3.** (a) $y = 2(3)^x + 5$ (b) $y = -3\left(\frac{1}{4}\right)^x + 48$

**4.** (a) **4** (b) **5/4** (c) **$-3/2$**
(d) **6** (e) **2/3** (f) **$-3/2$**


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**7.** Given the function $f(x) = a(4)^{x-1} + d$, an expression for the $y$-intercept is:

Exam Style

A. $a + d$

B. $4a + d$

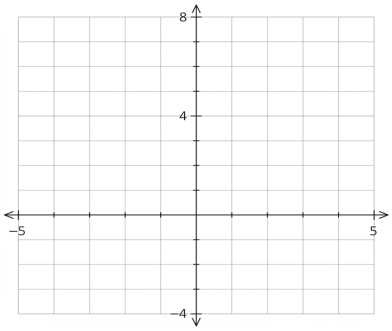
C. $\frac{a}{4} + d$

D. $\frac{4}{a} + d$

**8.** A student wishes to algebraically determine the $x$-intercept of the function $y = 2(\frac{1}{8})^{x-1} - 32$. One of steps in his correct solution lists the linear equation $a = -3x + b$, where the values of $a$ and $b$ are, respectively, ____ and ____ .

Exam Style

**9.** For the function $f(x) = log_{1/3}(9 - 3x)$, provide a detailed sketch, and indicate the domain, range, asymptote, and coordinates of any intercepts. *Use an algebraic process to determine the graph characteristics, where applicable state in exact values only.*




**10.** The function $y = 2log_m(bx - 3)$ ; $m > 0, m \neq 1, b \neq 0$ has a domain:

Exam Style

A. $\{x | x > b/3 \ , x \in \mathbb{R}\}$

B. $\{x | x > b + 3, x \in \mathbb{R}\}$

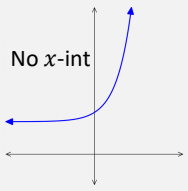
C. $\{x | x > 3 \ , x \in \mathbb{R}\}$

D. $\{x | x > 3/b, x \in \mathbb{R}\}$

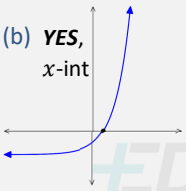
*Answers to Review Questions from Previous Page*

**5.**

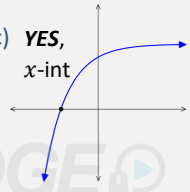
(a) No $x$-int



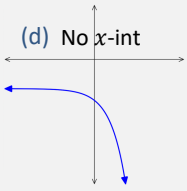
(b) ***YES***, $x$-int



(c) ***YES***, $x$-int



(d) No $x$-int



**6.** (a) $9/2$ (b) $1/64$ (c) $8$ (d) $3$

(e) $-9/2$ (f) $4/9$ (g) $13/4$ (h) $1/4$



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**11.** The function $\boldsymbol{y = log_2\,(bx + 0.5)}$ ; $b \neq 0$ has a $y$-intercept of:

Exam Style

A. $1/2$

B. $-1$

C. $b/2$

D. $-b$

**12.** An exponential function $\boldsymbol{f(x) = a(3)^x + k}$ ; $a \neq 0$ has range of $\{y > -18\,, y \in \mathbb{R}\}$, and an $x$-int. at $(2, 0)$.

A function $y = g(x)$ is obtained by vertically reflecting the graph of $f(x)$ about the $x$-axis, and horizontally stretching it about the $y$-axis by a factor of 3.

A function $y = k(x)$ is obtained by reflecting the graph of $f(x)$ about the line $y = x$.

(a) Determine an equation for the function $y = g(x)$

(b) State the range, and $x$ and $y$ intercepts for $y = g(x)$

(c) Determine the equation for the function $y = k(x)$

(b) State the domain, and $x$ and $y$ intercepts for $y = k(x)$

**13.** The equation $\boldsymbol{a log_b(xy) + 3 = c}$ can be written in exponential form as:

Exam Style

A. $b^{c-3} = x^a y^a$

B. $b^{c-3} = xy^a$

C. $b^{c-3} = \dfrac{xy}{a}$

D. $b^{c-3} = a^{xy}$

*Answers to Review Questions from Previous Page*

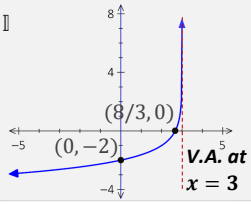
**7.** C **8.** 43 **9.** Domain $\{x|x < 3\,, x \in \mathbb{I}$

