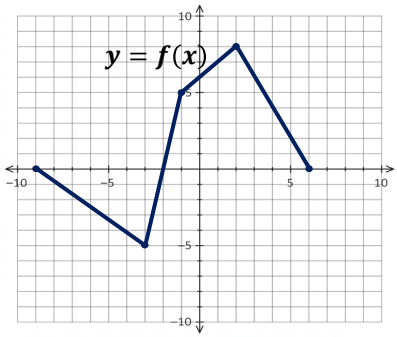
Chapter 1 Transformations REVIEW

1. The graph of $y = f(x)$ is shown below. State a mapping rule to sketch the graph of each transformed function and state the domain and range.

(a) $y = f(x-3) - 4$

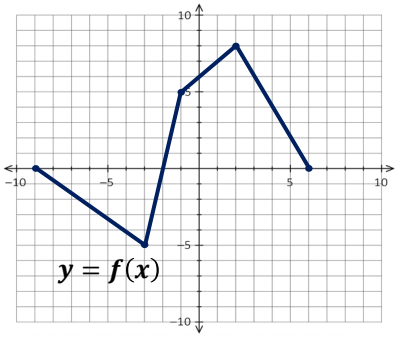


i _____ *Mapping Rule*

ii *D:* _____

iii *R:* _____

(b) $y = f(-x)$

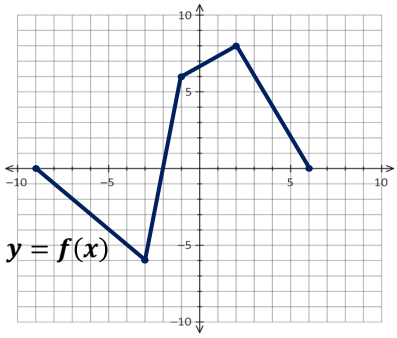


i _____ *Mapping Rule*

ii *D:* _____

iii *R:* _____

(c) $y = -\frac{1}{2}f(x)$

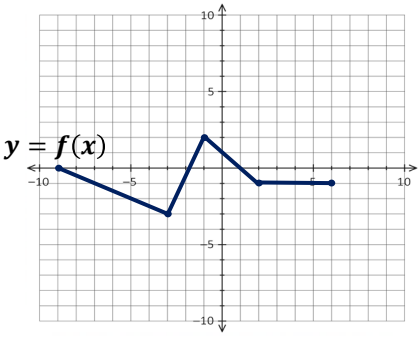


i _____ *Mapping Rule*

ii *D:* _____

iii *R:* _____

(d) $y - 2 = 3f(x)$

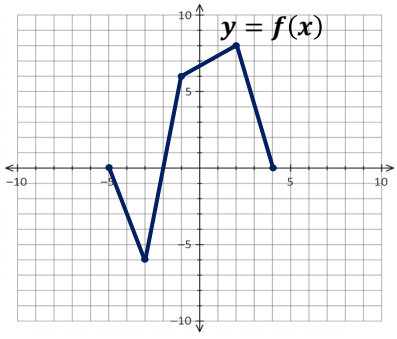


i _____ *Mapping Rule*

ii *D:* _____

iii *R:* _____

(e) $y = f[\frac{1}{2}(x-1)]$

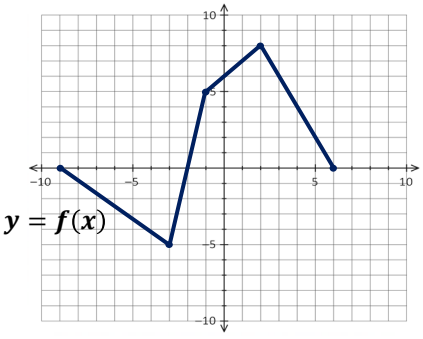


i _____ *Mapping Rule*

ii *D:* _____

iii *R:* _____

(f) $y = -f(3x-6) + 4$



i _____ *Mapping Rule*

ii *D:* _____

iii *R:* _____

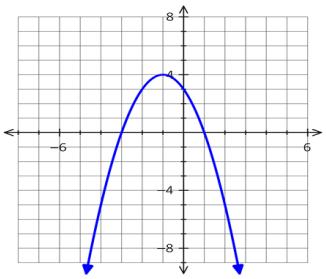
2. Given the function $f(x) = x^2 - 3x + 1$, determine a simplified equation, both in (i) terms of $f(x)$ and (ii) in term of x, for each transformed function of $f(x)$ described below:

(a) The graph is horizontally translated 1 unit right, and vertically translated 3 units down.

