***General Instructions :***

*Read the following instructions very carefully and strictly follow them :*

*(i)* *This question paper contains **38** questions. **All** questions are **compulsory**.*

*(ii)* *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

*(iii)* *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

*(iv)* *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

*(v)* *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

*(vi)* *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

*(vii)* *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

*(viii)* *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

*(ix)* *Use of calculators is **not** allowed.*

## SECTION A

*This section comprises multiple choice questions (MCQs) of 1 mark each.*

**1.** If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of x + y + z is :

(a) 10 (b) 6

(c) 8 (d) 0

**2.** If $A \cdot (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to :

(a) 12 (b) 9

(c) 3 (d) 27

**3.** A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) AB = O  (b) AB = – BA

(c) AB = BA  (d) BA = O

**4.** For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A' = \sqrt{3}\ I$, where

$$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} ?$$

(a) $\frac{\pi}{3}$  (b) $\frac{\pi}{6}$

(c) 0  (d) $\frac{\pi}{2}$

**5.** Let A be the area of a triangle having vertices $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$. Which of the following is correct ?

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm A$  (b) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 2A$

(c) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \frac{A}{2}$  (d) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = A^2$

**6.** $\int 2^{x+2} dx$ is equal to :

(a) $2^{x+2} + C$  (b) $2^{x+2} \log 2 + C$

(c) $\frac{2^{x+2}}{\log 2} + C$  (d) $2 \cdot \frac{2^x}{\log 2} + C$

7. $\int \frac{2\cos 2x - 1}{1 + 2\sin x} dx$ is equal to :

(a) $x - 2\cos x + C$ (b) $x + 2\cos x + C$

(c) $-x - 2\cos x + C$ (d) $-x + 2\cos x + C$

8. The solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is :

(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $\log x - \log y = C$

(c) $xy = C$ (d) $x + y = C$

9. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2} \sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$ ?

(a) 3 (b) 2

(c) 6 (d) not defined

10. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is :

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{2}$ (d) 0

11. $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that the projection of $\vec{a}$ on $\vec{b}$ is 0. The angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\pi$

(c) $\frac{\pi}{4}$ (d) 0

12. In $\Delta$ ABC, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$. If D is mid-point of BC, then vector $\overrightarrow{AD}$ is equal to :

(a) $4\hat{i} + 6\hat{k}$ (b) $2\hat{i} - 2\hat{j} + 2\hat{k}$

(c) $\hat{i} - \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{k}$

13. The value of $\lambda$ for which the angle between the lines

$\vec{r} = \hat{i} + \hat{j} + \hat{k} + p(2\hat{i} + \hat{j} + 2\hat{k})$ and

$\vec{r} = (1 + q)\hat{i} + (1 + q\lambda)\hat{j} + (1 + q)\hat{k}$ is $\frac{\pi}{2}$ is :

(a) $-4$ (b) $4$

(c) $2$ (d) $-2$

14. If $P(A \cap B) = \frac{1}{8}$ and $P(\bar{A}) = \frac{3}{4}$, then $P\left(\frac{B}{A}\right)$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

15. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is :

(a) 1 (b) 2

(c) any real number (d) 0

16. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$

(c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

**17.** The number of feasible solutions of the linear programming problem given as

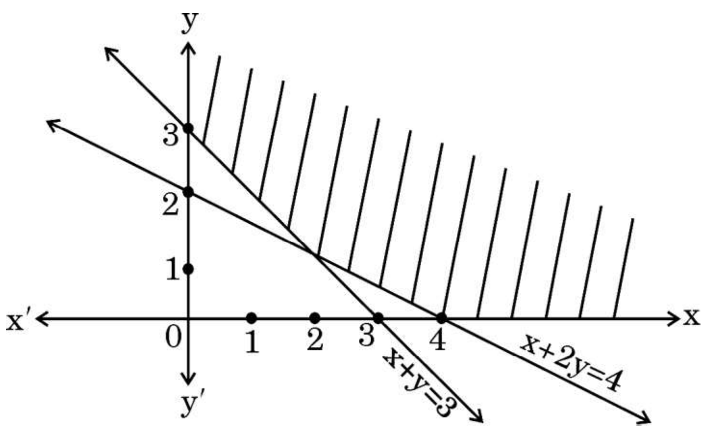
Maximize $z = 15x + 30y$ subject to constraints :

$3x + y \leq 12$, $x + 2y \leq 10$, $x \geq 0, y \geq 0$ is

(a) 1 (b) 2

(c) 3 (d) infinite

**18.** The feasible region of a linear programming problem is shown in the figure below :




Which of the following are the possible constraints ?

