General Instructions :

Read the following instructions very carefully and strictly follow them :

(i)   This question paper contains **38** questions. *All* questions are *compulsory*.

(ii) This question paper is divided into *five* Sections – *A*, *B*, *C*, *D* and *E*.

(iii) In *Section A,* Questions no. *1* to *18* are Multiple Choice Questions (MCQs) and questions number *19* and *20* are Assertion-Reason based questions of *1* mark each.

(iv) In *Section B,* Questions no. *21* to *25* are Very Short Answer (VSA) type questions, carrying *2* marks each.

(v)   In *Section C,* Questions no. *26* to *31* are Short Answer (SA) type questions, carrying *3* marks each.

(vi) In *Section D,* Questions no. *32* to *35* are Long Answer (LA) type questions carrying *5* marks each.

(vii) In *Section E,* Questions no. *36* to *38* are case study based questions carrying *4* marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is *not* allowed.

SECTION A

*This section comprises 20 Multiple Choice Questions (MCQs) carrying **1** mark each. 20×1=20*

1. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & -2 \end{bmatrix}$, then the value of $\left| A \, adj \, (A) \right|$ is :

    (A)   – 1   (B)   1

    (C)   2   (D)   3

2. For two matrices A and B, given that $A^{-1} = \frac{1}{4} B$, then inverse of $(4A)$ is :

    (A)   4B   (B)   B

    (C) $\frac{1}{4} B$ (D) $\frac{1}{16} B$

3. If X, Y and XY are matrices of order 2×3, $m \times n$ and 2×5 respectively, then number of elements in matrix Y is :

(A)   6
(B)   10
(C)   15
(D)   35

4. The number of discontinuities of the function f given by

$$f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ e^x, & \text{if } 0 \le x \le 1 \\ 2-x, & \text{if } x > 1 \end{cases}$$

is :

(A)   0
(B)   1
(C)   2
(D)   3

5. Let $y = f\left(\dfrac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\dfrac{dy}{dx}$ at $x = \dfrac{1}{2}$?

(A) $-\dfrac{1}{64}$
(B) $-\dfrac{1}{32}$
(C) -32
(D) -64

6. If $y = \log \sqrt{\sec \sqrt{x}}$, then the value of $\dfrac{dy}{dx}$ at $x = \dfrac{\pi^2}{16}$ is :

(A) $\dfrac{1}{\pi}$
(B) π

(C) $\dfrac{1}{2}$
(D) $\dfrac{1}{4}$

7. If $x = 3\cos\theta$ and $y = 5\sin\theta$, then $\dfrac{dy}{dx}$ is equal to :

(A) $-\dfrac{3}{5}\tan\theta$
(B) $-\dfrac{5}{3}\cot\theta$

(C) $-\dfrac{5}{3}\tan\theta$
(D) $-\dfrac{3}{5}\cot\theta$

8. For the function $f(x) = x^3$, $x = 0$ is a point of :

   (A)   local maxima
   (B)   local minima
   (C)   non-differentiability
   (D)   inflexion

9. The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is not differentiable at $x =$

   (A)   0
   (B)   1
   (C)   2
   (D) $\dfrac{3}{2}$

10. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is :

   (A) $\dfrac{2\pi}{3}$ cm/s
   (B) π cm/s
   (C) $\dfrac{4\pi}{3}$ cm/s
   (D) 2π cm/s

11. $\displaystyle\int_{-\pi/4}^{\pi/4} x^3 \cos^2 x \, dx$ is equal to :

   (A)   0
   (B) -1
   (C)   1
   (D)   2

12. $\displaystyle\int \dfrac{x-3}{(x-1)^3} e^x \, dx$ is equal to :

   (A) $\dfrac{2\,e^x}{(x-1)^3} + C$
   (B) $\dfrac{-2\,e^x}{(x-1)^2} + C$
   (C) $\dfrac{e^x}{(x-1)} + C$
   (D) $\dfrac{e^x}{(x-1)^2} + C$

13. The area (in sq. units) of the region bounded by the curve $y = x$, x-axis, $x = 0$ and $x = 2$ is :

(A) $\dfrac{3}{2}$ 　　　　　　　　(B) $\dfrac{1}{2}\log 2$

(C) 2 　　　　　　　　(D) 4

14. The number of arbitrary constants in the general solution of the differential equation $\dfrac{dy}{dx} + y = 0$ is :

(A) 0 　　　　　　　　(B) 1

(C) 2 　　　　　　　　(D) 3

15. What is the value of $\dfrac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$

for vectors $\vec{a} = 2\hat{i} - 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$?