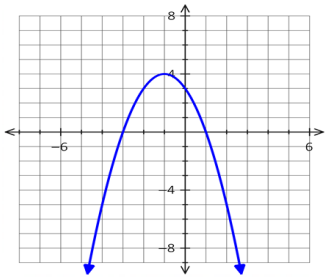
(b) The graph of $f(x)$ is reflected in the line $y = 0$, and vertically translated 3 units down.

3. Given the function $f(x) = -x^2 - 2x + 3$ shown below, (i) construct a mapping rule to sketch the graph of each transformed function described below, and determine a simplified equation, both in (ii) terms of $f(x)$ and (iii) in term of x. *Be sure to transform all points with integer coordinates.*

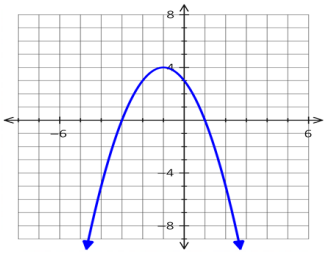
(a) The graph is vertically stretched about the x-axis by a factor of 2 and reflected in the line $y = 0$. ➔



(b) The graph of $f(x)$ is horizontally stretched by a factor of 2. ➔

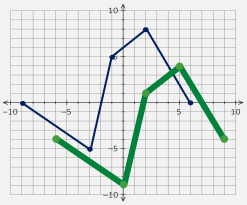


(c) The graph is reflected in the line $y = 0$, horizontally translated 4 units right, and vertically translated 3 units down. ➔

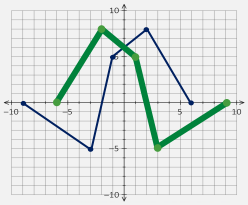


Answers from previous page

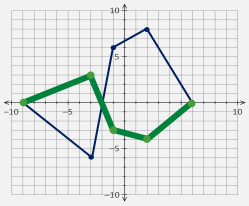
1. (a) i $(x, y) \to (x + 3, y - 4)$ ii $[-6, 9]$ iii $[-9, 4]$



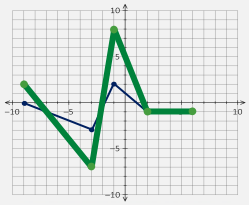
(b) i $(x, y) \to (-x, y)$ ii $[-6, 9]$ iii $[-5, 8]$



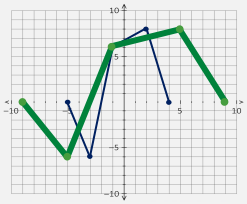
(c) i $(x, y) \to (x, -\frac{1}{2}y)$ ii $[-9, 6]$ iii $[-4, 3]$



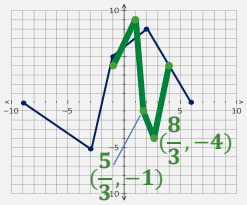
(d) i $(x, y) \to (x, 3y + 2)$ ii $[-9, 6]$ iii $[-7, 8]$



(e) i $(x, y) \to (2x + 1, y)$ ii $[-9, 9]$ iii $[-6, 8]$



(f) i $(x, y) \to (\frac{1}{3}x + 2, -y + 4)$ ii $[-1, 4]$ iii $[-4, 9]$



2. (a) i $y = f(x - 1) - 3$

ii $y = (x - 1)^2 - 3(x - 1) + 1 - 3$

simplifies to.... $y = x^2 - 5x + 2$

(b) i $y = -f(x) - 3$

ii $y = -(x^2 - 3x + 1) - 3$

simplifies to.... $y = -x^2 + 3x - 4$

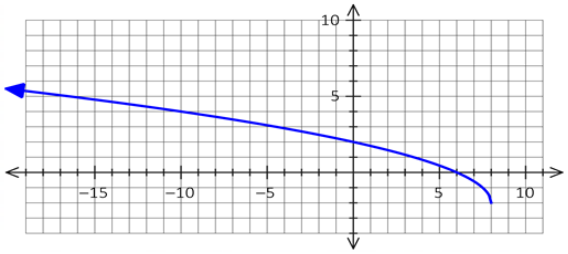
EDGE math30-1edge.com

4. The graph of $f(x) = \sqrt{-2(x-8)} - 2$ is shown below. The graph is stretched about the y-axis by a factor of 2 and reflected in the y-axis. (i) Sketch the graph of the transformed function, and determine a simplified equation, both in (i) terms of $f(x)$ and (ii) in term of x.

(a) Sketch the graph of the transformed function.