(a) $x + 2y \geq 4$, $x + y \leq 3$, $x \geq 0$, $y \geq 0$

(b) $x + 2y \leq 4$, $x + y \leq 3$, $x \geq 0$, $y \geq 0$

(c) $x + 2y \geq 4$, $x + y \geq 3$, $x \geq 0$, $y \geq 0$

(d) $x + 2y \geq 4$, $x + y \geq 3$, $x \leq 0$, $y \leq 0$

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

**19.** *Assertion (A) :* Range of $[\sin^{-1} x + 2 \cos^{-1} x]$ is $[0, \pi]$.

*Reason (R) :* Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

**20.** *Assertion (A) :* A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (– 1, – 2, 1) and (1, 2, 5).

*Reason (R):* Lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$.

## SECTION B

*This section comprises very short answer (VSA) type questions of 2 marks each.*

**21.** If $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{r} \times \hat{j}) \cdot (\vec{r} \times \hat{k}) - 12$.

**22.** If the angle between the lines $\frac{x-5}{\alpha} = \frac{y+2}{-5} = \frac{z + \frac{24}{5}}{\beta}$ and $\frac{x}{1} = \frac{y}{0} = \frac{z}{1}$ is $\frac{\pi}{4}$, find the relation between $\alpha$ and $\beta$.

**23.** If f(x) = a(tan x – cot x), where a > 0, then find whether f(x) is increasing or decreasing function in its domain.

**24.** (a) Evaluate : $3 \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2 \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

**OR**

(b) Draw the graph of $f(x) = \sin^{-1} x$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of f(x).

**25.** (a) If $y = x^{\frac{1}{x}}$, then find $\frac{dy}{dx}$ at x = 1.

**OR**

(b) If $x = a \sin 2t$, $y = a(\cos 2t + \log \tan t)$, then find $\frac{dy}{dx}$.

## SECTION C

*This section comprises short answer (SA) type questions of 3 marks each.*

**26.** (a) Find the general solution of the differential equation :

$$\frac{d}{dx}(xy^2) = 2y(1 + x^2)$$

**OR**

(b) Solve the following differential equation :

$$x e^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

**27.** Evaluate :

$$\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x} + \sqrt{4-x}}\, dx$$

**28.** Evaluate :

$$\int_1^e \frac{1}{\sqrt{4x^2 - (x \log x)^2}}\, dx$$

**29.** (a) Find :

$$\int \frac{\cos x}{\sin 3x}\, dx$$

**OR**

(b) Find :

$$\int x^2 \log (x^2 + 1)\, dx$$

**30.** Determine graphically the minimum value of the following objective function :

$z = 500x + 400y$

subject to constraints

$x + y \leq 200,$

$x \geq 20,$

$y \geq 4x,$

$y \geq 0.$

**31.** (a) A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

**OR**

(b) There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

## SECTION D

*This section comprises long answer (LA) type questions of 5 marks each.*

**32.** Show that a function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = \frac{5x - 3}{4}$ is both one-one and onto.

**33.** The area of the region bounded by the line $y = mx$ $(m > 0)$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

**34.** (a) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that $A^3 - 6A^2 + 7A + 2I = O$.

**OR**

(b) If $A = \begin{bmatrix} 3 & 2 \\ 5 & -7 \end{bmatrix}$, then find $A^{-1}$ and use it to solve the following system of equations :

$3x + 5y = 11, \; 2x - 7y = -3.$

35. (a) Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines.

**OR**

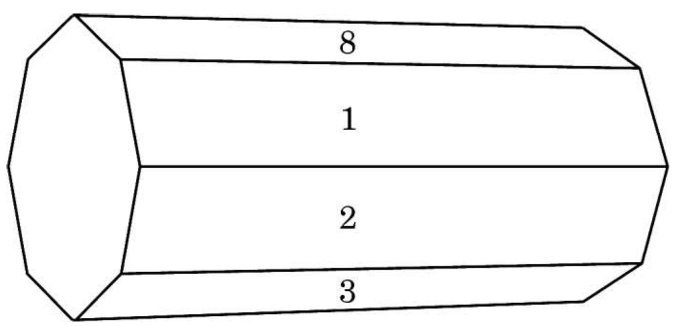
(b) Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(– 1, – 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

## SECTION E

*This section comprises 3 case study based questions of 4 marks each.*

### Case Study – 1

36. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.




