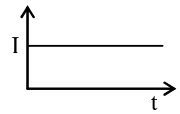
ALTERNATING CURRENT

1. DIRECT CURRENT ::

(i) If the magnitude and direction of current do not change with time then it is called direct current (DC).
Example : Current drawn from the cell.

(ii) Frequency of DC is Zero.

(iii) Graph is as shown in Figure.



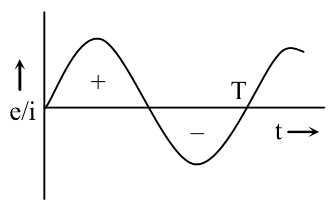
2. ALTERNATING CURRENT OR ALTERNATING EMF ::

(i) The emf or current whose magnitude changes continuously with time between Zero and a maximum value and whose direction reverses periodically.

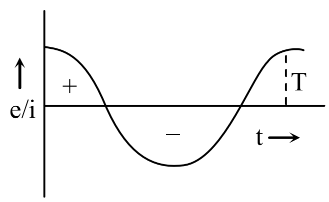
(ii) It has both negative and positive values.

(iii) The time taken to complete one cycle of variations is called the 'periodic-time' or time-period.

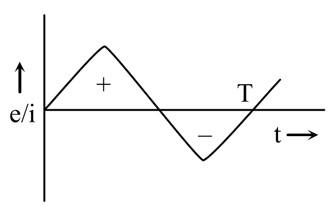
(iv) Graphical representation of same A.C.

(a)


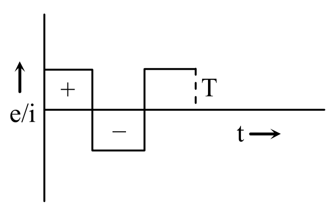
(Sinusoidal)

(b)


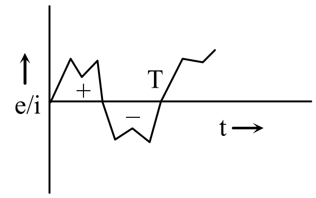
(Cosinusodial)

(c)


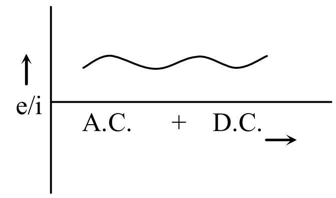
(Triangular)

(d)


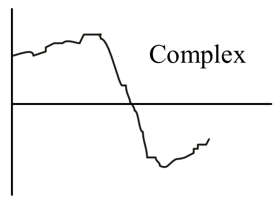
(Rectangular/T square)

(e)


(Complex)

(f)


(AC superimposed on DC)

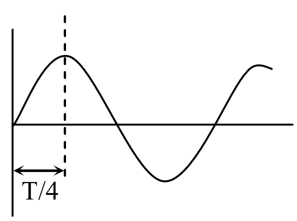
(g)


(v) AC/A emf is positive for half the cycle and negative for the rest half. Hence, Average value of AC/Aemf over a complete cycle is ZERO.

(vi) Area under the positive cycle is equal to area under negative cycle.

(vii) Ohm's law, Kirchoff's law and Joule's law are applicable for AC/Aemf.

(viii) Value of AC/Aemf reaches from Zero to maximum and vice-versa in time T/4, where T is the time-period of the cycle. Graphically,



(ix) The number of cycle completed by an AC/Aemf in one second is called the frequency. It is denoted by 'n'. $n = \frac{1}{T}$

(x) The value of AC is Zero and maximum two times each in a complete cycle.

(xi) The value of AC/Aemf is zero or maximum 2n times every second. The direction also changes '2n' times every second.

(xii) Rate of change of AC/A emf is maximum while changing the direction and minimum at the peak value.

(xiii) It's frequency is not zero.

(xiv) Generally, sinusoidal wave form is used as Alternating current/emf.

(xv) Direction of current flow is not shown in circuits employing AC/Aemf.

(xvi) Frequency of AC used in India is 50 Hz .

Ex.1 If the frequency of alternating emf is 50 Hz then the direction of emf changes in one second by -

(1) 50 times (2) 100 time

(3) 200 times (4) 500 times

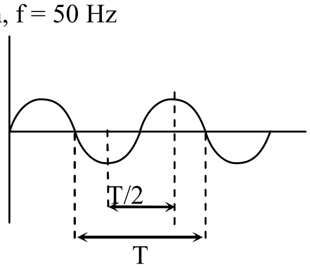
Sol. (2) In one cycle, direction of emf changes 2 times. The frequency is 50 Hz means there are 50 cycles per second and hence direction will change 100 times.

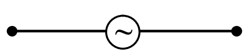
Ex.2 In the above example, what is the time taken to attain maximum positive from maximum negative value.

(1) $\frac{1}{100}$ sec (2) $\frac{1}{200}$ sec

(3) 100 sec. (4) 200 sec.

Sol. (1) Given, f = 50 Hz



$T = \frac{1}{f} = \frac{1}{50}$ sec 

according to the figure time taken

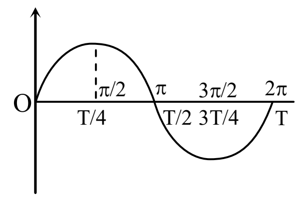
$= \frac{T}{2} = \frac{1}{100}$ sec and this is denoted by

3. SINUSOIDAL WAVE FORM ::

(i) Mathematical representation :

$E = E_0 \sin\omega t$	$I = I_0 \sin\omega t$
$E = E_0 \sin(\omega t + \phi)$	$I = I_0 \sin(\omega t + \phi)$
$E = E_0 \text{Cos}\omega t$	$I = I_0 \cos\omega t$
$E = E_0 \text{Cos}(\omega t + \phi)$	$I = I_0 \cos(\omega t + \phi)$
$E = a \text{Cos}\omega t + b \sin\omega t$	$I = a \cos\omega t + b \text{Cos}\omega t$

(ii) Graph:



4. MEASUREMENT OF AC CURRENT AND EMF ::

(i) DC ammeter and voltmeter are based on the principle of torque acting on a moving coil placed in a magnetic field. Hence it cannot be used for AC.

(ii) On AC measuring device should not depend on the direction of current.

(iii) Hot wire ammeter and Voltmeters are used for this purpose.

(iv) Hot wire instruments are based on heating effect of current. ($H = I^2Rt$).

(v) These devices gives the RMS values (not the amplitude) of voltage and current.

(vi) The scale of AC instruments is not linear unlike DC instruments.

(vii) Hot wire instruments can measure DC quantities also.

(viii) If AC is produced by a generator having a large number of poles then frequency of emf-

$$= \frac{\text{Frequency of rotation per second}}{2} = \frac{pn}{2}$$

(ix) L-C converter circuit is used to convert AC in to DC.

5. DEFINITIONS USED IN A.C. CIRCUITS ::

(i) Instantaneous value of A.C. (I)

(a) The electric current flowing at any instant of time in an A.C. circuit is defined as instantaneous value of A.C.

(b) A.C. is represented by the equation $I = I_0 \sin\omega t$

(ii) Amplitude of A.C. I_0 –

(a) The maximum value of A.C. is defined as amplitude or peak value of A.C.

(b) The value of A.C. becomes maximum twice in one cycle.

(iii) Average value of A.C. (or I) <I> or $\overline{I}$:

(a) The average of an A.C. during one cycle is defined as the average value of A.C.

(b) $<I> = \dfrac{\int_0^T I\,dt}{T}$

(c) For one complete cycle <I> = 0

(d) For half cycle $<I> = \dfrac{2I_0}{\pi} = 0.636\ I_0$

(iv) Frequency of A.C. (f) -

(a) The number of cycles completed by A.C. in one second is defined as its frequency.

(b) Its value is equal to the number of rotations made by the coil in magnetic field in one second

(c) It is equal to $\dfrac{\omega}{2\pi}$ or $\dfrac{1}{T}$

(d) The frequency of A.C. used in houses is 50Hz. It means that current flows for 50 times in one direction and 50 times in the opposite direction in an electric circuit.

(v) Time period A.C. (T) -

(a) The time during which A.C. completes one cycle is defined as its time period.

(b) Its value is equal to the time taken by the coil to complete one rotation in magnetic field.

(vi) Mean square value of A.C. $<I^2>$ -

(a) The mean of square of A.C. in one complete cycle is defined as mean square value of A.C.

(b) $<I^2> = \dfrac{1}{T}\int_0^T I^2\ dt$

(c) It value is $<I^2> = \dfrac{I_0^2}{2}$ which is not zero.

(vii) Root Mean square (R.M.S.) value or apparent value or effective value of A.C. (I_{rms})

(a) The square root of mean square value of A.C. is defined as R.M.S. value of A.C.

(b) It is equal to that direct current which produced same heating in a resistance as is produceed by the A.C. in same resistance during same time

(c) $I_{rms} = \sqrt{<I^2>} = \sqrt{\dfrac{1}{T}\int_0^T I_0^2 \sin^2 \omega t\, dt}$

(d) $I_{rms} = \dfrac{I_0}{\sqrt{2}} = 0.707\ I_0 = 70.7\% I_0$

(e) As this current produces same amount of heat or power in a resistance as is produces by D.C. hence it is also called the effective value of A.C.

(f) The A.C. ammeter and voltmeter measure its R.M.S. values because to measure alternating current or voltage, the instruments based on heating effect are required i.e. we measure $<I^2>$ instead of <I>

(viii) Peak to Peak value of A.C.

(a) The sum of maximum value of A.C. in positive half cycle and maximum value in negative half cycle is defined as peak to peak value of A.C.

(b) $I_{pp} = |+I_0| + |-I_0| = 2I_0$

(c) $I_{pp} = 2\sqrt{2}\ I_{rms} = 2.828\ I_{rms}$

(ix) Form factor of A.C. (F) –

The ratio of the R.M.S. value of A.C. to its average during half cycle is defined as the form factor of A.C.