(b) Determine an equation in terms of $f(x)$

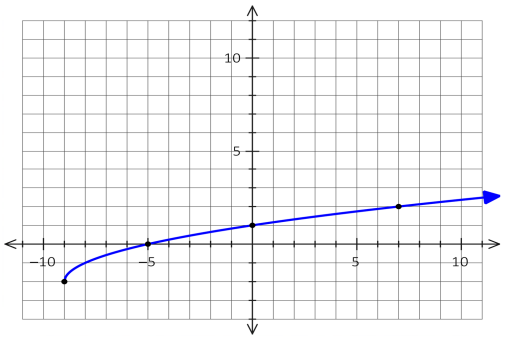
(c) … and in terms of x. *(Simplified)*



5. The graph of $f(x) = \sqrt{x+9} - 2$ is stretched about the x-axis by a factor of 3, reflected in the y-axis, horizontally translated 5 units left and vertically translated 6 units up.

(a) Construct a mapping rule to sketch the graph of the transformed function

(a) Determine a simplified equation, both in (i) terms of $f(x)$ and (ii) in term of x.



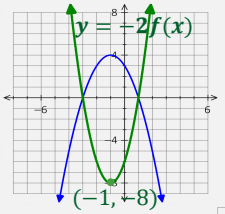
Answers from previous page

3. (a) i $(x, y) \to (x, -2y)$

ii $y = -2f(x)$

iii $y = 2x^2 + 4x - 6$

Invariant points on x-axis

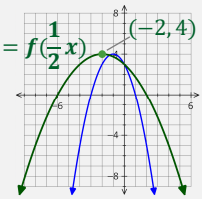


(b) i $(x, y) \to (2x, y)$ ii $y = f(\frac{1}{2}x)$

iii $y = -\left(\frac{1}{2}x\right)^2 - 2(\frac{1}{2}x) + 3$ *simplifies….*

$y = -\frac{1}{4}x^2 - x + 3$

Invariant point on y-axis



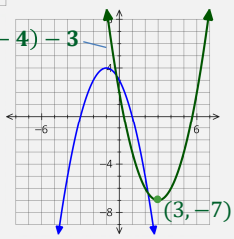
(c) i $(x, y) \to (x + 4, -y - 3)$

ii $y = -f(x - 4) - 3$

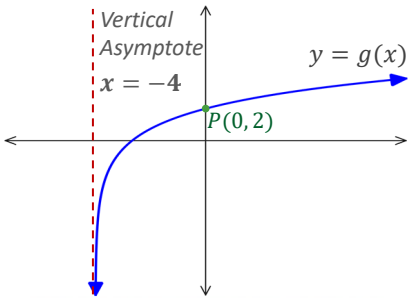
iii $y = -[-(x-4)^2 - 2(x-4) + 3] - 3$

simplifies….

$y = x^2 - 6x + 2$



6. The function $y = g(x)$, shown on the right, is transformed to $k(x) + 7 = 4\,g[-(x-2)]$.

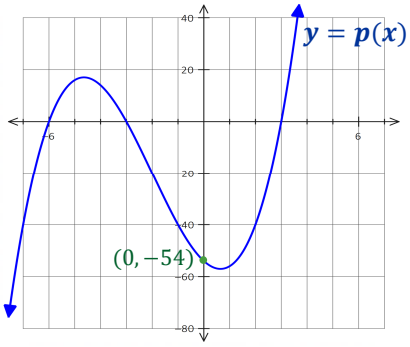


(a) State the domain of $k(x)$

(b) State the coordinates of the point P once transformed to $k(x)$.

7. Below is the graph of $\boldsymbol{p(x) = (x+6)(x+3)(x-3)}$. The x-intercepts have integer coordinates.

A new function $y = g(x)$ is obtained by stretching the graph of $p(x)$ about the line $x = 0$ by a factor of 1/3 and vertically stretching the graph by a factor of 1/6.



(a) Determine an equation for $y = g(x)$;

i *In terms of* $f(x)$*:*

ii *In terms of* x*:*

(b) Determine the coordinates of the (i) x and (ii) y-intercepts on the graph of $y = g(x)$.