The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table give the probability distribution of X.

| X : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| P(X) : | p | 2p | 2p | p | 2p | $p^2$ | $2p^2$ | $7p^2 + p$ |

Based on the above information, answer the following questions :

(i) Find the value of p. *1*

(ii) Find $P(X > 6)$. *1*

(iii) (a) Find $P(X = 3m)$, where m is a natural number. *2*

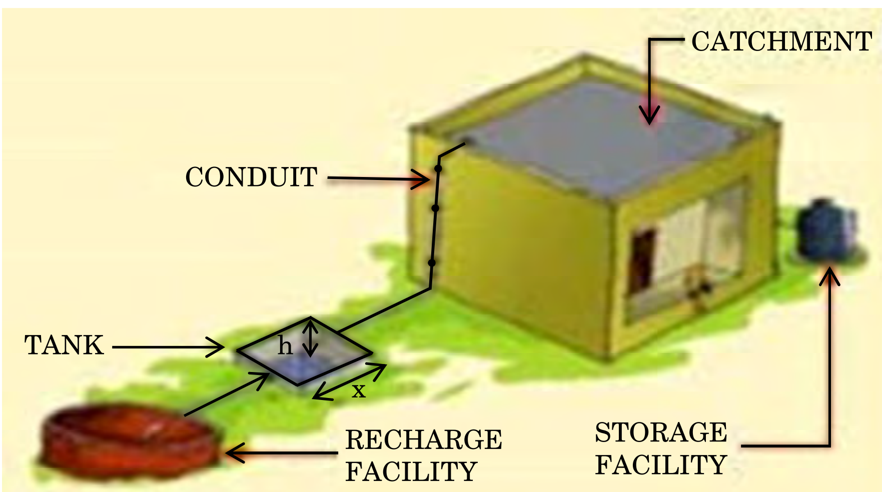
**OR**

(iii) (b) Find the mean $E(X)$. *2*

## Case Study – 2

**37.** In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 $m^3$. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 $h^2$, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM




Based on the above information, answer the following questions :

(i) Find the total cost C of digging the tank in terms of x. *1*

(ii) Find $\frac{dC}{dx}$. *1*

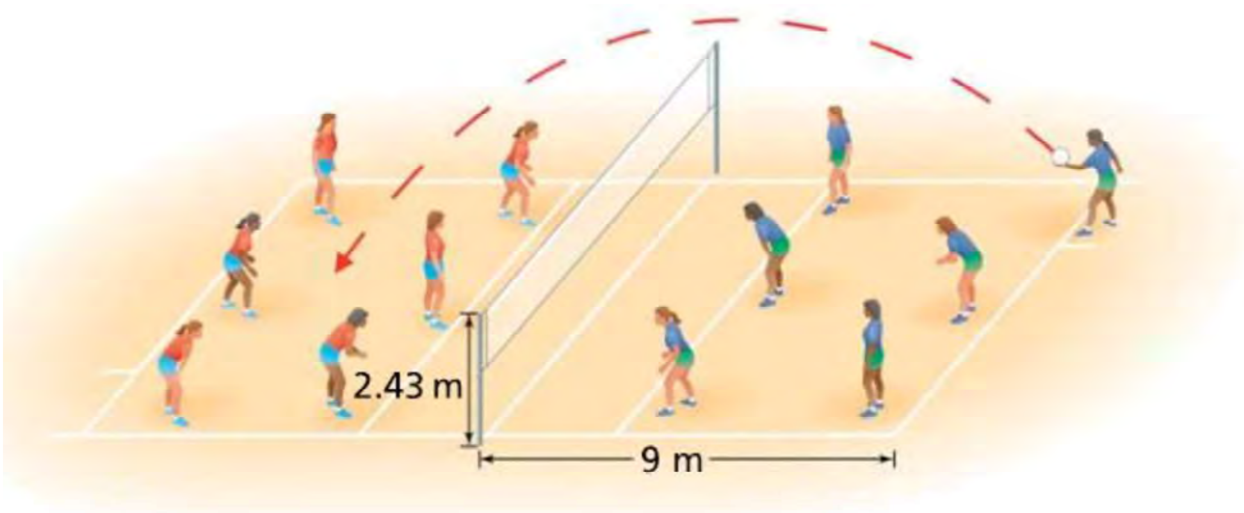
(iii) (a) Find the value of x for which cost C is minimum. *2*

**OR**

(iii) (b) Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$. *2*

## Case Study – 3

**38.** A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -\frac{7}{2}t^2 + \frac{13}{2}t + 1$, where h(t) is the height of ball at any time t (in seconds), $(t \geq 0)$.




Based on the above information, answer the following questions :

(i) Is h(t) a continuous function ? Justify. *2*

(ii) Find the time at which the height of the ball is maximum. *2*