(a) $F = \dfrac{I_{rms}}{<|I|>}$

(b) $F = \dfrac{I_0}{\sqrt{2}}\ \dfrac{\pi}{2I_0} = \dfrac{\pi}{2\sqrt{2}}$

6. PHASE ::

If a quantity is represented as

$X = X_0 \sin(\omega t \pm \phi)$, then

(i) phase = $(\omega t \pm \phi)$, then

Where, ωt= Instantaneous phase

ϕ= Initial phase

(ii) Instantaneous phase changes with time.

(iii) Initial phase is constant w.r.t time.

(iv) Phase determines the direction as well as magnitude of current/emf.

(v) Unit is Radian.

(vi) It is a DIMENSIONLESS quantity

(vii) Current and emf in a circuit have the same frequency but different phase unless the circuit is totally resistive.

6.1 Phase - difference:

(i) The difference between the phases of two currents and emf's is called phase-difference.

(ii) If, emf and current are given by :

$E = E_0 \sin(\omega t + \phi_1)$

$I = I_0 \sin(\omega t + \phi_2)$ Then,

phase of current relative to emf $= \phi_2 - \phi_1$

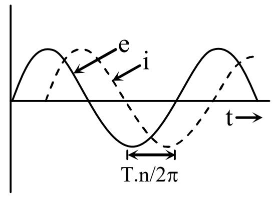
and phase of emf relative to current $= \phi_1 - \phi_2$

(iii) Phase-difference, generally, is given relative to current.

(iv) The quantity with higher phase is supposed to be leading and the other quantity is taken to be lagging.

Graphical-Representation:-

(i) If emf and currents are given by

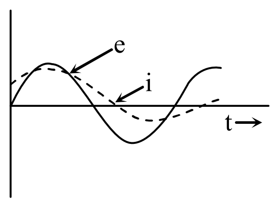


$E = E_0 \sin\omega t$

$I = I_0 \sin(\omega t - \phi)$

phase - difference = $0 - (-\phi) = \phi$

(ii) If current and emf are given by



$I = I_0 \sin(\omega t + \phi)$

$E = E_0 \sin(\omega t)$

Phase difference = $O - \phi = -\phi$

$\Rightarrow$ E is lagging and I is leading.

$\frac{t}{T} = \frac{\phi}{2\pi}$ i.e. change in phase after time

$t = \frac{2\pi t}{T} = \phi.$

Ex.3 The peak value of an A.C. emf given by $e = 3 \sin(314\ t) + 4 \cos(314\ t)$ where, e is in volts is -

(1) 5 (2) 3 (3) 4 (4) $\frac{5}{\sqrt{2}}$

Sol. (1) The given AC emf can be written in sinusiodal form, by using the substitution.

$3 = a \cos\theta,$

$4 = a \sin\theta$ ....(A)

Then $e = a\{\cos\theta \sin(314\ t) + \sin\theta \cos(314\ t)\}$

$= a \sin(314\ t + \theta)$

where, the peak value a is determined by squaring and adding the terms in eq. (A)

$a^2 = 3^2 + 4^2$ or $a = 5$

The peak value is 5.

Ex.4 For an ac current given by $I = 0.311 \sin(100\ \pi t - \pi/4)$ ampere, what is (i) peak value, (ii) frequency (iii) time period, (iv) initial phase and (iv) initial phase and (v) the instantaneous value at t = 0 sec?

Sol. Comparing with $I = I_0 \sin(\omega t + \phi)$, we have

(i) peak value $I_0 = 0.311$ A

(ii) angular frequency $\omega = 100\pi$.

$\therefore$ $f = \omega/2\pi = 50$ Hz.

(iii) time period $T = 1/50 = 0.02$ sec.

(iv) initial phase $\phi = -\pi/4 = -45^o$

(v) instantaneous value at t = 0,

$I = 0.311 \sin(0 - \pi/4) = -0.266$ A

Ex.5 If the household supply is at 220V, 50Hz, then the equation for instantaneous voltage, assuming V = 0 at t = 0 will be -

(1) 50 sin (220 πt) (2) 220 sin (50 πt)

(3) 311 sin (100 πt) (4) 100 sin (311 πt)

Sol. (3) Peak value $V_0 = \sqrt{2} \times 220 = 311$ V

The angular frequency $\omega = 2\pi \times 50 = 100\pi$

Thus the expression for instantaenous voltage is

$v = 311 \sin(100\ \pi t)$

Ex.6 The electric mains in a house are marked 220V–50Hz. Write down the equation for instantaneous voltage. Also find the time-period, peak to peak value and average over positive half cycle.

Sol. Equation for alternating voltage is

$E = E_0 \sin\omega t$

given, $E_{rms} = 220V$

$f = 50$

$E_0 = \sqrt{2}\ E_{rms} = 220 \times \sqrt{2} = 311V$

$\omega = 2\pi f = 100\pi \Rightarrow E = 311 \sin(100\ \pi t)$

Time – period $= \frac{1}{f} = \frac{1}{50} = 20$ ms

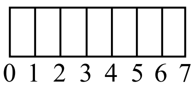
Peak to Peak value = $2E_0 = 2 \times 311 = 622$ V

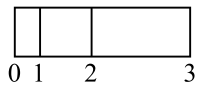
average over positive half cycle

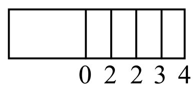
$= \frac{2}{\pi} E_0 = \frac{2}{\pi} \times 311 \approx 200V$

$\langle E \rangle = 0$

Ex.7 The current scale for AC ammeter is

(1) 

(2) 

(3) 

(4) None of these.

Sol. (2) Deflection in AC ammeter is directly proportional to the square of current and not the current

Ex.8 The RMS value of current for a current equation, $i = i_1 \cos\omega t + i_2 \sin\omega t$ is -

(1) $\frac{1}{\sqrt{2}}(i_1 + i_2)$ (2) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$

(3) $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$ (4) $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^2$

Sol. (3) $i = i_1 \cos\omega t + i_2 \sin\omega t = i_0 \sin(\omega t + \phi)$

Where $i_0 = \sqrt{i_1^2 + i_2^2}$ and $\phi = \tan^{-1}\left(\frac{i_1}{i_2}\right)$

$\Rightarrow \quad i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$

Note : Some of the alternatives can be excluded by careful observation. Use dimensional analysis. i.e. options (2) and (4) are having dimension of i^2 while required answer should have dimension of i.

Ex.9 An AC current is given by $I = I_0 + I_1 \sin\omega t$ then its rms value will be-

(1) $\sqrt{I_0^2 + 0.5 I_1^2}$ (2) $\sqrt{I_1^2 + 0.5 I_0^2}$

(3) 0 (4) $I_0/\sqrt{2}$

Sol. (1)

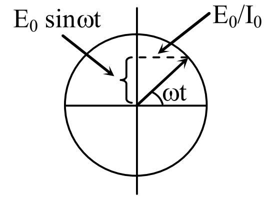
$I_{rms} = \sqrt{< I^2 >} = \{<I_0^2 + I_1^2 \sin^2\omega t + 2I_0 I_1 \sin\omega t>\}^{1/2}$

Now we use the fact that I_0, I_1 are constants as far as time is concerned, and

$< \sin\omega t > = 0$, $<\sin^2\omega t> = \frac{1}{2}$

Thus $I_{rms} = \{I_0^2 + I_1^2/2\}^{1/2} = \sqrt{I_0^2 + 0.5 I_1^2}$

7. VECTOR REPRESENTATION ::



(i) An anticlockwise rotating vector in XY plane represents a physical quantity.

(ii) Length of the vector is equal to the amplitude or peak value of the quantity.

(iii) Time taken to complete a circle is same as the time period 'T' of the quantity.

(iv) The instantaneous value of the alternating quantity is given by x or y component of the vector.

RESISTANCE (R) :

$$R = \frac{E}{I} = \frac{E_{rms}}{I_{rms}} = \frac{E_0}{I_0}$$

REACTANCE :

(i) The resistance offered by a capacitor or inductor coil is called reactance.

(ii) Reactance is that part of impedance in which the phase-difference between current and emf is $\frac{\pi}{2}$.

(iii) The value of reactance is given by ratio of potential difference and current flowing.

(iv) Power dissipation is a reactance is zero because angle between emf and current is 90^0.

(v) Reactance is of two types :

(i) Inductive Reactance (ii) Capacitive Reactance

$X_L = \omega L$ $X_c = \frac{1}{\omega C}$

(vi) To solve the circuit :

Resistance of wire = R

Resistance of inductor = $\omega L\, i$

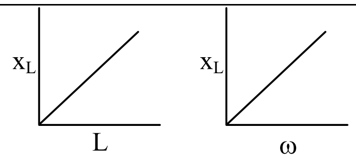
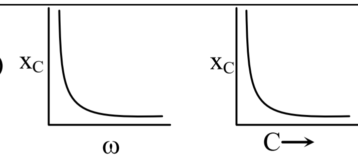
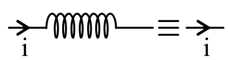
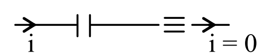
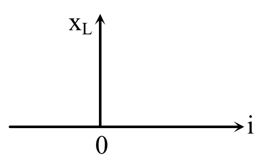
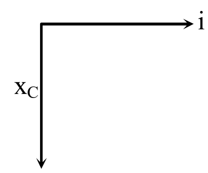
Resistance of capacitor = $-\frac{1}{\omega C} i$

Circuit is solved using laws of ohm, Kirchoff and complex algebra.

Note : (vii) Appearing here is ' iota ' of complex algebra . $i = \sqrt{-1}$.

(viii) Phase-difference between capacitive and inductive reactance is π .