8. Exam Style

A function $y = f(x)$ has a range of $(-\infty, 5]$. When the function is transformed to $y + 4 = -2f(x)$, the resulting function has a range of:

A. $(-\infty, -14]$

B. $[-14, \infty)$

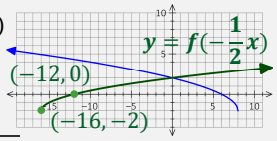
C. $(-\infty, -6]$

D. $[-6, \infty)$

Answers from previous page

4. (a) $(x, y) \to (-2x, y)$

(b) $y = f(-\frac{1}{2}x)$



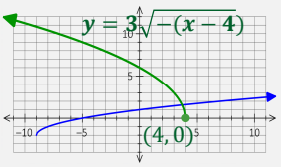
(c) $y = \sqrt{-2(-\frac{1}{2}x - 8) - 2}$ ➧ $y = \sqrt{x - 16} - 2$

5. (a) $(x, y) \to (-x - 5, 3y + 6)$

(b) $y = 3f[-(x+5)] + 6$

$y = 3(\sqrt{-(x+5)+9} - 2) + 6$

Simplifies to... $y = 3\sqrt{-(x-4)}$



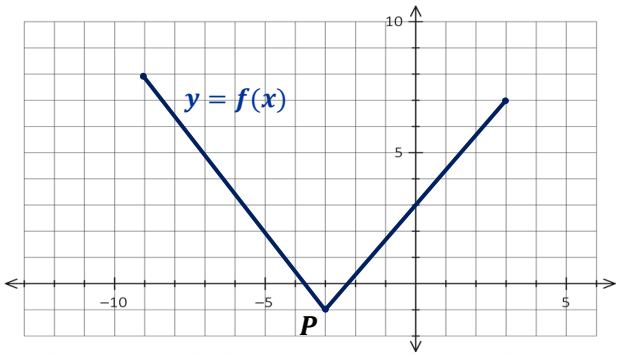
EDGE math30-1edge.com

9. A function $y = f(x)$ has a domain of $[-9, 15]$ and a range $[-8, 20]$. For each of the transformations of $f(x)$ described below, (i) provide a mapping rule and determine the new (ii) domain and (iii) range.

(a) $y = 2f(-x) - 10$ (b) $y = \frac{3}{4}f(\frac{3}{2}x - \frac{3}{2}) - 5$

10. The graph of $y = f(x)$ is shown on the right. The vertex is at point P.

For the graph of $y + 2 = \frac{1}{2}f(3x - 12)$, determine the:



i *Domain:*

ii *Range:*

iii *Coordinates of point corresponding to P on graph of $g(x)$:*

11. The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$, as shown below:

Exam Style

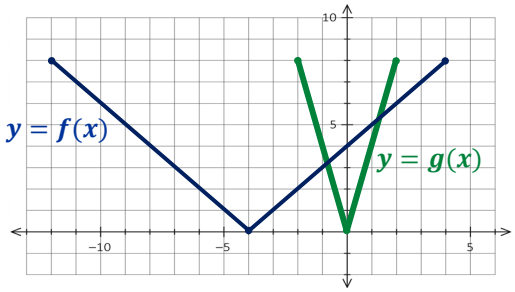
An equation for $g(x)$ in terms of $f(x)$ is:

A. $g(x) = f(4x - 1)$

B. $g(x) = f[\frac{1}{4}(x - 1)]$

C. $g(x) = f[4(x - 1)]$

D. $g(x) = f(\frac{1}{4}x - 1)$



12. Exam Style

A function $y = f(x)$ has a domain of $(-\infty, 6]$. When the function is transformed to $y = -f(2x + 4)$, the resulting function has a domain of:

A. $(-\infty, -1]$

B. $(-\infty, 1]$

C. $(-\infty, 10]$

D. $(-\infty, 16]$

Answers from previous page

6. (a) $(-\infty, 6)$ (b) $(2, 1)$

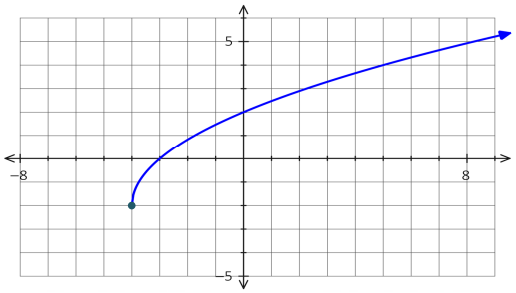
7. (a) i $g(x) = \frac{1}{6}p(3x)$ (b) i $(-2, 0)$, $(-1, 0)$, and $(1, 0)$ ii $(0, -9)$

8. B

ii $g(x) = \frac{1}{6}(3x + 6)(3x + 3)(3x - 3)$

Simplifies to... $g(x) = \frac{9}{2}(x + 2)(x + 1)(x - 1)$

13. The graph of $f(x) = 2\sqrt{x+4} - 2$ is shown on the right.



A transformed function is given by $y = f[-0.5(x+1)]$

(a) Construct a mapping rule for the transformation.

