JEE MATHS JULY 29 2025

If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is

(a) −7 (b) 6

(c) 7 (d) 5

Ans. (c) : Given,

$f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$

On differentiating w.r.t. x, we get –

$f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(x) \geq 0 \ \forall x \in (0, 1]$

$f'(x) \leq 0 \ \forall x \in [1, 5)$

$f'(x) = 0$ at $x = 1$

$3(1)^2 - 6(a-2) + 3a = 0$

$3 - 6a + 12 + 3a = 0$

$-3a = -15$

$a = 5$

$\therefore \ f(x) - 14 = (x-1)^2 (x-7)$

$$\frac{f(x)-14}{(x-1)^2} = (x-7)$$

Hence, $x - 7 = 0$

$x = 7$

An edge of a variable cube is increasing at the rate of 10cm/s. How fast the volume of the cube will increase when the edge is 5 cm long

(a) 750 cm^3/s (b) 75 cm^3/s

(c) 300 cm^3/s (d) 150 cm^3/s

(e) 25 cm^3/s

Ans. (a) : Volume of cube (v) = x^3 [x is edge of cube x = 5 cm]

$v = x^3$

$$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$$

$$\frac{dv}{dt} = 3 \times 5^2 \times 10$$

$$= 75 \times 10$$

$$\frac{dv}{dx} = 750 cm^3/sec$$

A cube is expanding in such a way that its edge is increasing at a rate of 2 inches per second. If its edge is 5 inches long, then the rate of change of its volume is

(a) 150 in^3/sec (b) 75 in^3/sec
(c) 50 in^3/sec (d) 30 in^3/sec
(e) 45 in^3/sec

Ans. (a) : Given,
$V = x^3$ [x = length, V= volume]

$$\frac{dV}{dt} = 3x^2 . \frac{dx}{dt} \qquad \left[\text{Given, } \frac{dx}{dt} = 2\right]$$

$$\frac{dV}{dt} = 3x^2 \times 2$$

$$\frac{dV}{dt} = 6x^2 \qquad [\because x = 5 \text{ inch}]$$

$$\frac{dV}{dt} = 6 \times 5^2$$

$$= 6 \times 5^2$$

$$\frac{dV}{dt} = 150 \text{ in}^3/\text{sec}$$

The function $f(x) = x^2 + 2x - 5$ is strictly increasing in the interval

(a) $[-1, \infty)$ (b) $(-\infty, -1)$
(c) $(-\infty, -1)$ (d) $(-1, \infty)$

Ans. (d) : Given,
$f(x) = x^2 + 2x - 5$
$\therefore$ $f'(x) = 2x + 2$
f(x) is an increasing function, if $f'(x) > 0$
$2x + 2 > 0$
$2x > -2$
$\therefore$ $x > -1$
Thus, f(x) is an increasing function for $x > -1$
i.e, $(-1, \infty)$

The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to the $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = 0$, is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

Ans. (b) : Let,

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) \text{ and}$$

$$z = \tan^{-1}\left(\frac{2\times\sqrt{1-x^2}}{1-2x^2}\right)$$

Putting $x = \tan\theta$, we get

$$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\tan\theta}{2}\right) = \frac{1}{2}\tan^{-1}(x)$$

$$\frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

Putting $x = \sin\theta$ in z , we get

$$z = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos 2\theta}\right) = \tan^{-1}(\tan 2\theta) = 2\theta = 2\sin^{-1}x$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{2}{\sqrt{1-x^2}}$$

Thus, $\frac{dy}{dz} = \frac{dy\,|\,dx}{dz\,|\,dx} = \frac{1}{4(1+x^2)}\sqrt{1-x^2}$

at $x = 0$

$$\left(\frac{dy}{dz}\right) = \frac{1}{4}$$

The number of terms in the expansion of $(x+y+z)^{10}$ is

(a) 142 (b) 11

(c) 110 (d) 66

Ans. (d) : The given expansion $(x+y+z)^{10}$

We know that,

The number of term in the expansion of $(x_1 + x_2 + x_3 + \dots + x_r)^n = {}^{n+r-1}C_n$

Where, n = exponent of the term

r = number of terms

Then, total number of terms –

$${}^{n+r-1}C_n = {}^{10+3-1}C_{10} = {}^{12}C_{10}$$

$$= \frac{12!}{(12-10)!10!}$$

$$= \frac{12\times 11\times 10!}{2!\times 10!}$$

So, total number of term $= \frac{132}{2} = 66$

The number of terms in the expansion of $(x^2+y^2)^{25} - (x^2-y^2)^{25}$ after simplification is

(a) 0 (b) 26

(c) 13 (d) 50

Ans. (c) : Given, expansion is,

$$(x^2+y^2)^{25} - (x^2-y^2)^{25}$$

We know that,

$$(a+b)^n - (a-b)^n = \frac{n+1}{2}$$

Where n is odd number.

So, the number of terms in the given expansion

$$\frac{n+1}{2} = \frac{25+1}{2} = 13$$