# All India 2024

# CBSE Board Solved Paper

***Time Allowed : 3 Hours*** **Maximum Marks : 80**

**General Instructions:**

(i) This question paper contains **38** questions. **All** questions are compulsory.

(ii) Question paper is divided into **Five** Sections - Sections **A, B, C, D** and **E**.

(iii) In Section **A** - Question Number **1** to **18** are Multiple Choise Questions (MCQ) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.

(iv) In Section **B** - Question Number **21** to **25** are Very Short Answer (VSA) type questions of **2** marks each.

(v) In Section **C** - Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.

(vi) In Section **D** - Question Number **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.

(vii) In Section **E** - Question Number **36** to **38** are case study based questions carrying **4** marks each where **2 VSA** type questions are of **1** mark each and **1 SA** type question is of **2** marks. Internal choice is provided in **2** marks question in each case study.

(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section - **B**, **3** questions in Section - **C**, **2** questions in Section - **D** and **2** questions in Section - **E**.

(ix) Use of calculators is **NOT** allowed.

## SECTION - A

**This section consists of 20 multiple choice questions of 1 mark each.**

**1.** If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{5\pi}{6}$ (d) $\frac{11\pi}{6}$

**2.** The vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = -3\hat{i} + 4\hat{j} + 4\hat{k}$ represents the sides of

(a) an equilaterlal triangle

(b) an obtuse - angled triangle

(c) an isosceles triangle

(d) a right - angled triangle

**3.** Let $\vec{a}$ be any vector such that $|\vec{a}|$ = a. The value of $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is :

(a) $a^2$ (b) $2a^2$

(c) $3a^2$ (d) 0

**4.** If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 + 7I = kA$, then the value of k is :

(a) 1 (b) 2

(c) 5 (d) 7

**5.** Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$. If AB = I, then the value of $\lambda$ is:

(a) $\frac{-9}{4}$ (b) $-2$

(c) $\frac{-3}{2}$ (d) 0

**6.** Derivative of $x^2$ with respect to $x^3$, is:

(a) $\frac{2}{3x}$ (b) $\frac{3x}{2}$

(c) $\frac{2x}{3}$ (d) $6x^5$

**7.** The function $f(x) = |x| + |x - 2|$ is

(a) continuous, but not differentiable at $x = 0$ and $x = 2$.

(b) differentiable but not continuous at $x = 0$ and $x = 2$.

(c) continuous but not differentiable at $x = 0$ only.

(d) neither continuous nor differentiable at $x = 0$ and $x = 2$.

**8.** The value of $\int_0^{\pi} \tan^2\left(\frac{\theta}{3}\right) d\theta$ is:

(a) $\pi + \sqrt{3}$ (b) $3\sqrt{3} - \pi$

(c) $\sqrt{3} - \pi$ (d) $\pi - \sqrt{3}$

**9.** The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = 0, x \neq 0$ is:

(a) $\frac{2}{x}$ (b) $x^2$

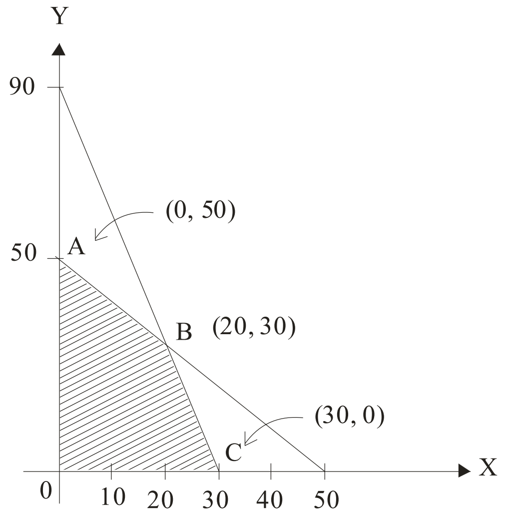
(c) $e^{\frac{2}{x}}$ (d) $e^{\log(2x)}$

**10.** The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to:

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 2 (d) 3

**11.** The maximum value of $Z = 4x + y$ for a L. P. P. whose feasible region is given below is:




(a) 50 (b) 110

(c) 120 (d) 170

**12.** The probability distribution of a random variable X is:

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| P(X) | 0.1 | k | 2k | k | 0.1 |

where k is some unknown constant.