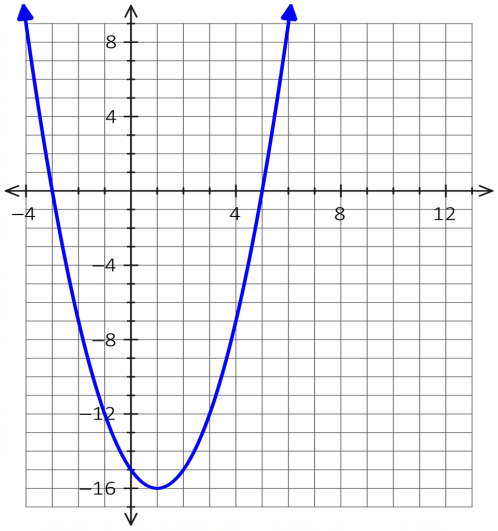
(b) Transform all points with integer coordinates to sketch the graph of the transformed function on the same grid.

(c) State the x-intercepts and domain for the graph of $y = g(x)$.

i *x-intercepts:* ii *Domain:*

(d) Write an equation for the transformed function, in terms of x.

14. The graph of $f(x) = (x-1)^2 - 16$ is shown on the right.



A function $y = g(x)$ is given by $g(x) = -\frac{1}{2}f(x-4)$

(a) Construct a mapping rule for the transformation from $y = f(x)$ to $y = g(x)$.

(b) Transform all points with integer coordinates to sketch the graph of $y = g(x)$ on the same grid.

(c) State the x and y-intercepts, and range for the graph of $y = g(x)$.

i *x-intercepts:* _____ iii *Range:* _____

ii *y-intercept:* _____

(d) Write an equation for $y = g(x)$, in terms of x.

Answers from previous page

9. (a) i $(x, y) \to (-x, 2y - 10)$ ii $[-15, 9]$ iii $[-26, 30]$ (b) i $(x, y) \to (\frac{2}{3}x + 1, \frac{3}{4}y - 5)$ ii $[-5, 11]$ iii $[-11, 10]$

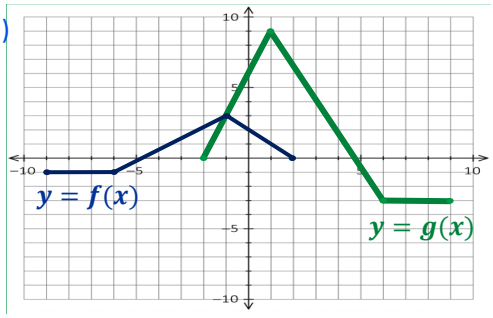
10. (a) i $[1, 5]$ ii $[-2.5, 2]$ iii $(3, -2.5)$

11. B **12.** B

EDGE math30-1edge.com

15. For each pair of graphs below, the graph of $y = g(x)$ represents a transformation of of $y = f(x)$ through any of stretches, reflections, and translations. Determine an equation for $y = g(x)$, in terms of $f(x)$.

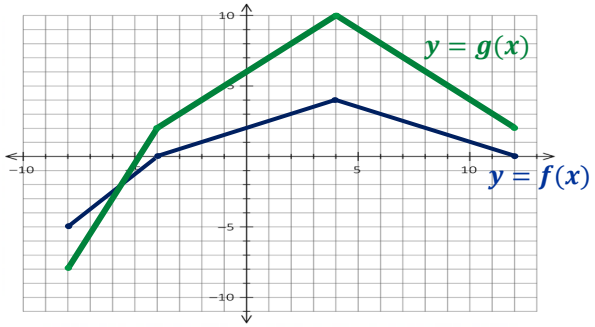
(a)



i *Mapping Rule:* _____

ii *Equation of* $g(x)$*:* _____

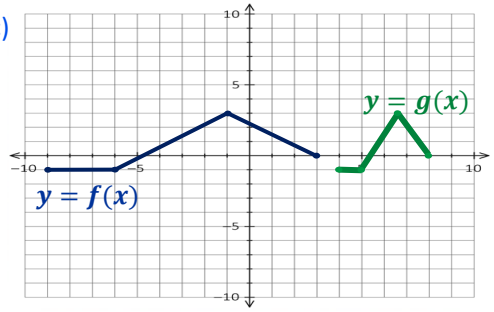
(b)



i *Mapping Rule:* _____

ii *Equation of* $g(x)$*:* _____

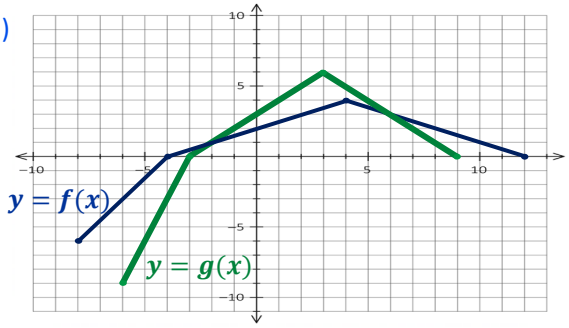
(c)



