JEE MATHS RELATIONS AND FUNCTIONS

301. If $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$ for all $x \in R-\{0\}$, then $f(x^4)$ is

a) $\frac{(1-x^4)(2x^4+3)}{5x^4}$ b) $\frac{(1+x^4)(2x^4-3)}{5x^4}$ c) $\frac{(1-x^4)(2x^4-3)}{5x^4}$ d) None of these

302. The domain of definition of the function $f(x)={}^{7-x}P_{x-3}$, is

a) $[3,7]$ b) $\{3,4,5,6,7\}$ c) $\{3,4,5\}$ d) None of these

303. Let $f(x)=x$ and $g(x)=|x|$ for all $x \in R$. Then, the function $\phi(x)$ satisfying $\{\phi(x)-f(x)\}^2+\{\phi(x)-g(x)\}^2=0$, is

a) $\phi(x)=x, x \in [0,\infty)$

b) $\phi(x)=x, x \in R$

c) $\phi(x)=-x, x \in (-\infty,0]$

d) $\phi(x)=x+|x|, x \in R$

304. The value of the function $f(x)=3\sin\left(\sqrt{\frac{\pi^2}{16}-x^2}\right)$ lies in the interval

a) $[-\pi/4,\pi/4]$ b) $[0,3/\sqrt{2}]$ c) $(-3,3)$ d) None of these

305. The period of the function $f(x)=|\sin x|+|\cos x|$ is

a) π b) $\pi/2$ c) 2π d) None of these

306. If $f(x)=(ax^2+b)^3$, then the function g such that $f(g(x))=g(f(x))$ is given by

a) $g(x)=\left(\frac{b-x^{1/3}}{a}\right)^{1/2}$ b) $g(x)=\frac{1}{(ax^2+b)^3}$ c) $g(x)=(ax^2+b)^{1/3}$ d) $g(x)=\left(\frac{x^{1/3}-b}{a}\right)^{1/2}$

307. Let R be the real line. Consider the following subsets of the plane $R \times R$

$S=\{(x,y): y=x+1 \text{ and } o<x<2\}$

$T=\{(x,y): x-y \text{ is an integer}\}$

Which of the following is true?

a) T is an equivalent relation on R but S is not
b) Neither S nor T is an equivalence relation on R
c) Both S and T are equivalence relations on R
d) S is an equivalence relations on R and T is not

308. Let $A=[-1,1]$ and $f: A \to A$ be defined as $f(x)=x|x|$ for all $x \in A$, then $f(x)$ is

a) Many-one into function
b) One-one into function
c) Many-one onto function
d) One-one onto function

309. If $f(x)=\frac{1-x}{1+x}, x \neq 0,-1$ and $\alpha=f(f(x))+f\left(f\left(\frac{1}{x}\right)\right)$, then

a) $\alpha>2$ b) $\alpha<-2$ c) $|\alpha|>2$ d) $\alpha=2$

310. Let R and S be two non-void relations on a set A. Which of the following statements is false?

a) R and S are transitive implies $R \cap S$ is transitive.

b) R and S are transitive implies $R \cup S$ is transitive.

c) R and S are symmetric implies $R \cup S$ is symmetric.

d) R and S are reflexive implies $R \cap S$ is reflexive.

311. $A = \{1, 2, 3, 4\}, B\{1, 2, 3, 4, 5, 6\}$ are two sets, and function $f: A \to B$ is defined by $f(x) = x + 2\ \forall x \in A$, then the function f is

a) Bijective b) Onto c) One-one d) Many-one

312. Let $f(x) = x + 1$ and $\phi(x) = x - 2$. Then the values of x satisfying $|f(x) + \phi(x)| = |f(x)| + |\phi(x)|$ are :

a) $(-\infty, 1]$ b) $[2, \infty)$ c) $(-\infty, -2]$ d) $[1, \infty)$

313. The domain of the function $f(x) = \frac{\sin^{-1}(3-x)}{\log_e(|x|-2)}$, is

a) $[2,4]$ b) $(2, 3) \cup (3, 4]$ c) $[2, 3)$ d) $(-\infty, -3) \cup [2, \infty)$

314. If $f(x) = \frac{1}{\sqrt{|x|-x}}$ then, domain of $f(x)$ is

a) $(-\infty, 0)$ b) $(-\infty, 2)$ c) $(-\infty, \infty)$ d) None of the above

315. The domain of definition of $f(x) = \log_{10}\{(\log_{10} x)^2 - 5 \log_{10} x + 6\}$, is

a) $(0, 10^2)$ b) $(10^3, \infty)$ c) $(10^2, 10^3)$ d) $(0, 10^2) \cup (10^3, \infty)$

316. If a function $f(x)$ satisfies the condition $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \neq 0$, then $f(x)$ equals

a) $x^2 - 2$ for all $x \neq 0$

b) $x^2 - 2$ for all x satisfying $|x| \geq 2$

c) $x^2 - 2$ for all x satisfying $|x| < 2$

d) None of these

317. The period of the function $f(x) = \sin\left(\frac{2x+3}{6\pi}\right)$, is

a) 2π b) 6π c) $6\pi^2$ d) None of these

318. $f: R \to R$ is a function defined by $f(x) = 10x - 7$. If $g = f^{-1}$, then $g(x) =$

a) $\frac{1}{10x - 7}$ b) $\frac{1}{10x + 7}$ c) $\frac{x + 7}{10}$ d) $\frac{x - 7}{10}$

