

Series &RQPS/S

Set – 3



प्रश्न-पत्र कोड
Q.P. Code

65/S/3

अनुक्रमांक

Roll No.

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परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 23 printed pages.
- Please check that this question paper contains 38 questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please write down the serial number of the question in the answer-book before attempting it.
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.



गणित MATHEMATICS



निर्धारित समय : 3 घण्टे

अधिकतम अंक : 80

Time allowed : 3 hours

Maximum Marks : 80

65/S/3

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P.T.O.



General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper contains **38** questions. **All** questions are **compulsory**.
- (ii) This question paper is divided into **five** Sections – **A, B, C, D** and **E**.
- (iii) In **Section A**, Questions no. **1** to **18** are Multiple Choice Questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.
- (iv) In **Section B**, Questions no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.
- (v) In **Section C**, Questions no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.
- (vi) In **Section D**, Questions no. **32** to **35** are Long Answer (LA) type questions carrying **5** marks each.
- (vii) In **Section E**, Questions no. **36** to **38** are case study based questions carrying **4** marks each.
- (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
- (ix) Use of calculator is **not** allowed.

SECTION A

This section comprises 20 Multiple Choice Questions (MCQs) carrying **1** mark each. $20 \times 1 = 20$

1. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$, then the value of $\det(A^{-1})$ is :

- (A) -1
- (B) 1
- (C) 0
- (D) 2

2. The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$

- (A) 0
- (B) 1
- (C) 2
- (D) $\frac{3}{2}$



3. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is :
- (A) $\frac{2\pi}{3}$ cm/s (B) π cm/s
- (C) $\frac{4\pi}{3}$ cm/s (D) 2π cm/s
4. $\int \frac{x-3}{(x-1)^3} e^x dx$ is equal to :
- (A) $\frac{2e^x}{(x-1)^3} + C$ (B) $\frac{-2e^x}{(x-1)^2} + C$
- (C) $\frac{e^x}{(x-1)} + C$ (D) $\frac{e^x}{(x-1)^2} + C$
5. The area (in sq. units) of the region bounded by the curve $y = x$, x -axis, $x = 0$ and $x = 2$ is :
- (A) $\frac{3}{2}$ (B) $\frac{1}{2} \log 2$
- (C) 2 (D) 4
6. What is the value of $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$ for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$?
- (A) $\frac{3}{7}$ (B) $\frac{7}{3}$
- (C) $\frac{4}{3}$ (D) $\frac{4}{7}$
7. If $f(x) = x^x$, then $f'(e)$ is equal to :
- (A) 0 (B) 2
- (C) e^e (D) $2e^e$



8. The function f given by $f(x) = x^3 - 3x^2 + 3x$, $x \in \mathbb{R}$ is increasing on :
- (A) $[1, \infty)$ (B) $(1, \infty)$
(C) $(-\infty, \infty)$ (D) $(-\infty, 1)$
9. If \vec{a} and \vec{b} are two vectors such that $\vec{a} \cdot \vec{b} > 0$ and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then the angle between \vec{a} and \vec{b} is :
- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
(C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$
10. For two matrices A and B , given that $A^{-1} = \frac{1}{4}B$, then inverse of $(4A)$ is :
- (A) $4B$ (B) B
(C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$
11. $\int_{\pi/6}^{\pi/3} \log(\tan x) dx$ is equal to :
- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$
(C) 0 (D) $\frac{\pi}{12}$
12. If X , Y and XY are matrices of order 2×3 , $m \times n$ and 2×5 respectively, then number of elements in matrix Y is :
- (A) 6 (B) 10
(C) 15 (D) 35



13. The number of discontinuities of the function f given by

$$f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ e^x, & \text{if } 0 \leq x \leq 1 \\ 2 - x, & \text{if } x > 1 \end{cases}$$

is :

- (A) 0 (B) 1
(C) 2 (D) 3

14. The integrating factor of the differential equation $(x^2 - 1) \frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$ is :

- (A) e^{2x} (B) $e^{x^2 - 1}$
(C) $\log(x^2 - 1)$ (D) $x^2 - 1$

15. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?