The probability that the random variable X takes the value 2 is:

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{4}{5}$ (d) 1

**13.** The function $f(x) = kx - \sin x$ is strictly increasing for

(a) $k > 1$ (b) $k < 1$

(c) $k > -1$ (d) $k < -1$

**14.** The Cartesian equation of a line passing through the point with position vector $\vec{a} = \hat{i} - \hat{j}$ and parallel to the line $\vec{r} = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$, is

(a) $\frac{x-2}{1} = \frac{y+1}{0} = \frac{z}{1}$ (b) $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$

(c) $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z}{0}$ (d) $\frac{x-1}{2} = \frac{y}{-1} = \frac{z-1}{0}$

**15.** If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is:

(a) 0 (b) 5

(c) 10 (d) 25

**16.** Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is:

(a) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(c) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (d) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

**17.** If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \ldots$ is:

(a) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

**18.** The integrating factor of the differential equation $(x+2y^2)\dfrac{dy}{dx} = y(y>0)$ is:

(a) $\dfrac{1}{x}$ (b) $x$

(c) $y$ (d) $\dfrac{1}{y}$

**Questions No. 19 & 20, are Assertion (A) and Reason (R) based questions carrying 1 marks each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R).**

**Select the correct answer from the codes (A), (B), (C) and (D) as given below:**

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

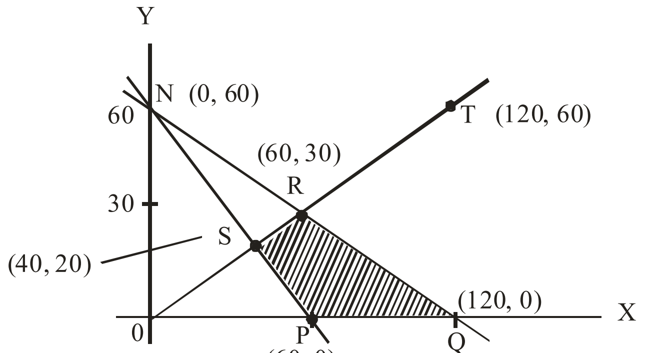
(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

**19.** **Assertion (A) :** The relation $R = \{(x, y) : (x+y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

**Reason (R) :** The number '2n' is composite for all natural numbers n.

**20.** **Assertion (A) :** The corner points of the bounded feasible region of a L. P. P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.




**Reason (R) :** The optimal solution of a LPP having bounded feasible region must occur at corner points.

## SECTION - B

**In this section there are 5 very short answer type questions of 2 marks each.**

**21.** (a) If $y = \cos^3(\sec^2 2t)$, find $\dfrac{dy}{dt}$.

**OR**

(b) If $x^y = e^{x-y}$, prove that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$.

**22.** The volume of a cube is increasing at the rate of $6 cm^3/s$. How fast is the surface area of cube increasing, when the length of an edge is 8 cm?

**23.** Show that the function f given by $f(x) = \sin x + \cos x$, is strictly decreasing in the interval $\left(\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right)$.

**24.** (a) Express $\tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right)$, where $\dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$ in the simplest form.

**OR**

(b) Find the principal value of

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right).$$

**25.** Find : $\displaystyle\int \frac{2x}{(x^2+1)(x^2-4)}dx.$

## SECTION - C

**In this section there are 6 short answer type questions of 3 marks each.**

**26.** Find $\dfrac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1}\sqrt{x}$ is given.

**27.** (a) Find the particular solution of the differential equation $\dfrac{dy}{dx} = y\cos 2x$, given that $y\left(\dfrac{\pi}{4}\right) = 2$.