319. If $f(x) = [x - 2]$, where $[x]$ denotes the greatest integer less than or equal to x, then $f(2.5)$ is equal to

a) $\frac{1}{2}$ b) 0 c) 1 d) Does not exist

320. The domain of definition of

$f(x) = \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3}$, is

a) $(10^3, 10^4)$ b) $[10^3, 10^4]$ c) $[10^3, 10^4)$ d) $(10^3, 10^4]$

321. The value of $n \in Z$ (the set of integers) for which the function $f(x) = \sin\frac{\sin nx}{\sin\left(\frac{x}{n}\right)}$ has 4π as its period is

a) 2 b) 3 c) 5 d) 4

322. The inverse of the function $f: R \to R$ given by $f(x) = \log_a\left(x + \sqrt{x^2+1}\right)$ $(a > 0, a \neq 1)$, is

a) $\frac{1}{2}(a^x + a^{-x})$ b) $\frac{1}{2}(a^x - a^{-x})$ c) $\frac{1}{2}\left(\frac{a^x + a^{-x}}{a^x - a^{-x}}\right)$ d) Not defined

323. The domain of definition of the function

$f(x) = x \cdot \frac{1 + 2(x+4)^{-0.5}}{2 - (x+4)^{0.5}} + (x+4)^{0.5} + 4(x+4)^{0.5}$ is

a) R b) $(-4, 4)$ c) R^+ d) $(-4, 0) \cup (0, \infty)$

324. If $f(x) = \frac{ax}{x+1}, x \neq -1$, for what value of a is $f[f(x)] = x$?

a) $\sqrt{2}$ b) $-\sqrt{2}$ c) 1 d) -1

325. The period of the function $f(x) = \operatorname{cosec}^2 3x + \cot 4x$ is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) π

326. The domain of the definition of the function $f(x) = \sqrt{1 + \log_e(1-x)}$ is

a) $-\infty < x \leq 0$ b) $-\infty < x \leq \frac{e-1}{e}$ c) $-\infty < x \leq 1$ d) $x \geq 1 - e$

327. The range of the function $\sin(\sin^{-1} x + \cos^{-1} x), |x| \leq 1$ is

a) [-1, 1] b) [1, -1] c) {0} d) {1}

328. The range of $f(x) = \cos x - \sin x$ is

a) [-1, 1] b) (-1, 2) c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ d) $[-\sqrt{2}, \sqrt{2}]$

329. The range of function $f(x) = x^2 + \frac{1}{x^2+1}$

a) $[1, \infty)$ b) $[2, \infty)$ c) $\left[\frac{3}{2}, \infty\right)$ d) None of these

330. If n is an integer, the domain of the function $\sqrt{\sin 2x}$ is

a) $\left[n\pi - \frac{\pi}{2}, n\pi\right]$ b) $\left[n\pi, n\pi + \frac{\pi}{4}\right]$ c) $[(2n-1)\pi, 2n\pi]$ d) $[2n\pi, (2n+1)\pi]$

331. If $f: R \to R$ is defined by $f(x) = x - [x] - \frac{1}{2}$ for all $x \in R$, where $[x]$ denotes the greatest integer function, then $\left\{x \in R : f(x) = \frac{1}{2}\right\}$ is equal to

a) Z b) N c) ϕ d) R

332. Suppose $f: [-2, 2] \to R$ is defined by

$f(x) = \begin{cases} -1, \text{for } -2 \leq x \leq 0 \\ x - 1 \text{ for } 0 \leq x \leq 2 \end{cases}$, then $\{x \in [-2, 2] : x \leq 0 \text{ and } f(|x|) = x\}$ is equal to

a) $\{-1\}$ b) {0} c) $\left\{-\frac{1}{2}\right\}$ d) ϕ

333. If $f: R \to R$ is defined by $f(x) = \sin x$ and $g: (1, \infty) \to R$ is defined by $g(x) = \sqrt{x^2 - 1}$, then $gof(x)$ is

a) $\sqrt{\sin(x^2-1)}$ b) $\sin\sqrt{x^2-1}$ c) $\cos x$ d) Not defined

334. Let R and C denote the set of real numbers and complex numbers respectively. The function $f: C \to R$ defined by $f(z) = |z|$ is

a) One to one b) Onto

c) Bijective d) Neither one to one nor onto

335. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is

a) $\frac{f(x)+1}{f(x)+3}$ b) $\frac{3f(x)+1}{f(x)+3}$ c) $\frac{f(x)+3}{f(x)+1}$ d) $\frac{f(x)+3}{3f(x)+1}$