- (A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$
(C) -32 (D) -64

16. The vectors $\vec{a} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$ and $\vec{b} = 3\hat{i} - 6\hat{j} + \hat{k}$ are collinear if value of λ is :

- (A) -30 (B) 30
(C) $\frac{3}{2}$ (D) $\frac{2}{3}$

17. The vector equation of the line passing through the origin and perpendicular to the lines $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ and $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$ is :

- (A) $\vec{r} = \lambda\hat{i}$ (B) $\vec{r} = \lambda\hat{j}$
(C) $\vec{r} = \lambda\hat{k}$ (D) $\vec{r} = \lambda(\hat{i} + \hat{k})$



18. If $y = \log \sqrt{\sec \sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is :

(A) $\frac{1}{\pi}$

(B) π

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : $\cos^{-1} \left(\cos \frac{13\pi}{6} \right)$ is equal to $\frac{\pi}{6}$.

Reason (R) : The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

20. Assertion (A) : If R and S are two events such that $P(R | S) = 1$ and $P(S) > 0$, then $S \subset R$.

Reason (R) : If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.

SECTION B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. Find the value of

$$\tan^{-1} \left(\tan \frac{3\pi}{5} \right) + \cos^{-1} \left(\cos \frac{13\pi}{6} \right) + \sin^{-1} \left(-\frac{1}{2} \right).$$



22. (a) Find :

$$\int \frac{x^3 - 1}{x^3 - x} dx$$

OR

- (b) Evaluate :

$$\int_{-4}^0 |x + 2| dx$$

23. (a) If $y = (\sin^{-1} x)^2$, then find $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx}$.

OR

- (b) If $y^x = x^y$, then find $\frac{dy}{dx}$.

24. Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of $f(x)$.

25. Find the Cartesian equation of the line passing through the origin, perpendicular to y-axis and making equal acute angles with x and z axes.

SECTION C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. (a) Find :

$$\int \frac{dx}{\cos x \sqrt{\cos 2x}}$$

OR



(b) Find :

$$\int \frac{5x - 3}{\sqrt{1 + 4x - 2x^2}} dx$$

27. (a) If $y = e^{a \cos^{-1} x}$, then show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$.

OR

(b) Find $\frac{dy}{dx}$, if $y = x^{\cos x} - 2^{\sin x}$.

28. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?

29. Find the general solution of the differential equation $\frac{dy}{dx} = xy \log x \log y$.

30. Find all vectors of magnitude $8\sqrt{14}$ units that are perpendicular to the vectors $2\hat{i} - \hat{k}$ and $2\hat{j} + 3\hat{k}$.

31. (a) Find a matrix A such that

$$A \begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}.$$

Also, find A^{-1} .

OR

(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$,

show that $A^3 = A^{-1}$.



SECTION D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. Prove that the relation R in the set of integers Z defined as

$R = \{(a, b) : 2 \text{ divides } (a + b)\}$ is an equivalence relation. Also, determine [3].

33. (a) Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

OR

- (b) Find the point of intersection of the lines

$$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k}), \text{ and}$$

$$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k}).$$

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

34. Solve the following linear programming problem graphically :

$$\text{Minimise } Z = 6x + 7y$$

subject to constraints

$$x + 2y \geq 240$$

$$3x + 4y \leq 620$$

$$2x + y \geq 180$$

$$x, y \geq 0.$$

35. (a) Using integration, find the area of the region bounded by the curve

$$y = \sqrt{4-x^2}, \text{ the lines } x = -\sqrt{2} \text{ and } x = \sqrt{3} \text{ and the } x\text{-axis.}$$

OR

- (b) Using integration, evaluate the area of the region bounded by the curve

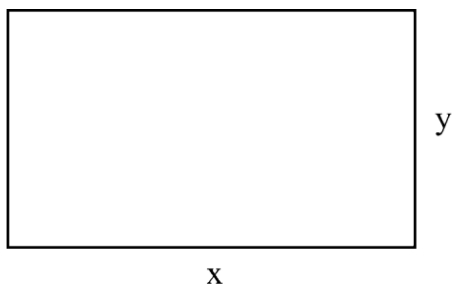
$$y = x^2, \text{ the lines } y = 1 \text{ and } y = 3 \text{ and the } y\text{-axis.}$$

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2 . If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2 .