Range $\{y \in \mathbb{R}\}$

Vert. Asym: at $x = 3$

$x$-intercept $(8/3, 0)$

$y$-intercept $(0, -2)$




**10.** D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**14.** The expression $(5^{2logx})(5^{2logx})$ can be written in exponential form as:

Exam Style

A. $5^{(2logx)^2}$

B. $5^{(logx)^4}$

C. $5^{(log\ x^4)}$

D. $5^{log(2x^2)}$

**15.** A function $\boldsymbol{f(x) = log_2(x)}$ is transformed by horizontally stretching the graph about the line $x = 0$ by a factor of 1/8. Show how this transformation can also be obtained by vertically translating the graph of $f(x)$ by a factor of $\boldsymbol{k}$ and state the value of $\boldsymbol{k}$.

**16.** The graph of $\boldsymbol{y = log_{\frac{1}{2}}(x + 8)}$ is transformed to $\boldsymbol{y = f(x)}$ with the following transformations applied:

- A horizontal stretch by a factor of 2
- A reflection about the $y$-axis
- A translation 2 units to the left

Use an algebraic process to determine each of the following characteristics of $y = f(x)$

(a) Domain (b) $x$-intercept (c) $y$-intercept

**17.** **NR**

Exam Style

If $\boldsymbol{log_{1/4}P = 0.25}$ and $\boldsymbol{sec\theta = P}$, then the largest value of $\theta$ on $[0, 2\pi)$ has the form $\frac{a\pi}{b}$, where the value of $a$ is ____ (first digit of your answer) and the value of $b$ is (second digit).

*Answers to Review Questions from Previous Page*

**11.** B

**12.** (a) $\boldsymbol{g(x) = -2(3)^{x/3} + 18}$ (b) Range $\{y|y < 18, y \in \mathbb{R}\}$ $x$-int. $(6, 0)$ $y$-int. $(0, 16)$

**13.** A

(c) $\boldsymbol{k(x) = log_3[\frac{1}{2}(x + 18)]}$ (d) Domain $\{x|x > -18, x \in \mathbb{R}\}$ $x$-int. $(-16, 0)$ $y$-int. $(0, 2)$



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**18.** Write each of the following expressions as simplified single logarithms.

(a) $5log_b a - \frac{log_b c}{2} - 3$

(b) $2log(xy) - \left[\frac{1}{2}log\left(\frac{x^4 z}{9}\right) - \frac{1}{3}log(y^6 z^2)\right]$

**19.** If $\boldsymbol{log_3 A = m}$, then find an expression for each of the following:

(a) $log_3(9A^3)$

(b) $log_3(\frac{\sqrt{27}}{A^2})$

(c) $log_3(\frac{\sqrt[3]{A^2}}{3})$

**20.** Given that $\boldsymbol{log_4 A = 22}$ and $\boldsymbol{log_4 B = 20}$, evaluate each of the following:

(a) $log_4(16AB^2)$

(b) $log_4\left(\frac{32A^3}{B}\right)$

(c) $log_4\left(\frac{\sqrt{A}}{8\sqrt[4]{B}}\right)$

*Answers to Review Questions from Previous Page*

**14.** C

**15.** Apply horiz. stretch $\boldsymbol{y = log_2(8x)}$ ➔ $\boldsymbol{y = log_2 8 + log_2 x}$ ➔ $\boldsymbol{y = 3 + log_2 x}$ ***Same as a vert. translation 3 up***

**16.** (a) $\{\boldsymbol{x}|\boldsymbol{x} < \boldsymbol{14}, \boldsymbol{x} \in \mathbb{R}\}$ (b) $(\boldsymbol{12}, \boldsymbol{0})$ (c) $(\boldsymbol{0}, \boldsymbol{log_{0.5} 7})$ **17.** $\boldsymbol{74}$


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**21.** If $\boldsymbol{log_2 3 = 4x}$, $\boldsymbol{log_2 5 = 3y}$, and $\boldsymbol{log_{\sqrt{2}} n^3 = 12}$, determine an expression for each of the following:

(a) $log_2(45n^4)$ (b) $log_2(2700\sqrt{n}\,)$ (c) $log_2\left(\frac{\sqrt{75}}{8n^2}\right)$

**22.** Use an algebraic process to solve the equation $3\left(4^{x+1}\right) = 60$, correct to the nearest hundredth.

*Verify graphically on your calculator.*

**23.** For the equation $\boldsymbol{6^{2x} = 10^{x-2}}$, use an algebraic process to:

(a) Determine an approximate solutions, correct to the nearest hundredth.