336. The range of the function $f(x) = \tan\sqrt{\frac{\pi^2}{9} - x^2}$ is

a) [0, 3] b) $[0, \sqrt{3}]$ c) $(-\infty, \infty)$ d) None of these

337. The domain of the function $f(x) = \operatorname{cosec}^{-1}[\sin x]$ in $[0, 2\pi]$, where $[\cdot]$ denotes the greatest integer function, is

a) $[0, \pi/2) \cup (\pi, 3\pi/2]$ b) $(\pi, 2\pi) \cup \{\pi/2\}$ c) $(0, \pi] \cup \{3\pi/2\}$ d) $(\pi/2, \pi) \cup (3\pi/2, 2\pi)$

338. Let R be the relation on the set R of all real numbers defined by aRb if $|a-b| \leq 1$, then R is

a) Reflexive and symmetric b) Symmetric only

c) Transitive only d) Anti-symmetric only

339. The domain of the function $f(x) = \log_e(x - [x])$ is

a) R b) $R - Z$ c) $(0, +\infty)$ d) Z

340. If $f: [0, \infty] \to [0, \infty]$ and $f(x) = \frac{x}{1+x}$, then f is

a) One-one and onto b) One-one but not onto

c) Onto but not one-one d) Neither one-one nor onto

341. The function $f: R \to R$ given by $f(x) = x^3 - 1$ is

a) A one-one function b) An onto function

c) A bijection d) Neither one-one nor onto

342. Let $[x]$ denote the greatest integer $\leq x$. If $f(x) = [x]$ and $g(x) = |x|$, then the value of $f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right)$ is

a) 2 b) -2 c) 1 d) -1

343. The domain of the function $f(x) = \frac{\cos^{-1} x}{[x]}$ is

a) $[-1, 0) \cup \{1\}$ b) [-1, 1] c) [-1, 1) d) None of these

344. The set of values of x for which of the function $f(x) = \frac{1}{x} + 2^{\sin^{-1}x} + \frac{1}{\sqrt{x-2}}$ exists is

a) R b) $R - \{0\}$ c) ϕ d) None of these

345. If $f(x)$ satisfies the relation $2f(x) + f(1-x) = x^2$ for all real x, then $f(x)$ is

a) $\frac{x^2+2x-1}{6}$ b) $\frac{x^2+2x-1}{3}$ c) $\frac{x^2+4x-1}{3}$ d) $\frac{x^2-3x+1}{6}$

346. If the function $f(x)$ is defined by $f(x) = a + bx$ and $f^r = fff....$ (repeated r times), then $f^r(x)$ is equal to

a) $a + b^r x$ b) $ar + b^r x$ c) $ar + bx^r$ d) $a\left(\frac{b^r - 1}{b-1}\right) + b^r x$

347. If $f(x) = \frac{x-1}{x+1}$, then $f(2x)$ is

a) $\frac{f(x)+1}{f(x)+3}$ b) $\frac{3f(x)+1}{f(x)+3}$ c) $\frac{f(x)+3}{f(x)+1}$ d) $\frac{f(x)+3}{3f(x)+1}$

348. If $f(x)$ is an odd periodic function with period 2, then $f(4)$ equals

a) 0 b) 2 c) 4 d) -4

349. The domain of definition of

$f(x) = \sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)} \times \frac{1}{x^2-36}$, is

a) $(-\infty, 0) - \{-6\}$ b) $(0, \infty) - \{1, 6\}$ c) $(1, \infty) - \{6\}$ d) $[1, \infty) - \{6\}$

350. The domain of the function $f(x) = \log_2(\log_3(\log_4 x))$ is

a) $(-\infty, 4)$ b) $(4, \infty)$ c) $(0, 4)$ d) $(1, \infty)$

ANSWERS

301)	**a**	**302)**	**c**	**303)**	**a**	**304)**	**b**
305)	**b**	**306)**	**d**	**307)**	**a**	**308)**	**d**
309)	**c**	**310)**	**b**	**311)**	**c**	**312)**	**b**
313)	**b**	**314)**	**a**	**315)**	**d**	**316)**	**b**
317)	**c**	**318)**	**c**	**319)**	**b**	**320)**	**c**
321)	**a**	**322)**	**b**	**323)**	**d**	**324)**	**d**
325)	**d**	**326)**	**b**	**327)**	**d**	**328)**	**d**
329)	**a**	**330)**	**b**	**331)**	**c**	**332)**	**c**
333)	**d**	**334)**	**d**	**335)**	**b**	**336)**	**b**
337)	**b**	**338)**	**a**	**339)**	**b**	**340)**	**b**
341)	**c**	**342)**	**d**	**343)**	**a**	**344)**	**c**
345)	**b**	**346)**	**d**	**347)**	**b**	**348)**	**a**
349)	**c**	**350)**	**b**				

301 **(a)**

Given, $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ ...(i)

Replacing x by $\frac{1}{x}$, we get

$$2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1 \qquad \text{...(ii)}$$

