General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then :

(a) $x = 1,\ y = 2$ (b) $x = 2,\ y = 1$

(c) $x = 1,\ y = -1$ (d) $x = 3,\ y = 2$

2. The product $\begin{bmatrix}a & b\\-b & a\end{bmatrix}\begin{bmatrix}a & -b\\b & a\end{bmatrix}$ is equal to :

(a) $\begin{bmatrix}a^2+b^2 & 0\\0 & a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2 & 0\\(a+b)^2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}a^2+b^2 & 0\\a^2+b^2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}a & 0\\0 & b\end{bmatrix}$

3. If A is a square matrix and $A^2 = A$, then $(I + A)^2 - 3A$ is equal to :

(a) I (b) A

(c) $2A$ (d) $3I$

4. If a matrix $A = [1 \ \ 2 \ \ 3]$, then the matrix AA' (where A' is the transpose of A) is :

(a) 14 (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (d) $[14]$

5. The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is

(a) 0 (b) 1

(c) $x + y + z$ (d) $2(x + y + z)$

6. The function $f(x) = |x|$ is

(a) continuous and differentiable everywhere.

(b) continuous and differentiable nowhere.

(c) continuous everywhere, but differentiable everywhere except at $x = 0$.

(d) continuous everywhere, but differentiable nowhere.

7. If $y = \sin^2 (x^3)$, then $\frac{dy}{dx}$ is equal to :

(a) $2 \sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$

(c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2 (x^3)$

8. $\int e^{5 \log x} dx$ is equal to :

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$

(c) $5x^4 + C$ (d) $6x^5 + C$

9. If $\int_0^a 3x^2 \, dx = 8$, then the value of 'a' is :

(a) 2 (b) 4

(c) 8 (d) 10

10. The integrating factor for solving the differential equation $x \frac{dy}{dx} - y = 2x^2$ is :

(a) e^{-y} (b) e^{-x}

(c) x (d) $\frac{1}{x}$

11. The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are :

(a) 2, 2 (b) 1, 3

(c) 2, 3 (d) 2, degree not defined

12. A unit vector along the vector $4\hat{i} - 3\hat{k}$ is :

(a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$

(b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$

(c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$

(d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$

13. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when :

(a) $0 < \theta < \frac{\pi}{2}$

(b) $0 \leq \theta \leq \frac{\pi}{2}$

(c) $0 < \theta < \pi$

(d) $0 \leq \theta \leq \pi$

14. Distance of the point (p, q, r) from y-axis is :

(a) q

(b) $|q|$

(c) $|q| + |r|$

(d) $\sqrt{p^2 + r^2}$

15. The solution set of the inequation 3x + 5y < 7 is :

(a) whole xy-plane except the points lying on the line 3x + 5y = 7.

(b) whole xy-plane along with the points lying on the line 3x + 5y = 7.

(c) open half plane containing the origin except the points of line 3x + 5y = 7.

(d) open half plane not containing the origin.

16. Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$?

(a) (– 2, 4)

(b) (3, 2)

(c) (– 5, 6)

(d) (4, 2)

17. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then :

(a) $0 < a < 1$

(b) $a > 2$

(c) $a > 0$

(d) $a = \pm \sqrt{3}$

18. The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is :

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$

(c) $\frac{3}{20}$ (d) $\frac{4}{5}$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. *Assertion (A) :* All trigonometric functions have their inverses over their respective domains.