i *Mapping Rule:* _____

ii *Equation of* $g(x)$*:* _____

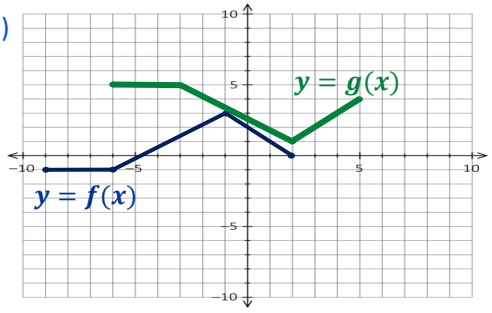
(d)



i *Mapping Rule:* _____

ii *Equation of* $g(x)$*:* _____

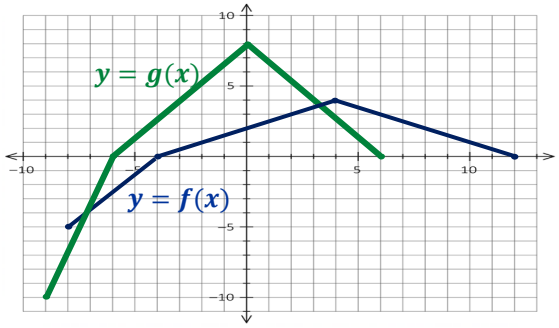
(e)



i *Mapping Rule:* _____

ii *Equation of* $g(x)$*:* _____

(f)



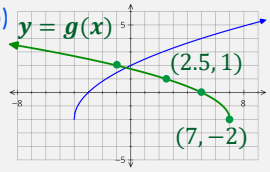
i *Mapping Rule:* _____

ii *Equation of* $g(x)$*:* _____

Answers from previous page

13. (a) $(x, y) \to (-2x - 1, y)$ (c) i $(5, 0)$ ii $(-\infty, 7]$

(b)



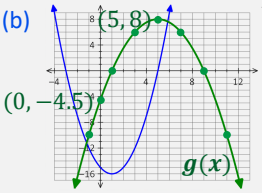
(d) $y = 2\sqrt{-0.5(x + 1) + 4} - 2$

simplifies to...

$g(x) = 2\sqrt{-0.5(x - 7)} - 2$

14. (a) $(x, y) \to (x + 4, -\frac{1}{2}y)$ (c) i $(1, 0)$ and $(9, 0)$ ii $(0, -4.5)$ iii $(-\infty, 8]$

(b)

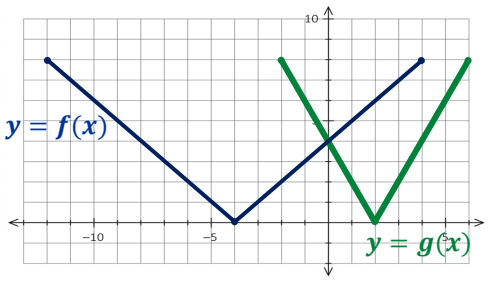


(d) $y = -\frac{1}{2}((x - 4 - 1)^2 - 16)$

simplifies to...

$g(x) = -\frac{1}{2}(x - 5)^2 + 8$

16. The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$, as shown:



(a) Use proper terminology to explain how the graph of $y = g(x)$ can be obtained by applying the following transformations to the graph of $y = f(x)$.

ii A stretch and a reflection

ii A stretch and a translation

(b) Provide a mapping rule for each sequence of transformations described above.

i _____ ii _____

(c) Based on the sequence of transformations described above, provide two possible equations for $g(x)$, in terms of $f(x)$.

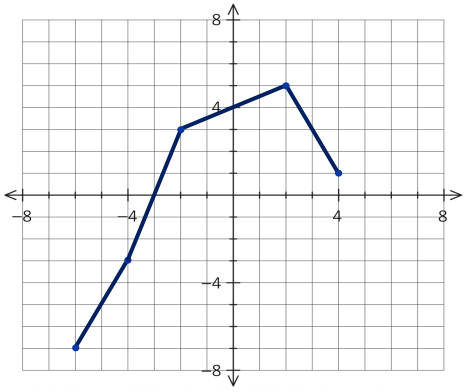
i _____ ii _____

17. The graph of $y = \sqrt{x}$ is transformed into the graph of $y = \sqrt{4x + 8}$.

(a) Provide a mapping rule and explain how the transformation above can be describes as a horizontal stretch about the y-axis and horizontal translation. Justify your answer by manipulating the equation.