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and subtracting Eq. (i) from Eq. (ii), we get

$$5f(x^2) = \frac{3}{x^2} - 1 - 2x^2$$

$$\Rightarrow \quad f(x^2) = \frac{1}{5x^2}(3 - x^2 - 2x^4)$$

$$\Rightarrow \quad f(x^4) = \frac{1}{5x^4}(3 - x^4 - 2x^8) \qquad \text{[Replacing } x \text{ by } x^2\text{]}$$

$$= \frac{(1 - x^4)(2x^4 + 3)}{5x^4}$$

302 **(c)**

The function $f(x) = {}^{7-x}P_{x-3}$ is defined only if x is an integer satisfying the following inequalities:
$(i) 7 - x \geq 0$ $(ii) x - 3 \geq 0$ $(iii) 7 - x \geq x - 3$
Now,

$$\left.\begin{array}{r} 7 - x \geq 0 \Rightarrow x \leq 7 \\ x - 3 \geq 0 \Rightarrow x \geq 3 \\ 7 - x \geq x - 3 \Rightarrow x \leq 5 \end{array}\right\} \Rightarrow 3 \leq x \leq 5$$

Hence, the required domain is $\{3, 4, 5\}$

303 **(a)**

We have,

$f(x) = x, g(x) = |x|$ for all $x \in R$ and $\phi(x)$ satisfies the relation

$[\phi(x) - f(x)]^2 + [\phi(x) - g(x)]^2 = 0$

$\Rightarrow \phi(x) - f(x) = 0$ and $\phi(x) - g(x) = 0$

$\Rightarrow \phi(x) = f(x)$ and $\phi(x) = g(x)$

$\Rightarrow f(x) = g(x) = \phi(x)$

But, $f(x) = g(x) = x$, for all $x \geq 0$ $[\because |x| = x \text{ for all } x \geq 0]$

$\therefore \phi(x) = x$ for all $x \in [0, \infty)$

304 **(b)**

We observe that $f(x) = 3\sin\left(\sqrt{\frac{\pi^2}{16} - x^2}\right)$ exists for

$$\frac{\pi^2}{16} - x^2 \geq 0 \Rightarrow -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

The least value of $\frac{\pi^2}{16}-x^2$ is 0 for $x=\pm\frac{\pi}{4}$ and the greatest value is $\frac{\pi^2}{16}$ for $x=0$. Therefore, the greatest value of $f(x)$ occurs at $x=0$ and the least value occurs at $x=\pm\pi/4$

Thus, greatest and least values of $f(x)$ are

$$f(0)=3\sin\left(\sqrt{\frac{\pi^2}{16}}\right)=3\sin\frac{\pi}{4}=\frac{3}{\sqrt{2}}\text{and, } f\left(\frac{\pi}{4}\right)=3\sin 0=0$$

Hence, the value of $f(x)$ lie in the interval $[0,3/\sqrt{2}]$

ALITER For $x\in[-\pi/4,\pi/4]=Dom\,(f)$, we find that $\sqrt{\frac{\pi^2}{16}-x^2}\in[0,\pi/4]$

Since $\sin x$ is an increasing function on $[0,\pi/4]$

$$\therefore \sin x\le\sin\sqrt{\frac{\pi^2}{16}-x^2}\le\sin\pi/4$$

$$\Rightarrow 0\le 3\sin\sqrt{\frac{\pi^2}{16}-x^2}\le\frac{3}{\sqrt{2}}\Rightarrow 0\le f(x)\le\frac{3}{\sqrt{2}}$$

305 **(b)**

$$f\left(\frac{\pi}{2}+x\right)=\left|\sin\left(\frac{\pi}{2}+x\right)\right|+\left|\cos\left(\frac{\pi}{2}+x\right)\right|$$

$$=|\cos x|+|\sin x|\text{for all } x.$$

Hence, $f(x)$ is periodic with period $\frac{\pi}{2}$.

306 **(d)**

It can be easily checked that $g(x)=\left(\frac{x^{1/3}-b}{a}\right)^{1/2}$ satisfies the relation $fog\,(x)=gof\,(x)$

307 **(a)**

Since, $(1,2)\in S$ but $(2,1)\notin S$

$\therefore$ $S$is not symmetric.

Hence, S is not an equivalent relation.

Given, $T=\{(x,y):(x-y)\in I\}$

Now, $xTx\Rightarrow x-x=0\in I$, it is reflexive relation

Again, $xTy\Rightarrow(x-y)\in I$

$\Rightarrow y-x\in I\Rightarrow yTx$it is symmetric relation.

Let xTy and yTz

$\therefore x-y=I_1$ and $y-z=I_2$

Now, $x - z = (x - y) + (y - z) = I_1 + I_2 \in I$

$\Rightarrow x - z \in I$

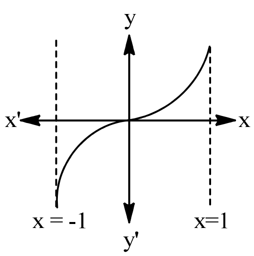
$\Rightarrow \; xTz$

$\therefore$ T is transitive.

Hence, T is an equivalent relation.

308 **(d)**

$$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$



Since, $-1 \leq x \leq 1$, therefore $-1 \leq f(x) \leq 1$

$\therefore$ Function is one-one onto.

