All India *2020*

CBSE Board Solved Paper

Time Allowed : 3 Hours   *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them:

   (i)   This question paper comprises **four** Sections A, B, C and D. This question paper carries **36** questions. **All** questions are compulsory.

   (ii) **Section A:** Questions no. **1** to **20** comprises of **20** questions of **1** mark each.

   (iii) **Section B:** Questions no. **21** to **26** comprises of **6** questions of **2** marks each.

   (iv) **Section C:** Questions no. **27** to **32** comprises of **6** questions of **4** marks each.

   (v)   **Section D:** Questions no. **33** to **36** comprises of **4** questions of **6** marks each.

   (vi) There is no overall choice in the question paper. However, an internal choice has been provided in **3** questions of **one** mark, **2** questions of **two** marks, **2** questions of **four** marks and **2** questions of **six** marks. Only one of the choices in such questions have to be attempted.

   (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

   (viii) Use of calculators is not permitted.

SECTION - A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1.   If A is a square matrix of order 3 and |A| = 5, then the value of |2A′| is

   (a) −10   (b)   10

   (c) −40   (d)   40

2.   If A is a square matrix such that $A^2 = A$, then

   $(I - A)^3 + A$ is equal to

   (a) I   (b)   0

   (c) I − A   (d)   I + A

3.   The principal value of $\tan^{-1}(\tan \frac{3\pi}{5})$ is

   (a) $\frac{2\pi}{5}$ (b) $\frac{-2\pi}{5}$

   (c) $\frac{3\pi}{5}$ (d) $\frac{-3\pi}{5}$

4.   If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

   (a) 0   (b)   1

   (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

5.   The vector equation of the line passing through the point (−1, 5, 4) and perpendicular to the plane $z = 0$ is

   (a) $\vec{r} = -\hat{i} + 5\hat{j} + 4\hat{k} + \lambda(\hat{i} + \hat{j})$

   (b) $\vec{r} = -\hat{i} + 5\hat{j} + (4 + \lambda)\hat{k}$

   (c) $\vec{r} = \hat{i} - 5\hat{j} - 4\hat{k} + \lambda\hat{k}$

   (d) $\vec{r} = \lambda\hat{k}$

6.   The number of arbitrary constants in the particular solution of a differential equation of second order is (are)

   (a) 0   (b)   1

   (c) 2   (d)   3

7. $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x\,dx$ is equal to

(a) -1
(b) 0
(c) 1
(d) 2

8. The length of the perpendicular drawn from the point $(4, -7, 3)$ on the y-axis is

(a) 3 units
(b) 4 units
(c) 5 units
(d) 7 units

9. If A and B are two independent events with $P(A) = \dfrac{1}{3}$ and $P(B) = \dfrac{1}{4}$, then $P(B'|A)$ is equal to

(a) $\dfrac{1}{4}$
(b) $\dfrac{1}{3}$
(c) $\dfrac{3}{4}$
(d) 1

10. The corner points of the feasible region determined by the system of linear inequalities are $(0, 0)$, $(4, 0)$, $(2, 4)$ and $(0, 5)$. If the maximum value of $z = ax + by$, where $a, b > 0$ occurs at both $(2, 4)$ and $(4, 0)$, then

(a) $a = 2b$
(b) $2a = b$
(c) $a = b$
(d) $3a = b$

Fill in the blanks in question numbers 11 to 15.

11. A relation R in a set A is called _____, if $(a_1, a_2) \in$ R implies $(a_2, a_1) \in$ R, for all $a_1, a_2 \in$ A.

12. The greatest integer function defined by $f(x) = [x]$, $0 < x < 2$ is not differentiable at $x =$ _____.

13. If A is a matrix of order 3×2, then the order of the matrix A′ is _____.

OR

A square matrix A is said to be skew-symmetric, if _____.

14. The equation of the normal to the curve $y^2 = 8x$ at the origin is _____.

OR

The radius of a circle is increasing at the uniform rate of 3 cm/sec. At the instant when the radius of the circle is 2 cm, its area increases at the rate of _____ cm²/s.