(ix) Comparative study of the two reactances :

INDUCTIVE	CAPACITIVE
(i) $X_L = \omega L$	(i) $X_C = \frac{1}{\omega C}$
(ii) 	(ii) 
(iii) X_L for DC = 0 ($\omega = 0$)	(iii) X_C for DC = ∞ ($\omega = 0$)
(iv) No resistance is offered by an inductor coil to DC. It simply behaves as closed circuit open 	**(iv) Infinite resistance is offered by a capacitor to DC. It be circuit haves as** 
(v) Unit : ohm	(v) unit : ohm
(vi) To solve the circuit, $X_L = \omega L i$	(vi) To solve the circuit, $X_C = -\frac{1}{\omega C}\ i$
(vii) Vector representation : 	(vii) Vector representation : 
(viii) $X_L = \omega L + \frac{\pi}{2} i$ $\therefore$ Voltage leads the current by $\frac{\pi}{2}$	(viii) $X_C = \omega C - \frac{\pi}{2} i$ $\therefore$ Current leads the voltage by $\frac{\pi}{2}$

IMPEDANCE (Z) ::

(i) When both resistance and reactance are present in the circuit, the effective resistance offered to the flow of current by the combination is called impedance .

(ii) $Z = Z \mid \phi = Z(\cos\phi + i \sin \phi)$

Where $Z \cos\phi$ resistive part

and $Z \sin\phi$ is reactive part .

ϕ is the angle of voltage relative to current.

$\Rightarrow Z \cos \phi = R$ ……..(A)

$\Rightarrow Z \sin \phi = (X_L - X_C)$ ……..(B)

or $Z = R + (X_L - X_C)\ i$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

(using complex algebra).

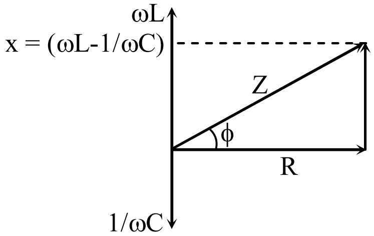
(iii) Magnitude of Z is given by

$$\frac{E}{I} = \frac{E_{rms}}{I_{rms}} = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

(iv) Angle ϕ is given by

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \text{ (From A \& B)}$$

(v) Phase representation :



SUSCEPTANCE

(i) The reciprocal of reactance is called susceptance.

(ii) Susceptance = $\frac{1}{X} = \left(\frac{1}{X_L - X_C}\right)$

Where $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

(iii) units : mho or $(\text{ohm})^{-1}$

(iv) Inductive susceptance = $\frac{1}{\omega L}$

(v) Capacitive susceptance = ωC

ADMITTANCE :

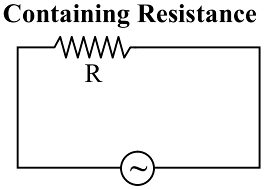
(i) The reciprocal of impedance is called admittance .

(ii) Admittance = $\frac{1}{Z} = \frac{1}{\sqrt{R^2 + (X_L - X_C)^2}}$

(iii) Unit : mho or $(\text{ohm})^{-1}$

8. DIFFERENT TYPES OF A. C. CIRCUITS

(A) Circuit Containing Resistance Only :



$E = E_0 \sin\omega t$

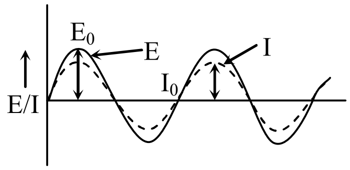
(i) Reactance = 0
Impedance = Resistance = R

(ii) Current I = $\left(\frac{E}{R}\right) = \frac{E_0 \sin\omega t}{R} = I_0 \sin\omega t$

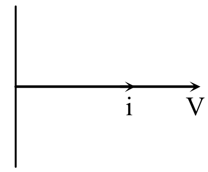
where $I_0 = \frac{E_0}{R}$

(iii) Current and voltage across a resistance are in the same phase .

(iv) Graphically,



(v) Phaser representation



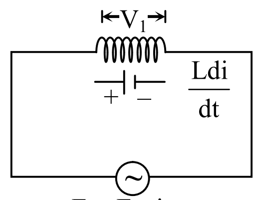
(vi) power loss = $I^2_{rms} R = \frac{E^2_{rms}}{R}$

$= I_{rms}E_{rms} = \frac{I_0 E_0}{2}$

(vii) No energy storage .

(B) Circuit containing inductance only :

(i) Impedance = Reatance = ωL



$E = E_0 \sin\omega t$

(ii) $V_L = L\frac{di}{dt} = E_0 \sin\omega t$

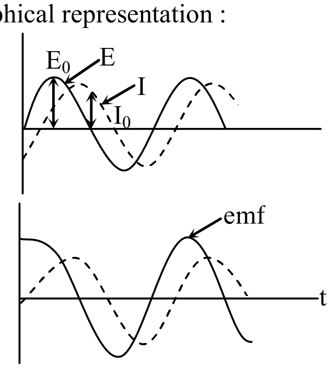
$I = \frac{-E_0 \cos\omega t}{\omega L}$

$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

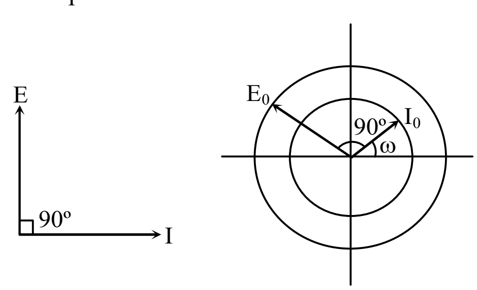
where $I_0 = \frac{E_0}{\omega L}$

Voltage leads the current by $\frac{\pi}{2}$.

(iii) Graphical representation :



(iv) Vector Representation :



(v) Energy loss = 0

because phase difference between I and E is $\frac{\pi}{2}$.

Energy loss is given by $I_{rms}\,E_{rms}\cos\phi$

(vi) Energy stored = $\frac{1}{2}LI^2$

This energy is stored in the magnetic field of coil .

(vii) Average energy stored = $\frac{1}{4}\,LI_0^2 = \frac{1}{2}\,LI^2_{rms}$

Ex.10 An AC circuit contains an inductance only, with L = 200 mH. If the frequency is 50 Hz and peak value of emf is 31.4 volt, then the inductive reactance and the current amplitude in the circuit will respectively be-

(1) 62.8Ω, 5A (2) 62.8Ω, 0.5A
(3) 6.28Ω, 0.05A (4) 6.28Ω, 0.5A

Sol. (2) $X_L = \omega L = 2\pi \times 50 \times 200 \times 10^{-3} = 20\pi = 62.8$ ohm.

$I_0 = E_0/X_L = 31.4/62.8 = 0.5$ A.

(C) Circuit Containing Capacitance only :

(i) Impedance (Z) = Reactance (X) = $X_C = \frac{1}{\omega C}$

(ii) $I = C\frac{dV_C}{dt}$ and V_C = voltage across capacitor

here, $V_C = E_0 \sin\omega t$

$I = \omega CE_0 \cos\omega t$

$I = \frac{E_0}{(1/\omega C)}\sin(\omega t + \frac{\pi}{2})$

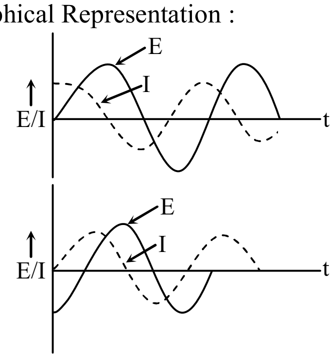
$I = I_0 \sin(\omega t + \frac{\pi}{2})$

where $I_0 = \omega CE_0 = \frac{E_0}{(1/\omega C)}$

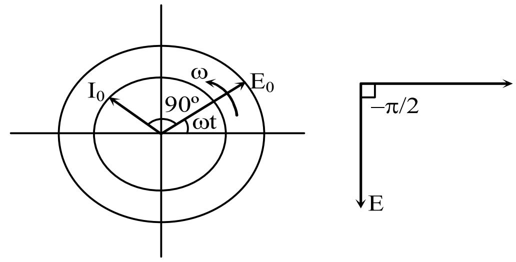
⇒ current leads the voltage by 90^0

(iii) phase of voltage lags the current by $\frac{\pi}{2}$

(iv) Graphical Representation :



(v) Vector Representation :



(vi) Energy loss = 0.

because angle between E and I is π/2 .

(vii) Energy stored = $1/2CV^2$

This energy is stored in the electric field of the capacitor.

Average energy stored = $1/2\ CE^2_{rms} = 1/4\ CE_0^2$

Ex.11 Let f = 50 Hz, and C = 100 μF in an AC circuit containing a capacitor only. If the peak value of the current in the circuit is 1.57 A, The expression for the instantaneous voltage across the capacitor will be-

(1) E = 50 sin (100 πt – π/2)
(2) E = 100 sin (50 πt)
(3) E = 50 sin 100 πt
(4) E = 50 sin (100 πt + π/2)

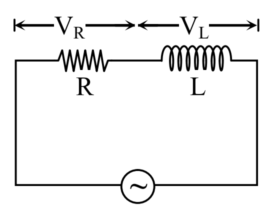
Sol. (1) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = \frac{100}{\pi}$ ohm.

$E_0 = I_0 X_C = 1.57 \times \frac{100}{3.14} = 50$ volt.

Therefore, E = 50 sin (100π t - π/2)
(If we assume the current is zero at t = 0, and then increases).

(D) L-R series circuit :

(i) Resistance = R



$E = E_0 \sin\omega t$

Reactance = ωL i (i ≠ current, iota $i = \sqrt{-1}$)

Z = Impedance = R + ω L i

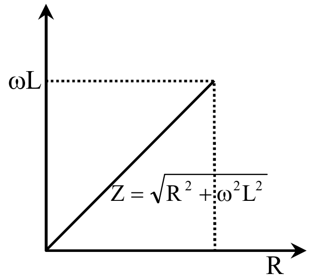
(ii) $E = E_0 \sin\omega t$

$I = I_0 \sin(\omega t - \phi)$

where, $I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{\omega^2 L^2 + R^2}}$

and $\phi = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\left(\frac{V_L}{V_R}\right)$

(iii) Voltage equation :



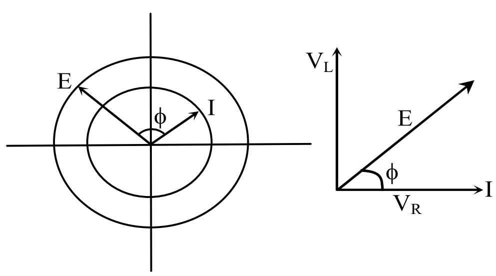
$V = V_R + V_L\, i$ V - applied potential

$|V| = \sqrt{V_R^2 + V_L^2}$ V_R - p.d. across resistance

V_L - p.d. across inductance

V_L leads V_R by $\pi/2$

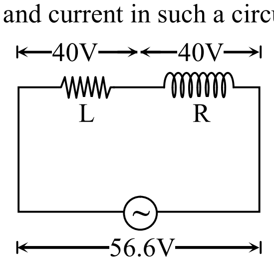
(iv) Vector Representation :



(v) Energy dissipitated $= \frac{1}{2} E_0 I_0 \cos\phi$

$= I_{rms} E_{rms} \cos\phi$

Ex.12 A student claimes that in a series LR circuit when the applied voltage is 56.6V, then the voltage across R and L are 40 volt each. Is the claim correct ? What is the phase difference between voltage and current in such a circuit ?