309 **(c)**

We have,

$$f(x) = \frac{1 - x}{1 + x}$$

$$\Rightarrow f(f(x)) = f\left(\frac{1 - x}{1 + x}\right) = \frac{1 - \frac{1 - x}{1 + x}}{1 + \frac{1 - x}{1 + x}} = x$$

Again,

$$f(x) = \frac{1 - x}{1 + x}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1 - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{x - 1}{x + 1}$$

$$\therefore f\left(f\left(\frac{1}{x}\right)\right) = f\left(\frac{x - 1}{x + 1}\right) = \frac{1 - \frac{x - 1}{x + 1}}{1 + \frac{x - 1}{x + 1}} = \frac{1}{x}$$

$\therefore \alpha = f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right) = x + \frac{1}{x}$

$\Rightarrow |\alpha| = \left|x + \frac{1}{x}\right| \geq 2$

310 **(b)**

Let $A = \{1, 2, 3\}$

Let two transitive relations on the set A are

$R = \{(1,1), (1,2)\}$

And $S = \{(2,2), (2,3)\}$

Now, $R \cup S = \{(1,1), (1,2), (2,2), (2,3)\}$

Here, $(1,2), (2,3) \in R \cup S \Rightarrow (1,3) \notin R \cup S$

$\therefore R \cup S$ is not transitive.

311 **(c)**

$f(1) = 3, f(2) = 4, f(3) = 5, f(4) = 6$

$\Rightarrow 1 \in B, 2 \in B$ do not have any pre-image in A

$\Rightarrow f$ is one-one and into

312 **(b)**

We observe that
$|f(x) + \phi(x)| = |f(x)| + |\phi(x)|$ is true, if
$f(x) \geq 0$ and $\phi(x) \geq 0$
OR
$f(x) < 0$ and $\phi(x) < 0$
$\Rightarrow (x > -1 \text{ and } x > 2)$ or $(x < -1 \text{ and } x < 2)$
$\Rightarrow x \in (2, \infty) \cup (-\infty, -1)$

313 **(b)**

We have, $f(x) = \dfrac{\sin^{-1}(3 - x)}{\log_e(|x| - 2)}$
$\sin^{-1}(3 - x)$ is defined for all x satisfying
$-1 \leq 3 - x \leq 1 \Rightarrow -4 \leq -x \leq -2 \Rightarrow x \in [2, 4]$
$\log_e(|x| - 2)$ is defined for all x satisfying
$|x| - 2 > 0 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$
Also, $\log_e(|x| - 2) = 0$ when $|x| - 2 = 1$ i.e., $x = \pm 3$
Hence, domain of $f = (2, 3) \cup (3, 4]$

314 **(a)**

$f(x)$ is defined

When $|x| > x$

$\Rightarrow \quad x < -x, x > x$

$\Rightarrow \ 2x < 0, (x > x$ is not possible)

$\Rightarrow \ x < 0$

Hence domain of $f(x)$is $(-\infty, 0)$.

315 **(d)**

We have,
$f(x) = \log_{10}\{(\log_{10} x)^2 - 5(\log_{10} x) + 6\}$
Clearly, $f(x)$ assumes real values, if
$(\log_{10} x)^2 - 5\log_{10} x + 6 > 0$ and $x > 0$
$\Rightarrow (\log_{10} x - 2)(\log_{10} - 3) > 0$ and $x > 0$
$\Rightarrow (\log_{10} x < 2$ or $\log_{10} x > 3)$and $x > 0$
$\Rightarrow (x < 10^2$or, $x > 10^3)$and $x > 0 \Rightarrow x \in (0, 10^2) \cup (10^3, \infty)$

316 **(b)**

We have,
$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$
$\Rightarrow f(y) = y^2 - 2$, where $y = x + \frac{1}{x}$
Now,
$x > 0 \Rightarrow y = x + \frac{1}{x} \geq 2$ and, $x < 0 \Rightarrow y = x + \frac{1}{x} \leq -2$
Thus, $f(y) = y^2 - 2$ for all y satisfying $|y| \geq 2$

317 **(c)**

Since $\sin x$ is a periodic function with period 2π and
$f(x) = \sin\left(\frac{2x+3}{6\pi}\right) = \sin\left(\frac{x}{3\pi} + \frac{1}{2\pi}\right)$
$\therefore f(x)$ is periodic with period $= \frac{2\pi}{1/3\pi} = 6\pi^2$

318 **(c)**

Let $f(x) = y$. Then,

$10x - 7 = y \Rightarrow x = \frac{y+7}{10} \Rightarrow f^{-1}(y) = \frac{y+7}{10}$

Hence, $f^{-1}(x) = \frac{x+7}{10}$

319 **(b)**

$\therefore\ f(2.5) = [2.5 - 2] = [0.5] = 0$

320 **(c)**

We have,

$f(x) = \sqrt{\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3}$

Clearly, $f(x)$ assumes real values, if

$\log_{10}(\log_{10} x) - \log_{10}(4 - \log_{10} x) - \log_{10} 3 \geq 0$

$\Rightarrow \log_{10}\left\{\frac{\log_{10} x}{3(4 - \log_{10} x)}\right\} \geq 0$