Reason (R) : The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

20. *Assertion (A) :* The lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are perpendicular, when $\vec{b_1} \cdot \vec{b_2} = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate :

$$\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$$

22. If $(x^2 + y^2)^2 = xy$, then find $\frac{dy}{dx}$.

23. Find the maximum and minimum values of the function given by $f(x) = 5 + \sin 2x$.

24. If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.

25. (a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}; \quad \frac{x}{-3} = \frac{y}{2} = \frac{z}{5}.$$

OR

(b) The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find :

$$\int \frac{x^2 + x + 1}{(x+1)^2 (x+2)} dx$$

27. (a) Evaluate :

$$\int_{\pi/4}^{\pi/2} e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$$

OR

(b) Evaluate :

$$\int_{-2}^{2} \frac{x^2}{1 + 5^x} dx$$

28. (a) Find :

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx$$

OR

(b) Evaluate :

$$\int_{0}^{\pi/2} \sqrt{\sin x} \cos^5 x \, dx$$

29. (a) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x+y}{x}, \; y(1) = 0.$$

OR

(b) Find the general solution of the differential equation

$$e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0.$$

30. Solve the following linear programming problem graphically :

Minimise : $z = -3x + 4y$

subject to the constraints

$$x + 2y \leq 8,$$
$$3x + 2y \leq 12,$$
$$x, y \geq 0.$$

31. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations

$x - y + 2z = 1;\ 2y - 3z = 1;\ 3x - 2y + 4z = 3.$

33. Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

34. (a) If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by (a, b) R (c, d), if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

35. (a) Show that the following lines do not intersect each other :

$$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5};\ \frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$$

OR

(b) Find the angle between the lines

$2x = 3y = -z$ and $6x = -y = -4z$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Let f(x) be a real valued function. Then its

- Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$
- Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal.

For the function $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

answer the following questions :

(i) What is R.H.D. of f(x) at x = 1 ? *1*

(ii) What is L.H.D. of f(x) at x = 1 ? *1*

(iii) (a) Check if the function f(x) is differentiable at x = 1. *2*

OR

(iii) (b) Find $f'(2)$ and $f'(-1)$. *2*

Case Study – 2

37. A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80.

Let : E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E : represent completing the construction work on time.

Based on the above information, answer the following questions :

(i) What is the probability that all the workers are present for the job ? *1*

(ii) What is the probability that construction will be completed on time ? *1*

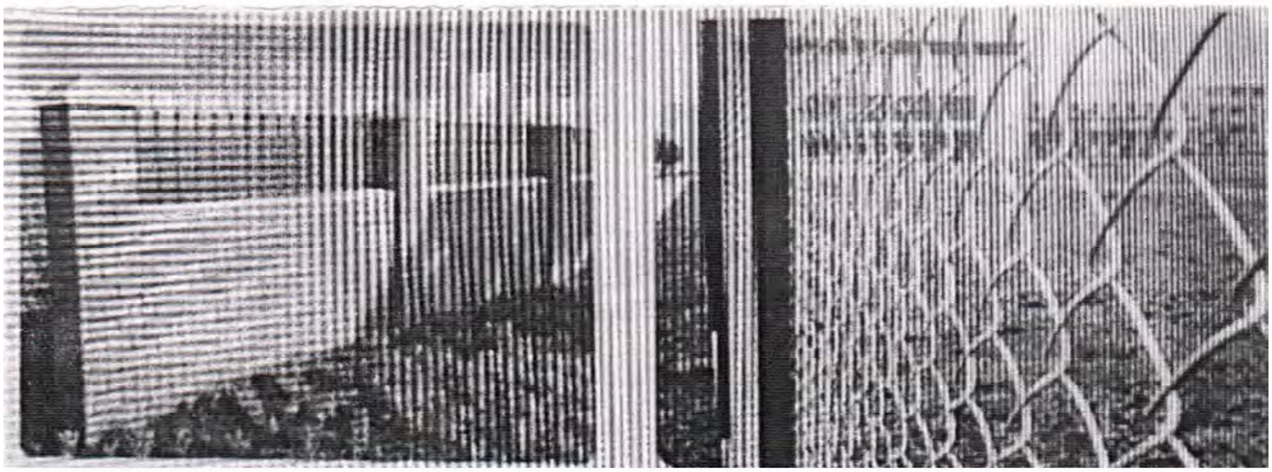
(iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2*

OR

(iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*

Case Study – 3

38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.



Based on the above information, answer the following questions :

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. *2*

(ii) Determine the maximum value of A(x). *2*