Sol. In a series LR circuit

$E = \sqrt{V_R^2 + V_L^2} = \sqrt{40^2 + 40^2} = 40\sqrt{2}$

$= 40 \times 1.414 = 56.6\text{V}.$

yes the claim is correct. The phase difference ϕ is such that

$\tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{40}{40} = 1$

or $\phi = 45^o$ or $\pi/4$.

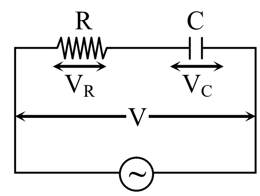
The phase angle is 45^o.

(E) R-C series circuit :

(i) Reactance (X), $X_C = \frac{-i}{\omega C}$ Resistance = R

Impedance, $Z = R - \frac{1}{\omega C} i,$

$|z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$, $\tan\phi = \frac{1}{(\omega C)R}$



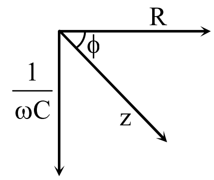
$E = E_0 \sin\omega t$

(ii) $E = E_0 \sin\omega t$

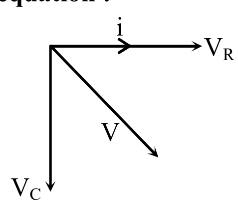
$I = I_0 \sin(\omega t + \phi)$ where $I_0 = E_0\omega C$ and

$\tan\phi = \frac{1}{(\omega CR)} = \frac{X_C}{R} = \frac{V_C}{V_R}$, ϕ is the phase of current relative to voltage.

(iii) Voltage lags the current by ϕ.



(iv) **Voltage equation :**

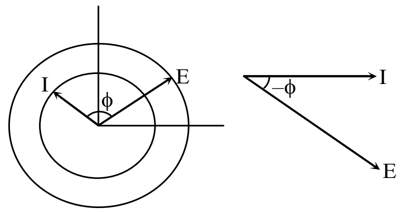


$V = V_R - V_C\, i$

$V = \sqrt{V_R^2 + V_C^2}$

(v) V_R (voltage across resistance) leads the V_C (voltage across capacitor) by $\frac{\pi}{2}$.

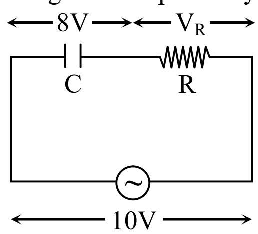
(vi) Vector Representation



(vii) Energy dissipation :

$\frac{1}{2} E_0 I_0 \cos\ \phi = E_{rms}\, I_{rms} \cos\phi$

Ex.13 In a series CR circuit shown in Fig, the applied voltage is 10V and the voltage across capacitor is found to be 8V. Then the voltage across R, and the phase difference between current and the applied voltage will respectively be –



(1) 6V, $\tan^{-1}\left(\frac{4}{3}\right)$ (2) 3V, $\tan^{-1}\left(\frac{3}{4}\right)$

(3) 6V, $\tan^{-1}\left(\frac{5}{3}\right)$ (4) None of the above.

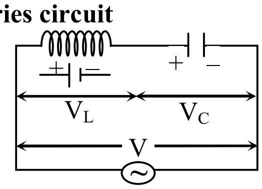
Sol. (1) Using $E = \sqrt{V_R^2 + V_C^2}$ we find $10^2 = V^2_R + 8^2$

or $V^2_R = 100 - 64 = 36$ or $V_R = 6V$

Thus voltage across resistance is 6V. The phase difference ϕ is, $\tan\phi = \frac{V_C}{V_R} = \frac{8}{6} = \frac{4}{3}$

$\phi = \tan^{-1}(4/3)$

(F) L-C series circuit



(i) Resistance = 0

Reactance $X = X_L - X_C$

$= \omega L - \frac{1}{\omega C}$

(ii) Impedance $= \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2} = \left|\omega L - \frac{1}{\omega C}\right|$

Case l : if $\omega L < \frac{1}{\omega C}$ or $\omega < \frac{1}{\sqrt{LC}}$

i.e. reactance is capacitive.

$\phi = \frac{-\pi}{2}$ or .

E lags the current by $\frac{\pi}{2}$.

Case 2: If $\omega L > \frac{1}{\omega C}$ or $\omega > \frac{1}{\sqrt{LC}}$

i.e. reactance is inductive $\phi = +\frac{\pi}{2}$ or E leads the current by $\frac{\pi}{2}$

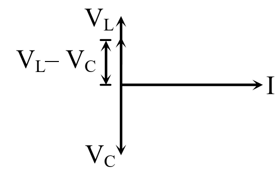
(iii) $E = E_0 \sin\omega t$

$I = I_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$ where , $I_0 = \frac{E_0}{\left|\omega L - \frac{1}{\omega C}\right|}$

+ and– signs will appear for $X_L > X_C$ and $X_L < X_C$ cases respectively.

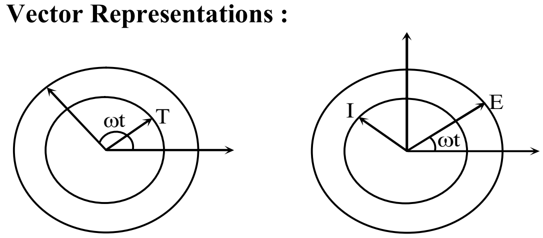
(iv) **Voltage equation**

$V = |V_L - V_C|$



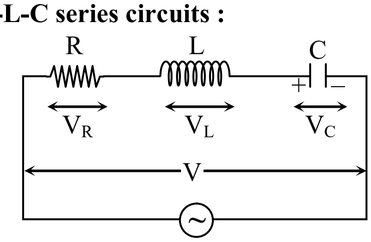
(v) Energy dissipation is zero because angle (ϕ) between current and voltage is 90^0 and loss is given by $E_{rms}\, I_{rms} \cos\phi$.

(vi) Vector Representations :



Inductive reactance Capacitive reactance

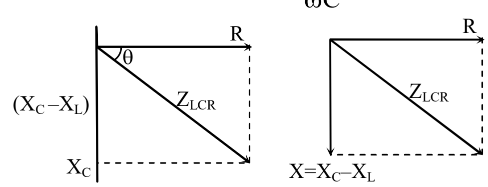
(G) **R-L-C series circuits :**



(i) Resistance = R

Reactance $= \omega L - \frac{1}{\omega C}$

Impedance $Z = R + \left(\omega L - \frac{1}{\omega C}\right) i$



$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

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$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{V_L - V_C}{V_R}$

(ii) $E = E_0 \sin\omega t \Rightarrow I = I_0 \sin(\omega t - \phi)$

where $I_0 = \dfrac{E_0}{|Z|} = \dfrac{E_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$ and

$\tan\phi = \dfrac{X_L - X_C}{R}$

(iii) Since $\tan\phi = \dfrac{X_L - X_C}{R}$

Case 1 : If $X_L > X_C$

$\phi = +\text{ve}$

$\Rightarrow$ Voltage leads the current and reactance is said to be inductive.

Case 2 : If $X_C > X_L$

$\phi = -\text{ve}$

Voltage lags the current and reactance is said to be capacitive.

(iv) Voltage equation :

$V = V_R + (V_L - V_C)\, i \qquad (i = \sqrt{-1})$

(v) if $E = E_0 \sin\omega t$

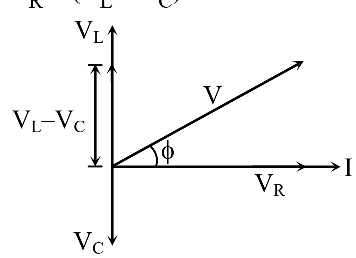
$V_R = IR = I_0 R \sin(\omega t - \phi)$

$V_L = \omega L\, I_0 \sin(\omega t - \phi + \pi/2) = \omega L\, I_0 \cos(\omega t - \phi)$

$V_C = \dfrac{1}{\omega C}\, I_0 \sin\left(\omega t - \phi - \dfrac{\pi}{2}\right)$

$= \dfrac{1}{-\omega C}\, I_0 \cos\left(\omega t - \phi\ \dfrac{\pi}{2}\right)$

$V = V_R + (V_L - V_C)\, i$



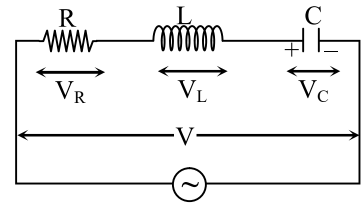
We see - that

$|V| = E$.

(vi) Energy disspitation :

$\dfrac{1}{2} E_0 I_0 \cos\phi = E_{rms} I_{rms} \cos\phi$.