$\Rightarrow \frac{\log_{10} x}{3(4 - \log_{10} x)} \geq 1$

$\Rightarrow \frac{4\log_{10} x - 12}{3(4 - \log_{10} x)} \geq 0$

$\Rightarrow \frac{\log_{10} x - 3}{\log_{10} x - 4} \leq 0$

$\Rightarrow 3 \leq \log_{10} x < 4 \Rightarrow 10^3 \leq x < 10^4 \Rightarrow x \in [10^3, 10^4)$

Hence, domain of $f = [10^3, 10^4)$

321 **(a)**

We observe that the periods of $\sin x$ and $\sin\frac{x}{n}$ are $\frac{2\pi}{|n|}$ and $2|n|\pi$ respectively

Therefore, $f(x)$ is periodic with period $2|n|\pi$

But, $f(x)$ has period 4π

$\therefore\ 2|n|\pi = 4\pi \Rightarrow |n| = 2 \Rightarrow n = \pm 2$

322 **(b)**

It can be easily checked that $f: R \to R$ given by $f(x) = \log_a(x + \sqrt{x^2 + 1})$ is a bijection

Now, $f\left(f^{-1}(x)\right) = x$

$\Rightarrow \log_a\left(f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1}\right) = x$

$\Rightarrow f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1} = a^x$ ...(i)

$\Rightarrow \frac{1}{f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1}} = a^{-x}$

$\Rightarrow -f^{-1}(x) + \sqrt{\{f^{-1}(x)\}^2 + 1} = a^{-x}$ ...(ii)

Subtracting (ii) from (i), we get

$2f^{-1}(x) = a^x - a^{-x}$

$\Rightarrow f^{-1}(x) = \frac{1}{2}(a^x - a^{-x})$

323 **(d)**

We have,

$$f(x) = x\frac{1 + \frac{2}{\sqrt{x+4}}}{2 - \sqrt{x+4}} + \sqrt{x+4} + 4\sqrt{x+4}$$

Clearly, $f(x)$ is defined for $x + 4 > 0$ and $x \neq 0$

So, Domain of $f(x)$ is $(-4, 0) \cup (0, \infty)$

324 **(d)**

$\because \quad f(f(x)) = f\left(\frac{\alpha x}{x+1}\right)$

$$= \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right) + 1} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$\Rightarrow \quad \frac{\alpha^2 x}{\alpha x + x + 1} = x$ [given]

$\Rightarrow \quad \alpha^2 = \alpha x + x + 1$

$\Rightarrow \quad \alpha^2 - 1 = (\alpha + 1)x$

$\Rightarrow \quad (\alpha + 1)(\alpha - 1 - x) = 0$

$\Rightarrow \quad \alpha + 1 = 0 \quad \Rightarrow \quad \alpha = -1 \qquad [\because \alpha - 1 - x \neq 0]$

325 **(d)**

$f(x) = \mathrm{cosec}^2 3x + \cot 4x$

Period of $\mathrm{cosec}^2 3x$ is $\frac{\pi}{3}$ and $\cot 4x$ is $\frac{\pi}{4}$.

$\therefore$ Period of $f(x) =$ LCM of $\left\{\frac{\pi}{3} \text{and} \frac{\pi}{4}\right\}$

$$= \frac{\text{LCM of } (\pi, \pi)}{\text{HCF of } (3, 4)} = \frac{\pi}{1} = \pi$$

326 **(b)**

Given, $f(x) = \sqrt{1 + \log_e(1 - x)}$

For domain, $(1 - x) > 0$ and $\log_e(1 - x) \geq -1$

$\Rightarrow \quad x < 1$ and $1 - x \geq e^{-1}$

$\Rightarrow \quad x < 1$ and $x \leq 1 - \frac{1}{e}$

$\Rightarrow \quad -\infty < x \leq \frac{e-1}{e}$

327 **(d)**

$\sin(\sin^{-1} x + \cos^{-1} x) = \sin\left(\frac{\pi}{2}\right) = 1$

$\therefore$ Range of $\sin(\sin^{-1} x + \cos^{-1} x)$ is 1.

328 **(d)**

Given, $f(x) = \cos x - \sin x$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\cos\left(\frac{\pi}{2} + x\right)$$

Since, $-1 \le \cos x \le 1 \Rightarrow -1 \le \cos\left(\frac{\pi}{4} + x\right) \le 1$

$\Rightarrow \quad -\sqrt{2} \le \sqrt{2}\cos\left(\frac{\pi}{4} + x\right) \le \sqrt{2}$

$\therefore$ Range is $[-\sqrt{2}, \sqrt{2}]$

329 **(a)**

Given, $f(x) = x^2 + \frac{1}{x^2+1}$

$$= (x^2 + 1) - \left(\frac{x^2}{x^2 + 1}\right)$$

$$= 1 + x^2\left(1 - \frac{1}{x^2 + 1}\right) \ge 1, \forall x \in R$$

Hence, range of $f(x)$is $[1, \infty)$.