**OR**

(b) Find the particular solution of the differential equation

$$\left(xe^{\frac{y}{x}} + y\right)dx = x\,dy, \text{ given that } y = 1 \text{ when } x = 1.$$

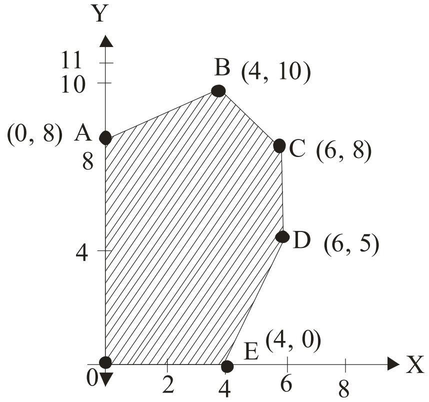
**28.** Find : $\displaystyle\int \sec^3\theta\, d\theta$

**29.** (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

**OR**

(b) A biased die is twice as likely to show an even number as an odd number. If such a dies is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

**30.** The corner points of the feasible region determined by the system of linear constraints are as shown in the following figure:




(i) If $Z = 3x - 4y$ be the objective function, then find the maximum value of Z.

(ii) If $Z = px + qy$ where $p, q > 0$ be the objective function. Find the condition on p and q so that maximum value of Z occurs at B(4, 10) and C(6, 8).

**31.** (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{1+\cos 2x+\sin 2x}$

**OR**

(b) Find : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}}+\frac{x}{\sqrt{1+x^2}}\right]dx$

## SECTION - D

In the section there are 4 long answer type questions of 5 marks each.

**32.** (a) Let $A = R - \{5\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one - one and onto.

**OR**

(b) Check whether the relation S in the set of real numbers R defined by S = {(a, b) : where $a - b + \sqrt{2}$ is an irrational number} is reflexive, symmetric or transitive.

**33.** (a) Find the distance between the line $\frac{x}{2}=\frac{2y-6}{4}=\frac{1-z}{-1}$ and another line parallel to it passing through the point (4, 0, –5).

**OR**

(b) If the line $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point (3, –4, 7).

**34.** Find $A^{-1}$, if $A=\begin{bmatrix}1 & 2 & 1\\ 2 & 3 & -1\\ 1 & 0 & 1\end{bmatrix}$. Hence, solve the following system of equations:

$x + 2y + z = 5$

$2x + 3y = 1$

$x - y + z = 8$

**35.** (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X - axis and the ordinates $x = -2$ and $x = 2$, using integration.

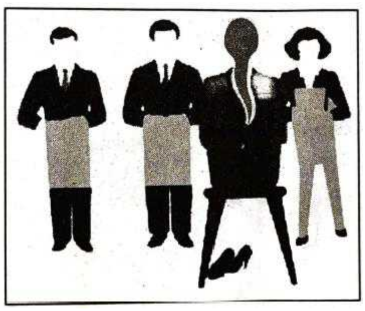
**OR**

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X - axis.

## SECTION - E

In this section, there are 3 case study based question of 4 marks each.

**36.** Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions:

(i) What is the probability that at least one of them is selected? **1**

(ii) Find $P(G \mid \overline{H})$ where G is the event of Jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected. **1**

(iii) Find the probability that exactly one of them is selected. **2**

**OR**

(iii) Find the probability that exactly two of them are selected. **2**

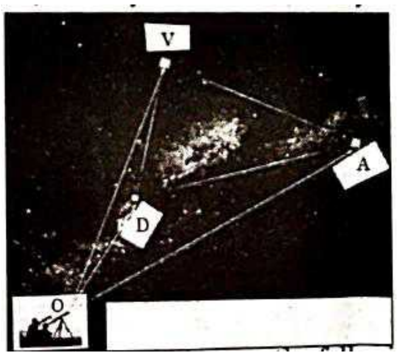
**37.** A store has been selling calculators at ₹ 350 each. A market survey indicated that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.


Based on the above information, answer the following questions:

(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result. **2**

(ii) What rebate in price of calculator should the store give to maximise the revenue? **2**

**38.** An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}, 7\hat{i}+5\hat{j}+8\hat{k}$ and $-3\hat{i}+7\hat{j}+11\hat{k}$ respectively.




Based on the above information, answer the following questions:

(i) How far is the star V from star A? **1**

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. **1**

(iii) Find the measure of $\angle VDA$. **2**

**OR**

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? **2**