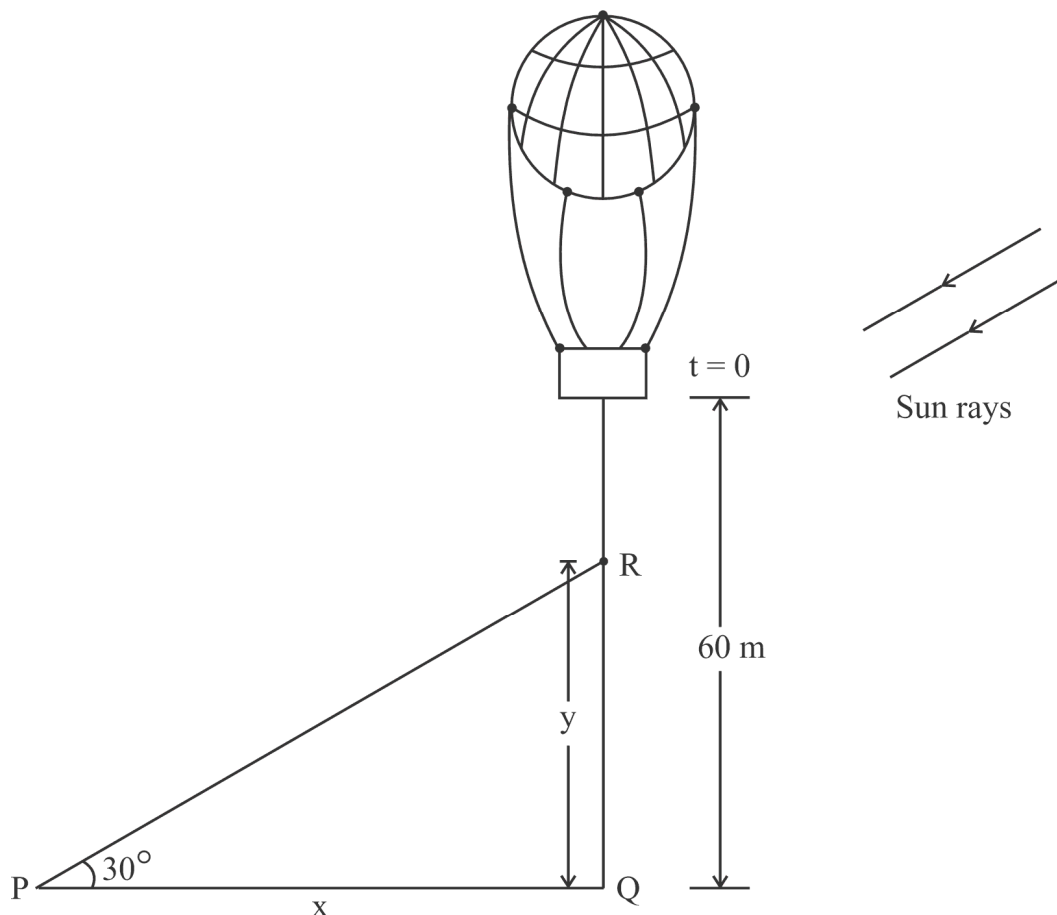


On the basis of the above information, answer the following questions :

- Formulate the linear equations in x and y to represent the given information.
- Find the dimensions of the plot of land by matrix method.

Case Study – 2

37. A sandbag is dropped from a balloon at a height of 60 metres.



Shadow path \rightarrow

When the angle of elevation of the sun is 30° , the position of the sandbag is given by the equation $y = 60 - 4.9 t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions :

- (i) Find the relation between x and y , where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.
- (ii) After how much time will the sandbag be 35 metres above the ground ?
- (iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

- (iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?