330 **(b)**

Let $y = \sqrt{\sin 2x} \Rightarrow 0 \le \sin 2x \le 1,$

$\Rightarrow \quad 0 \le 2x \le \frac{\pi}{2}$

$\Rightarrow \quad 0 \le x \le \frac{\pi}{4}$

$\Rightarrow \quad x \in \left[n\pi, n\pi + \frac{\pi}{4}\right]$

331 **(c)**

We have, $f(x) = x - [x] - \frac{1}{2}$

$\therefore f(x)=\frac{1}{2} \Rightarrow x-[x]=1$

But, for any $x \in R, 0 \leq x-[x]<1$

$\therefore x-[x] \neq 1$ for any $x \in R$

Hence, $\left\{x \in R : f(x)=\frac{1}{2}\right\}=\phi$

332 **(c)**

Since, $x \in [-2,2], \quad x \leq 0$ and $f(|x|)=x$

For $\quad -2 \leq x \leq 0$

$f(-x)=x \quad \Rightarrow \quad \leq(-x)-1=x \quad \Rightarrow \quad x=-\frac{1}{2}$

333 **(d)**

Given, $\quad f(x)=\sin x$

And $\quad g(x)=\sqrt{x^2-1}$

$\therefore$ Range of $f=[-1,1] \notin$ domain of $g=(1,\infty)$

$\therefore gof$ is not defined.

334 **(d)**

Given, $f: C \rightarrow R$ such that $f(z)=|z|$

We know modulus of z and $\bar{z}$ have same values, so $f(z)$ has many one.

Also, $|z|$ is always non-negative real numbers, so it is not onto function.

335 **(b)**

We have,

$f(x)=\frac{x-1}{x+1}$

$\Rightarrow \frac{f(x)+1}{f(x)-1}=\frac{2x}{-2}$ [Applying componendo-dividendo]

$\Rightarrow x=\frac{f(x)+1}{1-f(x)}$

$\therefore f(2x)=\frac{2x-1}{2x+1}=\frac{2\left\{\frac{f(x)+1}{1-f(x)}\right\}-1}{2\left\{\frac{f(x)+1}{1-f(x)}\right\}+1}=\frac{3f(x)+1}{f(x)+3}$

336 **(b)**

Given, $f(x)=\tan\sqrt{\frac{\pi}{9}-x^2}$

For $f(x)$ to be defined $\frac{\pi^2}{9}-x^2 \geq 0$

$\Rightarrow \quad x^2 \leq \frac{\pi^2}{9} \Rightarrow -\frac{\pi}{3} \leq 3 \leq \frac{\pi}{3}$

$\therefore$ Domain of $f=\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

The greatest value of $f(x)=\tan\sqrt{\frac{\pi^2}{9}-0}$, when $x=0$

And the least value of $f(x)=\tan\sqrt{\frac{\pi^2}{9}-\frac{\pi^2}{9}}$, when $x=\frac{\pi}{3}$

$\therefore$ The greatest value of $f(x)=\sqrt{3}$ and the least value of $f(x)=0$

$\therefore$ Range of $f=[0,\sqrt{3}]$.

337 **(b)**

We have,

$$[\sin x]=\begin{cases} 0, 0 \leq x < \pi/2 \\ 1, x=\pi/2 \\ 0, \pi/2 < x \leq \pi \\ -1, \pi < x < 2\pi \\ 0, x=\pi, 2\pi \end{cases}$$

And, $\operatorname{cosec}^{-1} x$ is defined for $x \in (-\infty,-] \cup [1,\infty)$

$\therefore f(x)=\operatorname{cosec}^{-1}[\sin x]$ is defined for $x=\frac{\pi}{2}$ and $x \in (\pi, 2\pi)$

Hence, domain of $\operatorname{cosec}^{-1}[\sin x]$ is $(\pi, 2\pi) \cup \left\{\frac{\pi}{2}\right\}$

338 **(a)**

aRa if $|a-a|=0<1$, which is true.

$\therefore$ It is reflexive.

Now, aRb,

$|a-b| \leq 1 \Rightarrow |b-a| \leq 1$

$\Rightarrow \quad aRb \Rightarrow bRa$

$\therefore$ It is symmetric.

339 **(b)**

Given

$f(x)=\log_e(x-[x])=\log_e\{x\}$

When x is an integer, then the function is not defined.

$\therefore$ Domain of the function $R - Z$.

340 **(b)**

Here, $f:[0,\infty] \to [0,\infty)$ *ie*, domain is $[0,\infty)$ and codomain is $[0,\infty)$.

For one-one $f(x) = \frac{x}{1+x}$

$\Rightarrow$ $$f'(x) = \frac{1}{(1+x)^2} > 0, \forall x \in [0,\infty)$$

$\therefore$ $f(x)$is increasing in its domain. Thus, $f(x)$ is one-one in its domain.

For onto (we find range)

$f(x) = \frac{x}{1+x}$ *ie*, $y = \frac{x}{1+x} \Rightarrow y + yx = x$

$\Rightarrow x = \frac{y}{1-y} \Rightarrow \frac{y}{1-y} \geq 0$ as $x \geq 0 \therefore 0 \leq y \neq 1$

ie, Range $\neq$ Codomain

$\therefore$ $f(x)$is one-one but not onto.

341 **(c)**

Given, $f(x) = x^3 - 1$

Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1^3 - 1 = x_2^3 - 1$

$\Rightarrow$ $x_1^3 = x_2^3$

$\Rightarrow$ $x_1 = x_2$

$\therefore$ $f(x)$is one-one. Also, it is onto as range of $f = R$

Hence, it is a bijection.