15. The position vectors of two points A and B are $\overrightarrow{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overrightarrow{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vector of a point P which divides the line segment joining A and B in the ratio 2 : 1 is _____.

Question numbers 16 to 20 are very short answer type questions.

16. If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find A (adj A).

17. Find : $\displaystyle\int x^4 \log x\,dx$

OR

Find: $\displaystyle\int \dfrac{2x}{\sqrt[3]{x^2+1}}\,dx$

18. Evaluate: $\displaystyle\int_{1}^{3} |2x-1|\,dx$

19. Two cards are drawn at random and one-by-one without replacement from a well-shuffled pack of 52 playing cards. Find the probability that one card is red and the other is black.

20. Find: $\displaystyle\int \dfrac{dx}{\sqrt{9-4x^2}}$

Question numbers 21 to 26 carry 2 marks each.

21. Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1}x$, $\dfrac{1}{\sqrt{2}} \le x \le 1$.

OR

Consider a bijective function $f = R_+ \to (7, \infty)$ given by $f(x) = 16x^2 + 24x + 7$, where R_+ is the set of all positive real numbers. Find the inverse function of f.

22. If $x = at^2$, $y = 2at$, then find $\dfrac{d^2 y}{dx^2}$.

23. Find the points on the curve $y = x^3 - 3x^2 - 4x$ at which the tangent lines are parallel to the line $4x + y - 3 = 0$.

24. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 6\hat{j} + 2\hat{k}$.

OR

Find the volume of the parallelopiped whose adjacent edges are represented by $2\vec{a}$, $-\vec{b}$ and $3\vec{c}$, where

$\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and $\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$.

25. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other.

26. The probability of finding a green signal on a busy crossing X is 30%. What is the probability of finding a green signal on X on two consecutive days out of three?

Question number 27 to 32 carry 4 marks each.

27. Let N be the set of natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if $ad = bc$ for all $a, b, c, d \in$ N. Show that R is an equivalence relation.

28. If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\dfrac{dy}{dx}$.

29. Find: $\int \sec^3 x \, dx$

30. Find the general solution of the differential equation
    $y \, e^y \, dx = (y^3 + 2x \, e^y) dy$.

OR

Find the particular solution of the differential equation

$x\dfrac{dy}{dx} = y - x \tan\left(\dfrac{y}{x}\right)$, given that $y = \dfrac{\pi}{4}$ at $x = 1$.

31. A furniture trader deals in only two items – chairs and tables. He has ₹ 50,000 to invest and a space to store at most 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and table, respectively. Formulate the above problem as an LPP to maximise the profit and solve it graphically.

32. There are two bags, I and II. Bag I contains 3 red and 5 black balls and Bag II contains 4 red and 3 black balls. One ball is transferred randomly from Bag I to Bag II and then a ball is drawn randomly from Bag II. If the ball so drawn is found to be black in colour, then find the probability that the transferred ball is also black.

OR

An urn contains 5 red, 2 white and 3 black balls. Three balls are drawn, one-by-one, at random without replacement. Find the probability distribution of the number of white balls. Also, find the mean and the variance of the number of white balls drawn.

Question numbers 33 to 36 carry 6 marks each.

33. If A $= \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the

following system of the equations:

$x + 2y - 3z = 6$

$3x + 2y - 2z = 3$

$2x - y + z = 2$

OR

Using properties of determinants, prove that

$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$= (a - b)(b - c)(c - a)(a + b + c)(a^2 + b^2 + c^2)$.

34. Using integration, find the area of the region bounded by the triangle whose vertices are (2, –2), (4, 5) and (6, 2).

35. Show that the height of the right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one-third of the height of the cone, and the greatest volume of the cylinder is $\dfrac{4}{9}$ times the volume of the cone.

36. Find the equation of the plane that contains the point A(2, 1, –1) and is perpendicular to the line of intersection of the planes $2x + y - z = 3$ and $x + 2y + z = 2$. Also find the angle between the plane thus obtained and the y-axis.

OR

Find the distance of the point P(–2, –4, 7) from the

point of intersection Q of the line

$\vec{r} = (3\hat{i} - 2\hat{j} + 6\hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane

$\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 6$. Also write the vector equation of the

line PQ.