Ex.14 In the following circuit, $E = E_0 \sin\omega t$, $E_0 = 200$ volt, $R = 20\ \Omega$, $L = .1$ H , $C = 10.6$ F. The current in amp at f = 0 and f = is respectively

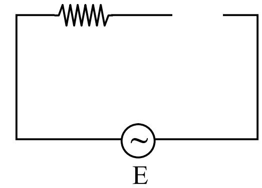


(1) 0, 10 (2) 10, 0
(3) 10, 10 (4) 0, 0

Sol. (4) Inductive reactance $= 2\pi f L = X_L$

Capacitive reactance $= \dfrac{1}{2\pi f C} = X_C$

at f = 0, $X_L = 0$ and $X_C = \infty$ So we have an open circuit as shown



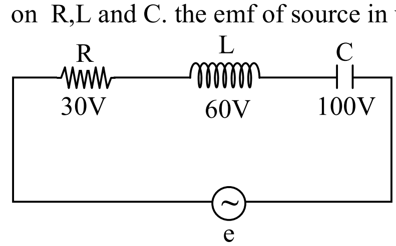
at $f = \infty$, $X_L = \infty$

and $X_C = 0$ again

we have an open circuit $\Rightarrow i = 0$

in both the cases, i = 0

Ex.15 In the given fig. the potential difference is shown on R,L and C. the emf of source in volts is –

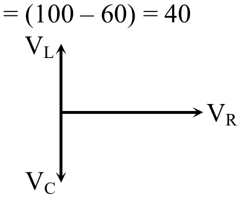


(1) 190 (2) 70 (3) 50 (4) 40

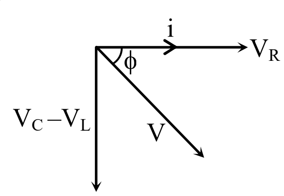
Sol. (4) From the vector representation

$V = \sqrt{V_R^2 + (V_C - V_L)^2}$

$V = (100 - 60) = 40$



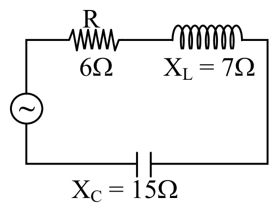
Note : the angle ' θ ' between current and emf will be given by



$$\theta = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right) = \tan^{-1}\left(\frac{4}{3}\right)$$

The given circuit has a capacitive reactance and hence emf will lag the current by an angle 'θ'.

Ex.16 For the series network shown in fig, find out (i) the peak current, (ii) impedance (iii) power factor, (iv) power consumed and (v) the expression for the instantaneous current, if e = 100 sin (314 t) volts (given $\tan^{-1}(4/3) = 53.13°$.)



Sol (i) Inpedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{6^2 + (15-7)^2} = 10\Omega.$

(ii) peak current

$I_0 = E_0/Z = 100/10 = 10$ ampere.

(iii) power factor $\cos\phi = \frac{R}{Z} = \frac{6}{10} = 0.6$

(iv) power $P = E_{rms}\, I_{rms} \cos\phi = E_0\, I_0 \cos\phi/2$

$= 100 \times 10 \times 0.6/2 = 300$ Watt

(v) The phase angle $\phi = \cos^{-1}(0.6)$

or $\phi = 53.13°$. The expression for the instantaneous current is

$i = I_0 \sin(\omega t + \phi) = 10 \sin(314\, t + 53.13°)$

The plus sign is used because $X_C > X_L$.

The current leads in a capacitive circuit.

9. POWER IN A.C. CIRCUIT

(i) The rate of work done or work done per second in an electric circuit is called power.

(ii) According to Joule,

power in an electric circuit = VI

work done in time ' t ' = VIt

(iii) unit - watt or joule / sec., horse power (HP)

1 watt = 1 joule / sec.

1 HP = 746 watt.

9.1 THREE TYPES OF POWER

(A) Instantaneous power :

(1) The power at any instant of time in an electric circuit is called instantaneous power.

(2) if instantaneous current and voltage representative are -

$I = I_0 \sin(\omega t)$ and

$E = E_0 \sin(\omega t + \phi)$, then

Instantaneous power = E I

$P_{inst} = (I_0 \sin\omega t)(E_0 \sin(\omega t + \phi))$

(B) Average power :

(i) The average of power dissipitated over one cycle is called average power.

(ii) $E = E_0 \sin(\omega t + \phi)$

$I = I_0 \sin\omega t$

$P(t) = E_0 I_0 \sin(\omega t + \phi) \sin\omega t$

$$\langle P \rangle = \frac{\int_0^T p(t)dt}{\int_0^T dt}$$

$$= \frac{1}{2} E_0 I_0 \cos\phi = E_{rms} I_{rms} \cos\phi$$

where ϕ is the angle between E and I .

(iii) **Case : 1** Circuit is totally resistive - i.e. $\phi = 0$

$\langle P \rangle = E_{rms} I_{rms}$

Case : 2 Circuit is pure inductive or capacitive -

i. e. $\phi = \pm\frac{\pi}{2}$

$\langle P \rangle = 0$

(iv) Maximum power dissipitation is in pure resistive circuit .

(C) Apparent power :

(i) The product of apparent voltage and apparent current in an electric circuit is called apparent power.

(ii) Apparent power $= I_{rms} E_{rms} = \frac{E_0 I_0}{2}$

(iii) This is always positive .

Ex.17 A bulb of 220 - 100 watt is connected with 110 volt of source, then loss of power is –

(1) 100 watt (2) 50 watt

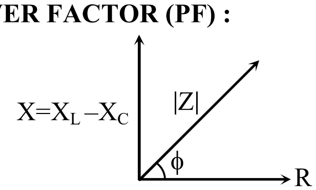
(3) 25 watt (4) 2 watt

Sol. (C) Resistance of the bulb $= \frac{(220)^2}{100} = (22)^2$

loss of power $= \frac{V^2}{R} = \frac{(110)^2}{(22)^2} = \frac{(11)^2 \times 100}{(22)^2}$

$= \frac{100}{4} = 25$ watt .

9.2 POWER FACTOR (PF) :



(i) The ratio of apparent power and average power is called power factor

(ii) PF = $\dfrac{E_{rms} I_{rms} \cos\phi}{E_{rms} I_{rms}} = \cos\phi$

(iii) since, $R = |Z| \cos\phi$

$\cos\phi = \dfrac{R}{|Z|} = PF$

(iv) power factor depends upon the components of the circuit.

(v) (a) If pure resistance , $\cos\phi = 1 = PF$.
i. e. average power = apparent power .

(b) If pure inductive or capacitive $\left(\phi = \pm\dfrac{\pi}{2}\right)$.

$PF = \cos\phi = 0$

(vi) The value of power factor lies between 0 and 1.

(vii) It is a dimensionless quantity.

(viii) Power factor is I in case of resonance which will be explained later.

9.3 WATTLESS CURRENT :

(i) The component of current which does not contribute to the average power dissipitation is called wattless current.

(ii) The average of wattless component over one cycle is zero over one cycle. i. e.
$< E_0 I_0 \sin\omega t \cos\omega t \sin\phi > = 0$

(iii)Instantaneous, value of wattless current $= (I_0 \sin\phi) \cos\omega t$

(iv) Amplitude of wattless current = $(I_0 \sin\phi)$

(v) The phase difference of wattless current and voltage is always $\dfrac{\pi}{2}$. That's why only it does not contribute to the average power dissipitation.

(vi) RMS value of wattless current = $I_{rms} \sin\phi$

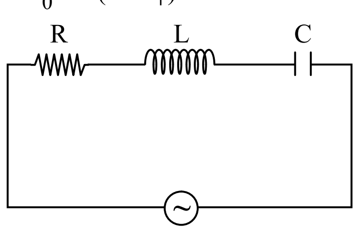
or $\dfrac{I_0}{\sqrt{2}} \sin\phi$

$\sin\phi = \dfrac{|x|}{|Z|} = \dfrac{|X_L - X_C|}{|Z|}$

(vii) Power component of current = $(I_0 \cos\phi) \sin\omega t$ and phase difference between this component and voltage is zero.

10. RESONANCE

(i) $I = I_0 \sin(\omega t - \phi)$



$E = E_0 \sin\omega t$

Where $I_0 = \dfrac{E_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

and $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$

(ii) We see that, current change with frequency (ω) . For a particular frequency, current in the circuit is maximum . This state is called resonance and the frequency is called resonance frequency.

(iii) In case of resonance,

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad X_L = X_C$$

$$\Rightarrow \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}} = f_r$$

(iv) at $f = f_r$, circuit is in resonance .

(a) Z = R

(b) power factor = 1

(c) $I = \dfrac{E_0}{R} \sin\omega t$

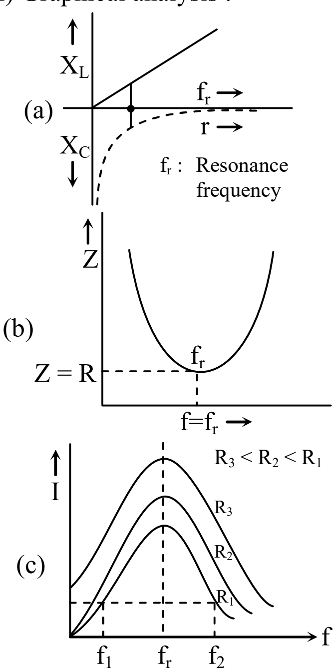
(d) phase difference between voltage and current is zero.

(e) Impedence Z is minimum = R.

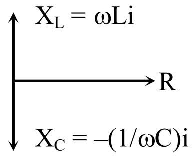
(v) phase difference between voltages across capacitor and inductor is π.

$$V_L = -V_C$$

(vi) Graphical analysis :



(vii) Vector Representation :



(viii) Frequencies at which value of current reduces to $\frac{I_0}{\sqrt{2}}$ are called half-power frequencies.

(ix) The internal between half power frequencies is called band-width.

Band Width = $f_2 - f_1$

(x) The value of Band width is independent of R.

(xi) Resonance is 'sharp' at low resistances.

Ex.18 The band width of a series resonant circuit is 400 Hz. If the resonant frequency is 4000 Hz, the value of X_L at resonance, L and C will respectively be-

(1) 100Ω, 0.04H, 0.4 μF

(2) 10Ω, 0.4H, 0.04 μF

(3) 100Ω, 0.4H, 0.4 μF

(4) None of these.

Sol.(1) Band width BW = f_r/Q

or Q = 4000/400 = 10

$(X_L)_{resonance} = QR = 10 \times 10 = 100\Omega$.