342 **(d)**

Given $f(x) = [x]$ and $g(x) = |x|$

Now, $f\left(g\left(\frac{8}{5}\right)\right) = f\left(\frac{8}{5}\right) = \left[\frac{8}{5}\right] = 1$

And $g\left(f\left(-\frac{8}{5}\right)\right) = g\left(\left[-\frac{8}{5}\right]\right) = g(-2) = 2$

$\therefore \quad f\left(g\left(\frac{8}{5}\right)\right) - g\left(f\left(-\frac{8}{5}\right)\right) = 1 - 2 = -1$

343 **(a)**

$\because \quad f(x) = \frac{\cos^{-1} x}{[x]}$

For $f(x)$ to be defined $-1 \leq x \leq 1$ and $[x] \neq 0 \Rightarrow x \notin [0, 1)$

$\therefore$ Domain of $f(x)$is $[-1, 0) \cup \{1\}$.

344 **(c)**

Let $f(x) = g(x) + h(x) + u(x)$, where

$g(x) = \frac{1}{x}, h(x) = 2^{\sin^{-1} x}$ and $u(x) = \frac{1}{\sqrt{x-2}}$

The domain of $g(x)$ is the set of all real numbers other than zero i.e. $R - \{0\}$

The domain of $h(x)$ is the set $[-1, 1]$ and the domain of $u(x)$ is the set of all reals greater than 2, i.e., $(2, \infty)$

Therefore, domain of $f(x) = R - \{0\} \cap [-1, 1] \cap (2, \infty) = \phi$

345 **(b)**

Given, $2f(x) + f(1-x) = x^2$ ...(i)

Replacing x by $(1-x)$, we get

$$2f(1-x) + f(x) = (1-x)^2$$

$$\Rightarrow \quad 2f(1-x) + f(x) = 1 + x^2 - 2x \quad \text{...(ii)}$$

On multiplying Eq. (i) by 2 and subtracting from Eq. (ii), we get

$$3f(x) = x^2 + 2x - 1 \Rightarrow f(x) = \frac{x^2 - 2x - 1}{3}$$

346 **(d)**

$f(x) = a + bx$

$\therefore \quad f\{f(x)\} = a + b(a + bx) = a(1 + b)b^2 x$

$\Rightarrow \quad f[f\{f(x)\}] = f\{a(1+b) + b^2 x\} = a(1 + b + b^2) + b^3 x$

$$\therefore \quad f^r(x) = a(1 + b + b^2 + \cdots + b^{r-1}) + b^r x$$

$$= a\left(\frac{b^r - 1}{b - 1}\right) + b^r x$$

347 **(b)**

We have,

$$f(x) = \frac{x-1}{x+1}$$

$$\Rightarrow \frac{f(x)+1}{f(x)-1} = \frac{2x}{-2}$$

$$\Rightarrow x = \frac{f(x)+1}{1-f(x)}$$

$$\therefore f(2x) = \frac{2x-1}{2x+1} = \frac{2\left\{\frac{f(x)+1}{1-f(x)}\right\}-1}{2\left(\frac{f(x)+1}{1-f(x)}\right)+1} = \frac{3f(x)+1}{f(x)+3}$$

348 **(a)**

Since, $f(-x) = -f(x)$ and $f(x+2) = f(x)$

$\therefore \quad f(x) = f(0)$ and $f(-2) = f(-2+2) = f(0)$

Now, $f(0) = f(-2) = -f(2) = -f(0)$

$\Rightarrow 2f(0) = 0 \Rightarrow f(0) = 0$

$\therefore \quad f(4) = f(2) = f(0) = 0$

349 **(c)**

We observe that $\frac{1}{x^2-36}$ is not defined for $x = \pm 6$

Also, $\sqrt{\log_{0.4}\left(\frac{x-1}{x+5}\right)}$ is a real number, if

$$0 < \frac{x-1}{x++5} \leq 1$$

$$\Rightarrow 0 < \frac{x-1}{x+5} \text{and} \frac{x-1}{x+5} \leq 1$$

$$\Rightarrow (x-1)(x+5) > 0 \text{ and } 1 - \frac{6}{x+5} \leq 1$$

$$\Rightarrow (x < -5 \text{ or } x > 1) \text{ and } -\frac{6}{x+5} \leq 0$$

$$\Rightarrow (x < -5 \text{ or } x > 1) \text{and } x + 5 > 0$$

$$\Rightarrow (x < -5 \text{ or } x > 1) \text{and } x > -5$$

Hence, domain of $f(x) = (1, \infty) - \{6\}$

350 **(b)**

Given, $f(x) = \log_2(\log_3(\log_4 x))$

We know, $\log_a x$ is defined, if $x > 0$

For $f(x)$ to be defined.

$\log_3 \log_4 x > 0, \log_4 x > 0$ and $x > 0$

$\Rightarrow \quad \log_4 x > 3^0 = 1, x > 4^0 = 1$ and $x > 0$

$\Rightarrow \quad x > 4, x > 1$ and $x > 0$

$\Rightarrow \quad x > 4$