(b) The exact solution can be written in the form $-\frac{logA}{log\left(\frac{B}{C}\right)}$

Determine the values of $A$, $B$, and $C$.

*Answers to Review Questions from Previous Page*

**18.** (a) $\boldsymbol{log_b\left(\frac{a^5}{b^3c^{1/2}}\right)}$ (b) $\boldsymbol{log\left(3y^4z^{1/6}\right)}$

**19.** (a) $\boldsymbol{3m+2}$ (b) $\boldsymbol{\frac{3}{2}-2m}$ (c) $\boldsymbol{\frac{2}{3}m-1}$

**20.** (a) $\boldsymbol{64}$ (b) $\boldsymbol{48.5}$ (c) $\boldsymbol{4.5}$


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**24.** The number of hours, $H$, that cream stays fresh decreases exponentially as the temperature of the surrounding air, $n$ (in $°C$) increases. The relationship is $H = 160(0.1)^{0.038n}$.

(a) Determine how long cream will stay fresh if it is kept at $2°C$, correct o the nearest whole hour.

(b) If cream is left out in a room stays fresh for 16 hours, use an algebraic process to determine the temperature of the room, correct to the nearest tenth of a degree.

**25.** A radioactive substance is decaying at rate of 1.15% every hour.

(a) If initially there is 100% of a sample, set up and algebraically solve an equation to determine how long it would take to decay to half the initial amount, correct to the nearest tenth of an hour.

(b) Use your result from (a) to determine an alternative exponential decay function, in the form $A = 100(b)^{t/p}$, where the initial amount is multiplied by $b$ every $p$ hours.

(c) In (a) and (b) above we saw that we can model the amount the substance decays to using two different types of equations, each with a different value for $b$. Use each to predict how much of the substance will remain after 24 hours, correct to the nearest whole percentage.

**26.** The effective percentage of oxygen in the air decreases as a function of altitude. At sea level (0 km elevation), the oxygen count is 20.9, while on top of Mount Everest (8.85 km) the oxygen count is 6.9.

**Fill in the blank:** Every _____ km in altitude, the effective percentage of oxygen in the air decreases by half. *Round to the nearest tenth.*

*Answers to Review Questions from Previous Page*

**21.** (a) $\boldsymbol{8x + 3y + 8}$ (b) $\boldsymbol{12x + 6y + 4}$ (c) $\boldsymbol{2x + 3y - 7}$ **22.** $\boldsymbol{x \approx 1.16}$

**23.** (a) $x = -3.60$ (b) $\boldsymbol{A = 100, B = 18, C = 5}$


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**27.** A $1000 investment is made in a fund that promises the invested amount will double in 9 years.

(a) If that rate of return continues, set up and algebraically solve an equation to predict how long it will take for the investment to reach $5000. (Nearest tenth of a yr)

(b) Use an algebraic process to determine the average annual rate of return (nearest tenth of a %) for the investment, if it is compounded:

i Annually i Monthly

**28.** Use an algebraic process to solve each of the following equations.

(a) $log(x+5) - log(x+1) = log3x$ (b) $log_4(x-4) + log_4(x+2) = 2$

**29.** Earthquake A measured 7.2 on the Richter Scale.

Determine the magnitude of Earthquake B, if it had one-third the intensity of Earthquake A. *Round to the nearest tenth.*

**30.** Two friends are having a discussion about Donald Trump. At the start of their conversation the intensity of sound for their discussion measured 59 dB. Near the end, the conversation had 25 times the sound intensity. Determine the decibel level of their conversation near the end, to the nearest whole number.

*Answers to Review Questions from Previous Page* ***and this page***

**24.** (a) $134$ hrs (b) $26.3$°C **26.** $5.5$ km

**25.** (a) Solve: $50 = 100(0.9885)^t$ ➔ $t \approx 59.9$ **hrs** (b) Alternative equation: $A = 100(1/2)^{t/59.9}$

(c) Evaluate: $A = 100(0.9885)^{24}$ and $A = 100(1/2)^{24/59.9}$. ➔ Both give $\approx 76\%$

**27.** (a) $20.9$ yrs (b) i $8.0\%$ ii $8.3\%$ **28.** (a) $1$ (b) $6$ **29.** $6.7$ on Richter **30.** $73$ dB


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