$L = X_L/\omega_r = 100/2 \times \pi \times 4000 = 1/8\pi$ henry $= 0.04$ H

Since at resonance $X_L = X_C$

Thus $X_C = \frac{1}{\omega_r C} = 100$

or $C = \frac{1}{2\pi \times 4000 \times 100} = 10^{-5}/8\pi$ farad

$= 0.4\ \mu F$

Ex.19 A series R-L-C circuit has a resonant frequency of 12 KHz. If R = 5Ω and X_L at resonance is 300 Ω, The cut off frequencies will be –

(1) 11,900 Hz, 12,100 Hz

(2) 9,000 Hz, 10,000 Hz

(3) 200 Hz, 210 Hz

(4) None of the above.

Sol.(1) $Q = \frac{(X_L)_{res}}{R} = \frac{300}{5} = 60$

Thus $BW = \frac{f_r}{Q} = \frac{12 \times 1000}{60} = 200$ HZ

$f_1 = f_r - (BW/2) = 12{,}000 - 100 = 11{,}900$ Hz

$f_2 = f_r + (BW/2) = 12{,}100$ Hz.

11. QUALITY FACTOR (Q)

(i) If ω_0 = Resonance Angular frequency.

then $Q = \frac{\text{Stored energy}}{\text{Energy loss during a cycle}}$

$$= \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

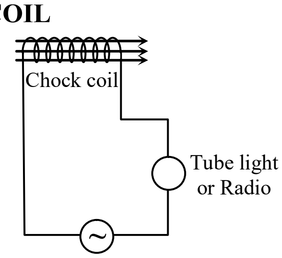
(ii) ' Q ' is dimensionless quantity

(iii) ' Q ' represents the sharpness of resonance higher value of ' Q ' represents sharper resonance.

General :

Phase difference	**Possible components**
(i) $\phi = 0$	(a) Only R (b) L-C-R Resonance circuit
(ii) $\phi = +\frac{\pi}{2}$	(a) Only L (b) L-C series circuit ($X_L > X_C$)
(iii) $\phi = -\frac{\pi}{2}$	(a) Only C (b) L-C Series circuit ($X_C > X_L$)
(iv) $0 < \phi < 90^0$	(a) L-R Series circuit (b) L-C-R series circuit ($X_L > X_C$)
(v) $0 > \phi > 90^0$	(a) C-R Series circuit (b) L-C-R series circuit ($X_C > X_L$)

12. CHOKE COIL



(i) This is an inductance-coil with high self inductance and low resistance.

(ii) The current in an alternating current circuit may be reduced by means of choke-coil involving minimum loss of energy.

(iii) Choke-coil is much more appropriate than resistance because, resistance involves loss of energy while choke coil involves very low energy loss ($\phi = \frac{\pi}{2}$)

(iv) Choke-coil cannot be used for DC.

(v) Choke-coil for different frequencies are made by using different substances in their core.

(vi) Choke-coil is used in tube-lights, motor etc. in series.

(vii) **Losses in choke-coil :**

(a) **Heat Loss :** The resistance R of a choke-coil is not exactly zero, which gives Joule-heating.

(b) **Eddy current losses :** Can be reduced by laminating the iron-core.

(c) **Hysteres is loss :**

13. COMBINATION OF ELECTRIC BULBS

(i) When a bulb of power P is connected to a mains line of V volt, then

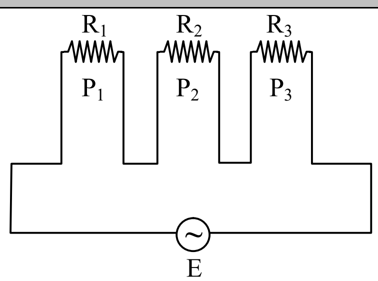
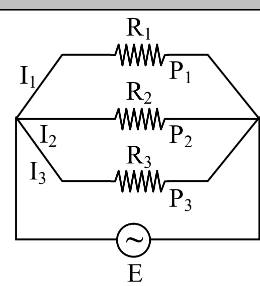
(a) The current flowing in the bulb $I = \frac{P}{V}$

(b) Resistance of bulb $R = \frac{V^2}{P} = \frac{P}{I^2}$

(c) Heat generated in the bulb $H = \frac{V^2t}{4.2R} = \frac{I^2Rt}{4.2}$

(ii) A bulb marked 100W - 200 V means that it is connected to a 200 V line,m then 100 watt power will be consumed (i.e. 100 Joule of electric energy will be converted into heat and light per second).

(iii) Comparative study of series and parallel combinations of bulbs-

Series combination	Parallel combination
(i) 	(i) 
(ii) In this combination the bulb of highest wattage will shine maximum.	(ii) In this combination the bulb of highest wattage will shine maximum.
(iii) In series combination the current through all the bulbs will be the same	(iii) In parallel combination the voltage across all the bulbes will be the same.
(iv) The resistance of lowest wattage bulb is maximum and hence heat generated in it $\left(H = \frac{I^2Rt}{4.2}\right)$ will be maximum and it will glow maximum and it will glow maximum	(iv) Resistance of highest wattage bulb is minimum and hence maximum heat $\left(H = \frac{V^2t}{4.2R}\right)$ will be generated in it and it will glow maximum
(v) The glow of bulbs connected in series depends on the voltage across their ends.	(v) The glow of bulbs connected in parallel depends on the current flowing in them
(vi) If the voltage across the ends of the bulbs connected in series is increased then current in them increases and consequently the higher wattage bulb will keep on glowing whereas the lower wattage bulb will get fused.	(vi) If the current in the circuit of parallel combination of bulbs is increased then the lower wattage bulb will keep on glowing whereas the higher wattage bulbwill get fused.
(vii) The resultant power lost is given by $\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + ...$	(vii) The resultant power lost (P) is given by $P = P_1 + P_2 + P_3 + ...$
(viii) Resultant resistance of the combination $R = R_1 + R_2 + R_3 + ...$	(viii) Resultant resistance of the combination $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

POINTS TO REMEMEBR

1. The average value of alternating emf or current for complete cycle is zero. That is why a D.C. ammeter or a D.C. voltmeter connected in A.C. circuit does not show any deflection. The another reason for no deflection is that due to the property of inertia the needle cannot follow the changes in direction and hence does not deflect at all from its position.
2. Smaller the power factor, smaller is the energy loss.
3. For R circuit power factor = 1 for L circuit, C circuit and L - C circuit, the power factor is zero.
4. An ideal choke coil controls the current in an A.C. circuit without the loss of energy.
5. Larger the Q factor sharper is the resonance.
6. Choke coil has a high resistance to A.C. but low resistance to D.C.
7. The frequency of current produced by generator (f)
   (a) Frequency of A.C. or A.V. (f)
   $= \frac{\text{No. of poles} \times \text{rotations/sec}}{2}$
   (b) $f = \frac{n_R N_P}{2}$ Here N_P = No. of poles,
   n_R = rotations/sec.
   (c) In India and England the frequency of A.C. generated at 220 V is 50Hz, whereas in the U.S.A. the frequency of A.C. generated at 110V in 60Hz.
8. The pole reversion takes place twice in one cycle (rotation).
9. In one second the voltage or current becomes zero 2f times and the direction of A.C. or A.V. also changes 2f times.
10. The rate of change of A.C is minimum at that instant when they are near their peak values.
11. The rate of change of A.C. or A.V. is minimum at that instant when they change their direction.
12. The A.C. or A.V. takes T/4 time in reaching from zero to maximum or from maximum to zero value.
13. Representation of E_0, E_{rms}, E_{av}, E_{max} and E_{min}. In the figure QQ' $\rightarrow$ peak to peak value. At point P_1, P_2 maximum rate of chane of E P, Q minimum rate of change of E.
14. Representation of E, $E^2 < E^2 >$ and E_{rms}.
    (a) $E = E_0 \sin(\omega t \pm \theta)$
    (b) $E = E_0 \sin(2\pi ft \pm \theta)$
    (c) $E = E_0 \sin\left(\frac{2\pi t}{T} \pm \theta\right)$
    (d) $E = E_0 \sin \omega t = E_0 \sin 2\pi ft = E_0 \sin \frac{2\pi t}{T}$
    (e) $E_0 = BNA\omega$
    (f) $I_0 = \frac{E_0}{R} = NBA \frac{\omega}{R}$
15. Relation between I_{rms} and <1> or E_{rms} and <E>
    (a) $I_{rms} = \frac{< I > \pi}{2\sqrt{2}}$
    (b) $E_{rms} = \frac{< E > \pi}{2\sqrt{2}}$

SOLVED EXAMPLES

Ex.1 The instantaneous emf in an ac circuit is given by E = 50 sin (314 t) volts, where t is in seconds. In how much time the emf will become 25 volts, starting from zero,

(1) $\frac{1}{50}$ s (2) $\frac{1}{200}$ s

(3) $\frac{1}{314}$ s (4) $\frac{1}{600}$ s

Sol.(4) For instantaneous emf to be 25 V, we must have

$$25 = 50 \sin(314\,t)$$

or $\frac{1}{2} = \sin(314\,t)$ or $\frac{\pi}{6} = 314\,t$

or $t = \frac{\pi}{6\times 314} = \frac{1}{600}$ s.

Ex.2 The phase difference between current and voltage in an AC circuit is π/4 radian. If the frequency of AC is 50 Hz, then the phase difference is equivalent to the time difference–

(1) 0.78s (2) 15.7 ms

(3) .25s (4) 2.5 ms

Sol. (4) The time difference equivalent to phase difference $\Delta\phi$ is given by

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T}$$

Thus, $\Delta t = \left(\frac{T}{2\pi}\times\Delta\phi\right) = \frac{1}{50}\times\frac{1}{2\pi}\times\frac{\pi}{4}$

$= \frac{1}{400} = 2.5\times10^{-3}$ sec.