(b) Provide a mapping rule and explain how the transformation above can be describes as a vertical stretch about the x-axis and horizontal translation. Justify your answer by manipulating the equation.

18. The graph of $y = f(x)$ is shown on the right.



(a) Sketch the graph of $x = f(y)$ on the same grid

(b) State the coordinates of any invariant point(s)

(c) State the domain and range of both $y = f(x)$ and $x = f(y)$

$y = f(x)$ D: _____ $x = f(y)$ D: _____

R: _____ R: _____

(d) Provide two possible domains of $f(x)$ so that the inverse is a function.

Answers from previous page

15. (a) i $(x, y) \to (-x, 3y)$ ii $g(x) = 3f(-x)$

(b) i $(x, y) \to (x, 2y + 2)$ ii $g(x) = 2f(x) + 2$

(c) i $(x, y) \to (\frac{1}{3}x + 7, y)$ ii $g(x) = f[3(x - 7)]$

(d) i $(x, y) \to (0.75x, 1.5y)$ ii $g(x) = \frac{3}{2}f(\frac{4}{3}x)$

(e) i $(x, y) \to (x + 3, -y + 4)$ ii $g(x) = -f(x - 3) + 4$

(f) i $(x, y) \to (0.75x - 3, 2y)$ ii $g(x) = 3f[\frac{4}{3}(x + 3)]$

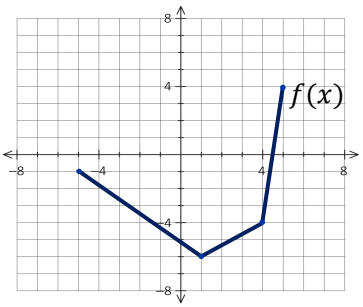
EDGE www.math30-1edge.com

19. (a) A point $P(-3,4)$ is on the graph of $y=f(x)$. Determine the coordinates of the point that corresponds to P on the graph of $y=-f(3x+9)-4$.

(b) A point $P(-3,4)$ is on the graph of $y=f(x)$. Determine the coordinates of the point that corresponds to P after the graph of $f(x)$ is horizontally stretched by a factor of 1/2, vertically stretched by a factor of 4, reflected in the line $y=0$, and horizontally translated 3 units left.

20. Exam Style

The graph of $y=f(x)$, as shown, is transformed to become the graph of $g(x)+3=-\frac{3}{2}f(x+3)$. The range of $y=g(x)$ is:



A. $[-7,-1/3]$

B. $[-12,3]$

C. $[-3,12]$

D. $[-9,6]$

21. Exam Style

The point $P(-8,2)$ lies on the graph of a function $y=f(x)$. Listed below are all possible coordinates of the new point corresponding to P after $f(x)$ undergoes a transformation.

Point 1 $(-6,2)$	**Point 2** $(-18,2)$	**Point 3** $(6,2)$	**Point 3** $(-10,-2)$
Point 4 $(-8,-1)$	**Point 5** $(2,-8)$	**Point 6** $(-8,2)$	**Point 6** $(-2,2)$

NR The correspoing point when the graph of $y=f(x)$ is

Reflected in the line $x=0$ and horizontally translated 2 units left is _____. *First digit of your answer*

Vertically stretched about the line $y=0$ by a factor of 2 and vertically translated 2 units down is _____. *Second digit of your answer*

Reflected in the line $y=x$ is _____. *Third digit of your answer*

Horizontally stretched about the line $x=0$ by a factor of $1/2$ and horizontally translated 2 units left is _____. *Second digit of your answer*

Answers from previous page

16. (a) i A horizontal stretch about the y-axis by a factor of 1/2, and a horizontal reflection about the y-axis.
i A horizontal stretch about the y-axis by a factor of 1/2, and a horizontal translation 4 units right.

(b) i $(x,y)\to(-\frac{1}{2}x,y)$ ii $(x,y)\to(\frac{1}{2}x+4,y)$ (c) i $y=f(-2x)$ ii $y=f[2(x-4)]$

17. (a) First re-write as $y=\sqrt{4(x+2)}$ $(x,y)\to(\frac{1}{4}x-2,y)$

A horizontal stretch about the y-axis by a factor of 1/4, and a horizontal translation 2 units left.

