All India 2017

CBSE Board Solved Paper

Time Allowed : 3 Hours ***Maximum Marks : 100***

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1-4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Questions **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long answer-**II** Type Questions carrying **6** marks each.
(vii) Please write down the serial number of the Question before attempting it.

SECTION - A

1. If for any 2 × 2 square matrix A, A (adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

2. Determine the value of 'k' for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k \quad , & x = 3 \end{cases}$$

3. Find : $\displaystyle\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} \text{dx}$

4. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2.5y + 5z = 20$.

SECTION - B

5. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

6. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $\left[-\sqrt{3}, 0\right]$.

7. The volume of a cube is increasing at the rate of 9cm^3/s. How fast is its surface area increasing when the length of an edge is 10 cm?

8. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

9. The x–coordinate of a point on the line joining the points P (2, 2, 1) and Q(5, 1, –2) is 4. Find its z-coordinate.

10. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red." Find if A and B are independent events.

11. Two tailors, A and B earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

12. Find : $\displaystyle\int \frac{dx}{5 - 8x - x^2}$

SECTION - C

13. If $\tan^{-1}\dfrac{x-3}{x-4} + \tan^{-1}\dfrac{x+3}{x+4} = \dfrac{\pi}{4}$, then find the value of x.

14. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 + 2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

15. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

OR

If $e^y(x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

16. Find : $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}\,d\theta$

17. Evaluate : $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$

OR

Evaluate : $\int_1^4 \{|x-1|+|x-2|+|x-4|\}\,dx$

18. Solve the differential equation $(\tan^{-1} x - y)\,dx = (1+x^2)\,dy$.

19. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

20. Find the value of λ, if four points with position vectors $3\hat{i} + 6\hat{j} + 9\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $4\hat{i} + 6\hat{j} + \lambda\hat{k}$ are coplanar.

21. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one students is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

23. Maximise $Z = x + 2y$

subject to the constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$x, y \geq 0$

Solve the above LPP graphically.

SECTION - D

24. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

25. Consider f : $R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

OR

Let A = $Q \times Q$ and let * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) $\in$A. Determine, whether * is commutative and associative. Then, with respect to *on A

(i) find the identity element in A

(ii) find the invertible elements of A

26. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4).

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

28. Find the particular solution of the differential equation $(x-y)\frac{dy}{dx} = (x+2y)$, given that $y = 0$ when $x = 1$.

29. Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1) crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3).

OR

A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.