Ex.3 The number of poles in an AC generator is 10, and the coil is rotating at the rate of 600 revolutions per minute. Then, the frequency of AC current produced by the generator is (in hertz)

(1) 10 (2) 50

(3) 1000 (4) 600

Sol.(2) The frequency of the ac is = number of rotations of the coil per second x number of pairs of pole

$= \frac{600}{60}\times 5 = 50.$

(10 poles means 5 pairs of poles (N–S).

Ex.4 A long solenoid connected to a 12V dc source passes a steady current of 2A. When the solenoid is connected to a source of 12V rms at 50 Hz, the current flowing is 1A rms. Then the inductance of the solenoid–

(1) 11 mH (2) 22 mH

(3) 33 mH (4) none of the above.

Sol.(3) From the first data, resistance of the solenoid is R = 12/2 = 6 ohm. From the second data, the impedance of the solenoid is

$Z = \frac{12V}{1A} = 12\Omega.$

Since $Z = \sqrt{R^2 + (\omega L)^2}$

or $Z^2 = R^2 + (\omega L)^2$, we have

$12^2 = 6^2 + (2\pi \text{ x } 50\,L)^2$

or $100\,\pi L = \sqrt{12^2 - 6^2} = \sqrt{108} = 10.4$

$L = \frac{10.4}{314} = 0.033$ henry.

Ex.5 A 110V, 60W lamp is run from a 220V ac mains using a capacitor in series with the lamp, instead of a resistor, then the voltage across the capacitor is about–

(1) 110V (2) 190V

(3) 220V (4) 311V

Sol.(2) $V_0 = \sqrt{V_R^2 + V_C^2}$, thus $V^2_C = V^2_0 - V^2_R$

$= 220^2 - 110^2 = 110^2\,(4 - 1)$

or $V_C = 110\,\sqrt{3} = 190$ volt

Ex.6 A 100 volt a.c. source of frequency 500 hertz is connected to a L–C–R circuit with L = 8.1 millihenry, C = 12.5 microfarad and R = 10 ohm, all connected in series. The potential difference across the resistance will be –

(1) 10V (2) 100V

(3) 50V (4) 500V

Sol.(2) Inductive reactance in the circuit is

$X_L = \omega L = 2\pi fL$

$= 2\times 3.14\times(500)\times(8.1\times10^{-3}) = 25.4$ ohm.

Capacitive reactance in the circuit is

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\times3.14\times500\times(12.5\times10^{-6})}$

$= 25.4$ ohm.

Since $X_L = X_C$, the impedance is

$Z = \sqrt{[R^2 + (X_L - X_C)^2]} = R = 10$ ohm.

$\therefore\ i_{rms} = \frac{E_{rms}}{Z} = \frac{100\,\text{volt}}{10\,\text{ohm}} = 10$ amp.

Hence, the p.d. across the resistance is

$i_{rms}\times R = 10$ amp $\times$ 10 ohm $= 100$ volt.

Ex.7 An ac circuit draws 5A at 160V and the power consumption is 600W. Then the power factor is –

(1) 1 (2) 0.75
(3) 0.50 (4) zero

Sol.(2) Power factor

$$\cos\phi = \frac{\text{Real power}}{V_{rms} I_{rms}}$$

or $$\cos\phi = \frac{600}{5\times160} = 0.75$$

Ex.8 A step down transformer operates on a 2.5 KV line and supplies a load with 80A. The ratio of the primary winding to the secondary winding is 20 : 1. Assuming 100 percent efficiency, the output power is –

(1) 200 KW (2) 100 KW
(3) 10 KW (4) none of the above.

Sol.(3) $P_{out} = V_s I_s$

Now $V_s = \frac{N_s}{N_p} \times V_p = \frac{1}{20} \times 2500 = 125$ V

Thus $P_{out} = 125 \times 80 = 10$ KW.

(It is assumed that the load is resistive, so that the power factor is unity).

Ex.9 An ac source of emf E = 200 sin (100 t) is connected to a choke coil of inductance 1 henry and resistance 100 Ω. The average power consumed is –

(1) 0 (2) 200W
(3) 141W (4) none of the above.

Sol.(4) The reactance $X_L = \omega L = 100 \times 1$

This impedance is $Z = \sqrt{R^2 + X_L^2} = 100\sqrt{2}$

The current (peak), $I_0 = \frac{E_0}{Z} = \frac{200}{100\sqrt{2}} = \sqrt{2}$ A

Thus, $P = E_{rms} I_{rms} \cos\phi = \frac{E_0 I_0}{2} \times \frac{R}{Z}$

$$= \frac{200\times\sqrt{2}}{2} \times \frac{100}{100\sqrt{2}} = 100\text{W}$$

Ex.10 A 2.5/π μF capacitor and a 3000–ohm resistance are joined in series to an a.c. source of 200 volt and 50 sec^{-1} frequency. The power factor of the circuit and the power dissipated in it will respectively be–

(1) 0.6, 0.06W (2) 0.06, 0.6W
(3) 0.6, 4.8W (4) 4.8, 0.6W.

Sol.(3) The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi\times50\times\left(\frac{25}{\pi}\times10^{-6}\right)} = 4000 \text{ ohm.}$$

The impedance of the circuit is

$$Z = \sqrt{(R^2 + X_C^2)} = \sqrt{[(3000)^2 + (4000)^2]}$$

= 500 ohm.

Power factor, $\cos\phi = \frac{R}{Z} = \frac{3000}{5000} = 0.6$

Power dissipation, $\bar{p} = V_{rms} \times I_{rms} \times \cos\phi$.

$$= V_{rms} \times \frac{V_{rms}}{z} \times \cos\phi.$$

$$= 200 \times \frac{2000}{5000} \times 0.6 = 4.8 \text{ watt.}$$

Ex. 11 A circuit drawn a power of 550 watt from a source of 220 volt, 50 hertz. The power factor of the circuit is 0.8 and the current lags in phase behind the potential difference. To make the power factor of circuit as 1.0, the capacitance required to be connected with it, will be –

(1) 70.4 μF (2) 75 μF
(3) 7.5 μF (4) 750 μF

Sol. (2) Initially, the current lags behind the potential difference. Hence the circuit contains resistance and inductance. The power of the circuit is –

$$P = V_{rms} \times i_{rms} \times \cos\phi.$$

But $i_{rms} = \frac{V_{rms}}{Z}$

where $Z = \sqrt{[R^2 + (\omega L)^2]}$ is the impedance of the circuit.

$$\therefore\ P = V_{rms} \times \frac{V_{rms}}{Z} \times \cos\phi.$$

or $$Z = \frac{(V_{rms})^2 \times \cos\phi}{P} = \frac{(220)^2 \times 0.8}{550} = 70.4 \text{ ohm.}$$

Power factor, $\cos\phi = \frac{R}{Z}$

$\therefore$ R = Z cos φ = 70.4 × 0.8 = 56.32 ohm.

Now $Z^2 = R^2 + (\omega L)^2$

$\therefore (\omega L)^2 = Z^2 - R^2 = (70.4)^2 - (56.4)^2 = 1784$

$\therefore\ \omega L = 42.2$ ohm.

The impedance of the circuit after inserting the capacitance is given by

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$$

Now, the power factor is given by $\cos\phi = \dfrac{R}{Z}$

$$= \frac{R}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}}$$

Clearly, for making power factor = 1.0, it must be that $\omega L = \dfrac{1}{\omega C}$ or $C = \dfrac{1}{\omega(\omega L)}$

But $\omega = 2\pi f = 2 \times 3.14 \times 50 = 314$

$\therefore C = \dfrac{1}{314 \times 42.2} = 75 \times 10^{-6}$ farad

= 75 micro farad.

Ex. 12 An L–C–R circuit has L = 10 mH. R = 3 ohm and C = 1μF connected in series to a source of 15 cos ωt volt. The current–amplitude and the average power dissipated per cycle at a frequency 10% lower than the resonant frequency will respectively be–

(1) 0.704A, 0.744W (2) 0.704A, 0.704W
(3) 7.04A, 7.44W (4) 70.4A, 74.4W

Sol.(1) Resonant frequency, $\omega_R = \dfrac{1}{\sqrt{(LC)}}$

$$= \frac{1}{\sqrt{(10\times10^{-3}\,H)\times(1\times10^{-6}\,F)}} = 10^4 \text{ per sec.}$$

The frequency 10% lower than this is

$\omega = 10^4 - 10^4 \times \dfrac{10}{100} = 9 \times 10^3$ per sec.

At this frequency, we have

$X_L = \omega L = 9 \times 10^3 \times (10 \times 10^{-3}) = 90$ ohm.

and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{9\times10^3\times(1\times10^{-6})}$

= 111.11 ohm.

∴ impedance,

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(3)^2 + (90 - 111.11)^2}$

= 21.32 ohm.

∴ current amplitude is

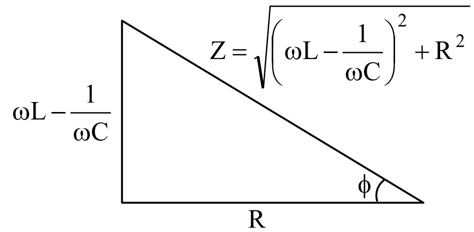
$$i_0 = \frac{E_0}{Z} = \frac{15}{21.32} = 0.704 \text{ ampere.}$$

Average power dissipated is $P = \dfrac{1}{2} E_0 i_0 \cos\phi$.

Here $\cos\phi = \dfrac{R}{Z} = \dfrac{3}{21.32} = 0.141$.

$\therefore P = \dfrac{1}{2} \times 15 \times 0.704 \times 0.141 = 0.744$ watt.

Ex. 13 A 750–hertz, 20–volt source is connected to a resistance of 100 ohm, an inductance of 0.1803 henry and a capacitance of 10 microfarad all in series. The time in which the resistance (thermal capacity = 2 joule/ºC) will get heated by 10ºC will be–



(1) 20 s (2) 200 s
(3) 348 s (4) 448 s

Sol.(3) Inductive reactance in the circuit is

$\omega L = 2 \times 3.14 \times 750 \times 0.1803 = 850$ ohm.

Capacitive reactance

$\dfrac{1}{\omega L} = \dfrac{1}{2\times3.14\times750\times(10\times10^{-6})}$

= 21.2 ohm.