(A) $\dfrac{3}{7}$ 　　　　　　　　(B) $\dfrac{7}{3}$

(C) $\dfrac{4}{3}$ 　　　　　　　　(D) $\dfrac{4}{7}$

16. The direction ratios of the line $\dfrac{x-1}{3} = \dfrac{2-y}{1} = \dfrac{3z}{2}$ are :

(A) 3, 1, 2 　　　　　　　　(B) 4, 3, 2

(C) 9, − 3, 2 　　　　　　　　(D) 9, 3, 2

17. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line $\vec{r} = 2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$ is :

(A) $\dfrac{x-1}{2} = \dfrac{y+3}{0} = \dfrac{z-2}{-1}$ 　　　(B) $\dfrac{x+1}{1} = \dfrac{y-3}{1} = \dfrac{z+2}{2}$

(C) $\dfrac{x+1}{2} = \dfrac{y-3}{0} = \dfrac{z+2}{-1}$ 　　　(D) $\dfrac{x-1}{1} = \dfrac{y+3}{1} = \dfrac{2-z}{-2}$

18. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b} > 0$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\dfrac{\pi}{4}$ 　　　　　　　　(B) $\dfrac{\pi}{3}$

(C) $\dfrac{2\pi}{3}$ 　　　　　　　　(D) $\dfrac{3\pi}{4}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is *not* the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* $\cos^{-1}\left(\cos\dfrac{13\pi}{6}\right)$ is equal to $\dfrac{\pi}{6}$.

Reason (R) : The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

20. *Assertion (A) :* If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$.

Reason (R) : If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.

SECTION B

This section comprises Very Short Answer (VSA) type questions of 2 marks each.

21. Find the value of $\cos^{-1}\left(\dfrac{1}{2}\right) - \tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right) + \operatorname{cosec}^{-1}(-2)$.

22. (a) If $y = (\sin^{-1} x)^2$, then find $(1 - x^2)\dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx}$.

OR

(b) If $y^x = x^y$, then find $\dfrac{dy}{dx}$.

23. Given that $f(x) = \dfrac{\log x}{x}$, find the point of local maximum of $f(x)$.

24. (a) Find :

$$\int \frac{x^3 - 1}{x^3 - x} \, dx$$

OR

(b) Evaluate :

$$\int_{-4}^{0} |x + 2| \, dx$$

25. Find the angle between the lines

$$\frac{5 - x}{-7} = \frac{y + 2}{-5} = \frac{z}{1} \text{ and } \frac{x}{1} = \frac{y}{2} = \frac{z}{3}.$$

SECTION C

This section comprises Short Answer (SA) type questions of 3 marks each.

26. (a) Find a matrix A such that

$$A \begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}.$$

Also, find A^{-1}.

OR

(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$,

show that $A^3 = A^{-1}$.

27. (a) If $x \sin (a + y) - \sin y = 0$,

prove that $\dfrac{dy}{dx} = \dfrac{\sin^2 (a + y)}{\sin a}$

OR

(b) Find $\dfrac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1} \sqrt{x}$.

28. (a) Find :

$$\int \frac{dx}{\cos x \sqrt{\cos 2x}}$$

OR

(b) Find :

$$\int \frac{5x - 3}{\sqrt{1 + 4x - 2x^2}}\, dx$$

29. Find the general solution of the differential equation

$y\, dx - x\, dy + (x \log x)\, dx = 0$.

30. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ represent the three sides of a triangle, then show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

31. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?

SECTION D

This section comprises Long Answer (LA) type questions of 5 marks each.

32. Check whether the relation S in the set of all real numbers ($\mathbb{R}$) defined by

S = {(a, b) : a ≤ b³} is reflexive, symmetric or transitive.

33. (a) Using integration, find the area of the region bounded by the curve

$y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

OR

(b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines y = 1 and y = 3 and the y-axis.

34. (a) Find the shortest distance between the lines $\dfrac{x-8}{3} = \dfrac{y+9}{-16} = \dfrac{z-10}{7}$ and

$\dfrac{x-15}{3} = \dfrac{y-29}{8} = \dfrac{z-5}{-5}$.

OR

(b) Find the point of intersection of the lines

$\vec{r} = \hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k})$, and

$\vec{r} = -3\hat{j} + 3\hat{k} + \mu(\hat{i} + 2\hat{j} - \hat{k})$.

Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

35. Solve the following linear programming problem graphically :

Minimise Z = 6x + 7y

subject to constraints

$x + 2y \geq 240$

$3x + 4y \leq 620$

$2x + y \geq 180$

x, y ≥ 0.
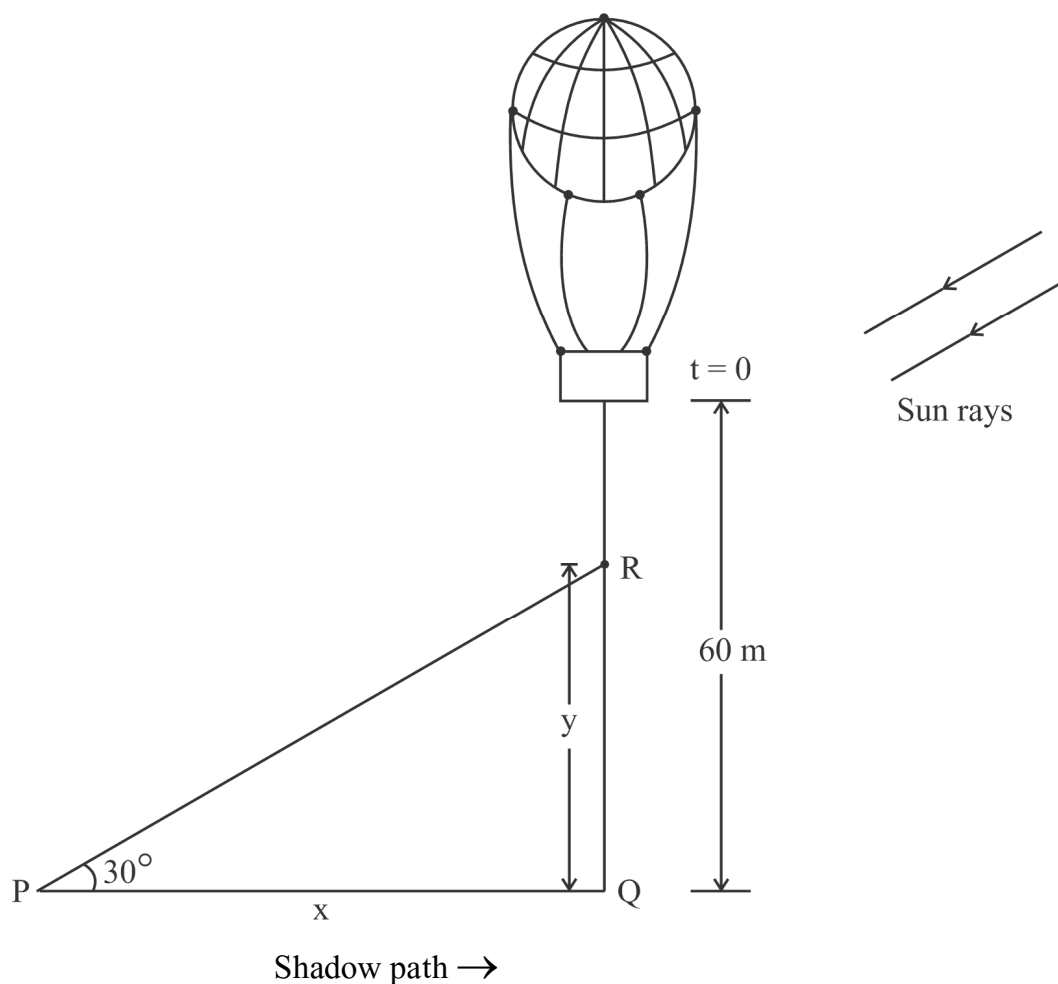
This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A sandbag is dropped from a balloon at a height of 60 metres.

Shadow path →

When the angle of elevation of the sun is $30°$, the position of the sandbag is given by the equation $y = 60 - 4.9\, t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.

On the basis of the above information, answer the following questions :

(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.

(ii) After how much time will the sandbag be 35 metres above the ground ?

(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

Case Study – 2

37. A salesman receives a commission for each sale he makes together with a fixed daily income. The number of sales he makes in a day along with their probabilities are given in the table below :

X :	0	1	2	3	4	5
P(X) :	0.42	3k	0.3	0.05	2k	0.03

His daily income Y (in ₹) is given by :

$$Y = 800X + 50$$

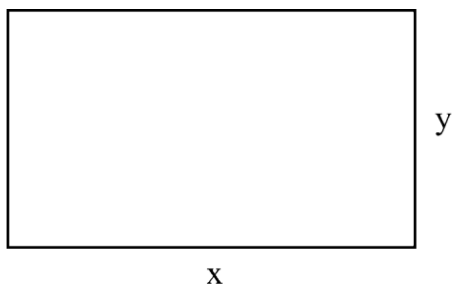On the basis of the above information, answer the following questions :

(i)   Find the value of k.

(ii)   Evaluate P(X ≥ 3).

(iii)   (a)   Calculate the expected weekly income of the salesman assuming he works five days per week.

OR

(iii)   (b)   Calculate the expected weekly income of the salesman assuming he works only for three days of the week.

Case Study – 3

38.   An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m². If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m².

On the basis of the above information, answer the following questions :

(i)   Formulate the linear equations in x and y to represent the given information.

(ii)   Find the dimensions of the plot of land by matrix method.