(b) First re-write as $y=\sqrt{4}\sqrt{x+2}$ ➔ $y=2\sqrt{x+2}$. $(x,y)\to(x-2,2y)$

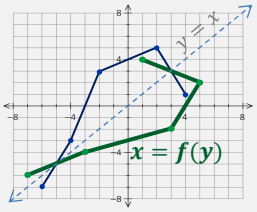
A vertical stretch about the x-axis by a factor of 2, and a horizontal translation 2 units left.

18. (a) $(x,y)\to(y,x)$ *so,* $(-6,-7)\to(-7,-6)$ *... and so on*

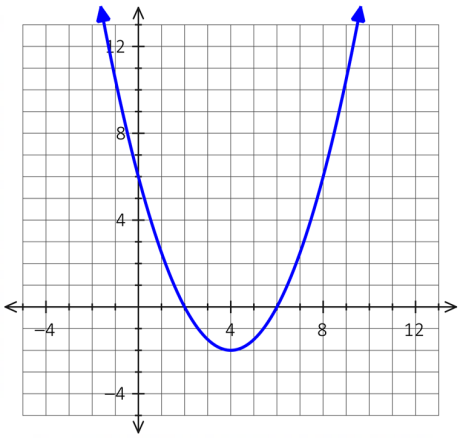
(b) $(-5,-5)$ and $(3,3)$ *on the line* $y=x$

(c) $f(x)$ *D:* $[-6,4]$ *R:* $[-7,5]$ $x=f(y)$ *D:* $[-7,5]$ *R:* $[-6,4]$

(d) *D:* $[-6,2]$ or $[2,4]$



22. The graph of $f(x) = 0.5(x-4)^2 - 2$ is shown on the right.



(a) Sketch the graph of the inverse of $f(x)$ by transforming all points with integer coordinates.

(b) State location and number of any invariant point(s).

(c) State the domain and range of both $y = f(x)$ and $x = f(y)$.

$y = f(x)$ D: _____ $x = f(y)$ D: _____

R: _____ R: _____

(d) State two possible domain restrictions so that the inverse is a function.

(e) Determine an equation for the inverse, in terms of x.

23. Exam Style

A function of $y = \sqrt{x}$ is transformed to $y + 1 = \sqrt{-3x + 9}$.

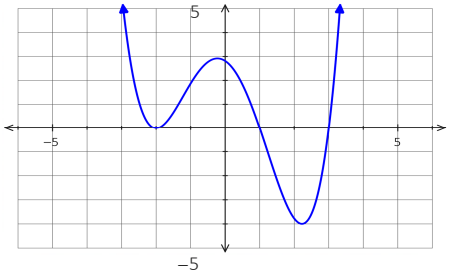
Below is a list of possible transformation descriptions.

0 Stretch about the line $x = 0$ by a factor of 3
1 Stretch about the line $x = 0$ by a factor of 1/3
2 Stretch about the line $y = 0$ by a factor of 3
3 Stretch about the line $y = 0$ by a factor of 1/3
4 Reflection in the line $x = 0$
5 Reflection in the line $y = 0$
6 Horizontal translation 9 units left
7 Horizontal translation 9 units right
8 Horizontal translation 3 units left
9 Horizontal translation 3 units right

NR A sequence of transformations that describes the changes to the graph of $y = \sqrt{x}$ is numbered , ____ , ____, _____, and ____.

24. Exam Style

The graph of $y = f(x)$ is shown below. The graph of $y = g(x)$ is obtained by reflecting the graph of $f(x)$ in the line $y = x$.



The number of invariant points in this transformation is:

A. 0
B. 1
C. 2
D. 3

Answers from previous page ***and this page***

19. (a) $(-4, -8)$ (b) $(-4.5, -16)$ **20.** **D** **21.** **3651**

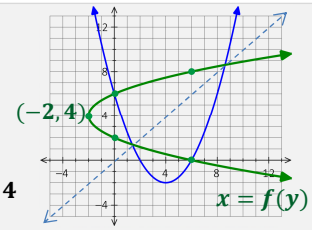
22. (a) $(x, y) \to (y, x)$ *so,* $(4, -2) \to (-2, 4)$ *... and so on*

(b) ***Two*** *invariant points on the line* $y = x$

(c) $f(x)$ D: $\{x \in \mathbb{R}\}$ R: $\{y | y \geq -2, y \in \mathbb{R}\}$

$x = f(y)$ D: $\{x | x \geq -2, x \in \mathbb{R}\}$ R: $\{x \in \mathbb{R}\}$

(d) D: $\{x | x \leq 4, x \in \mathbb{R}\}$ or $\{x | x \geq 4, x \in \mathbb{R}\}$ (e) Inverse: $y = \pm\sqrt{2(x+2)} + 4$



23. **149** **24.** **C**

EDGE math30-1edge.com