Impedance of the circuit,

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}}$$

$= [(100)^2 + (850 - 21.2)^2]^{1/2} = 835$ ohm.

Power dissipated,

$$P = V_{rms}\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}} i_{rms} \times \cos\phi$$

$= V_{rms} \times \dfrac{V_{rms}}{Z} \times \dfrac{R}{Z} = 20 \times \dfrac{20}{835} \times \dfrac{100}{835}$

= 0.0574 watt.

Heat produced in the resistance

= 2 joule/ºC × 10ºC = 20 joule.

Let this heat be prodced in t sec. Then

Pt = 20 joule

or $t = \dfrac{20 \text{ joule}}{0.0574 \text{ watt}} = 348$ second.

Ex.14 Kanha wants to calculate the current and power dissipated in an LCR series circuit . He connected 100Ω resistance to an AC source of peak value 200 V and angular frequency 300 radian/sec. When he removed only the capacitance, the current was found to be lagging behind the voltage by 60º. while on removing the inductance he found, the current leading the voltage by 60º. The value of peak current and the power dissipated obtained by him will be–

(1) 2A, 200W
(2) 4A, 100W
(3) 3A, 120W
(4) Nothing can be said.

Sol.(1) $\tan 60^\circ = \frac{\omega L}{R}$, $\tan 60^\circ = \frac{1/\omega C}{R}$.

$\therefore \quad \omega L = \frac{1}{\omega C}$.

$\therefore$ impedance of the circuit,

$$Z = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}} = R.$$

Current in the circuit, $i_0 = \frac{V_0}{Z} = \frac{V_0}{R} = \frac{200}{100} = 2A$

Average power, $\bar{p} = \frac{1}{2} = V_0\, i_0 \cos\phi$.

But $\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R} = 0$, $\therefore \cos\phi = 1$.

$\therefore \quad \bar{p} = \frac{1}{2} \times 200 \times 2 \times 1 = 200$ watt.

Ex. 15 In an alternating circuit connected to an emf of 100 volt and frequency 50 hertz, a resistance of 10 ohm and an inductance of $1/(10\pi)$ henry are connected in series. Find out the power dissipated in the circuit.

Sol. Power dissipated $P = V_{rms} \times I_{rms} \times \cos\phi$

$$= V_{rms} \times \frac{V_{rms}}{Z} \times \frac{R}{Z}$$

$$= \frac{V_{rms}^2 R}{Z^2} = \frac{V_{rms}^2 R}{R^2 + (\omega L)^2}.$$

(putting the values) P = 500 watt.

Ex. 16 In an LCR circuit, the capacitance is changed from C to 4C. For the same resonant frequency, the inductance should be changed from L to–

(1) 2L (2) 4L
(3) L/2 (4) L/4

Sol.(4) $\omega_r = \frac{1}{\sqrt{LC'}}$, $\omega_r' = \frac{1}{\sqrt{L'C'}}$,

For $\omega_r = \omega_r' = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L'(4C)}}$

or $LC = L'(4C)$ or $L' = L/4$

Ex. 17 A coil self–inductance 0.16 henry is connected to a condenser of capacity 0.81 μF. The frequency of A.C. that should be applied so that there is a resonance in the circuit (The resistance of the circuit is negligible) should be –

(1) 50 c/s (2) 60 c/s
(3) 442 c/s (4) 342 c/s

Sol.(3) Suppose the frequency in the alternating circuit is f. If the inductive reactance (ωL) is equal to the capacitive reactance ($1/\omega C$) in the circuit, then there is resonance in the circuit. Thus.

$$\omega L = \frac{1}{\omega C} \quad \text{or} \quad 2\pi f L = \frac{1}{2\pi f C}$$

or $f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)}$

Here L = 0.16 henry and C = 0.81 μF
$= 0.81 \times 10^{-6}$ farad.

$\therefore \quad f = \frac{1}{2 \times 3.14}\sqrt{\left(\frac{1}{0.16 \times (0.81 \times 10^{-6})}\right)}$

= 442 cycles/second.

Ex. 18 In an oscillatory circuit the value of self–inductance of the connected coil is 10 milli–henry. If the oscillatory frequency of the circuit is 1.0 megacycle/second then the capacity of the condenser connected in the circuit will be–

(1) 2.5 pF (2) 2.5 μF
(3) 0.25 pF (4) 0.25 μF

Sol.(1) If C be the capacity of the condenser connected in the circuit and L the self–inductance of the coil, then the resonant frequency of the circuit is given by

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)} \quad \text{or} \quad C = \frac{1}{L \times (2\pi f)^2}$$

Here L = 10 millihenry = 10×10^{-3} henry,
f = 1.0 megacycle/second
$= 1.0 \times 10^6$ cycle/second.

$$\therefore C = \frac{1}{(10 \times 10^{-3}) \times (2 \times 3.14 \times 1.0 \times 10^6)^2}$$

$= 2.5 \times 10^{-12}$ farad = 2.5 micro–micro farad (μ μF).

Ex.19 In series LCR circuit V = 100 sin(100πt) volt, I = 100sin(100 πt – π/3) mA. Then find out-
(i) rms voltage and current
(ii) Phase difference between V and I & frequency
(iii) Power factor and its nature
(iv) Average power loss
(v) Resistance, reactance & impedance

Sol. (i) rms voltage = $50\sqrt{2}$ volt

rms current = $50\sqrt{2}$ mA

(ii) π/3, 50 Hz

(iii) Power factor = $\cos \pi/3 = \frac{1}{2}$,

Nature : lagging

(iv) Average power loss = VI cosφ

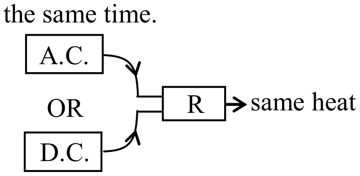
$= (50\sqrt{2})(50\sqrt{2}) \times 10^{-3}\left(\frac{1}{2}\right) = 2.5$ W

(v) R = 500 Ω; X = $500\sqrt{3}$ Ω;

$Z = \frac{V}{I} = 1000\ \Omega$

Ex.20 Define following terms related to A.C.
(a) R.M.S. value of current
(b) Wattless current
(c) Average value of A.C.
(d) Quality factor

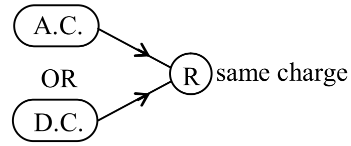
Sol. (a) The root mean square (r.m.s.) value of alternating current is defined as that value of steady current; which would generate the same amount of that is a given resistance in a given time, as is done by the alternating current, when applied to the same resistance for the same time.

If 

then D.C. is known as RMS of A.C.

(b) Wattless current : It is the part of current, which provides no contribution in real power in A.C. circuit. Wattless current = $I_{rms} . \sin\phi$

(c) The mean or average value of a.c. over any half cycle is defined as that value of steady current which would send the same amount of charge through a circuit in the time of half cycle (i.e. T/2) as is sent by the a.c. through the same circuit, in the same time

If 

(d) Quality factor : Q factor of A.C. circuit basically give an idea of stored energy and lost energy $Q \propto \frac{\text{stored energy}}{\text{lost energy}}$

OR

Q factor is the ratio of resonating frequency and band width and it also indicates about sharpness of resonance.

$Q = \frac{f_r}{\Delta f}$, high Q-factor gives sharp resonance.

Ex.21 What is choke coil. Write down its basic use with one advantage.

Sol. A coil of high inductance and low resistance is called choke coil.
Advantage → Minimum or negligible energy loss.
Use → It is used to control current in A.C. circuit.

Ex.22 Define half power frequencies in series LCR circuit.

Sol. Frequencies at which power becomes half of its maximum value.

OR

Frequencies at which current becomes $\frac{1}{\sqrt{2}}$ of its maximum value (or value at resonance)

OR

Half power frequencies are the frequencies at which reactance and resistance are equal.

Ex.23 Write the formula of quality factor and how the sharpness of current and frequency curve is varied with resistance in series L-C-R circuit. Show with figure.

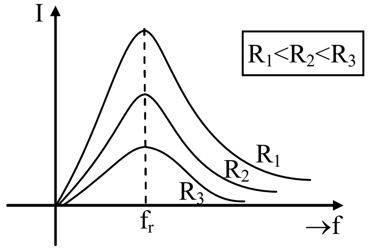
Sol. **Quality factor**

$$Q = \frac{2\pi \times \text{Average energy stored per cycle}}{\text{Average energy dissipated per cycle}}$$

$$Q = \frac{2\pi \times \frac{1}{2}LI_0^2}{\left(\frac{I_0^2 R}{2}\right) \times T} = \frac{\omega L}{R}$$

Sharpness $\propto \frac{1}{R}$

As resistance increases sharpness decreases.



Ex.24 Is a capacitor at very high frequencies acts like a short circuit ?

Sol. Yes

Ex.25 Is a circuit with a high Q value has a narrow resonance curve ?

Sol. Yes

Ex.26 The time required for a 50 Hz A.C. to change from zero to the r.m.s. value is ……….

Sol. 2.5×10^{-3} sec.

Ex.27 Average value of sinusoidal A.C. of peak value I_0 over 0 to π is ……….

Sol. $\frac{2I_0}{\pi}$

Ex.28 Ratio of real power to apparent power in series LCR series circuit is well known as ……….

Sol. Power factor

Ex.29 Why the multistrained wire is more suitable for flowing alternating current ?

Sol. Because A.C. always flow from the surface of the wire due to skin effect. So without using the extra material by dividing a single wire into multistrained wire we can increase the surface area.

Ex.30 What is the phase difference between

(i) Voltage across L and C in a series LCR circuit connected to an AC source

(ii) Current in L and C in a series LCR circuit connected to an AC source

Sol. (i) 180° or π rad (ii) 0° or same phase

Ex.31 An alternating current source $E = 100 \sin 1000t$ is connected through a inductor of 10 μH then write down the equation of current.

Sol. $10000 \sin\left(1000t - \frac{\pi}{2}\right)$ A