

Test / Exam Name: Polynomials

Standard: 9th

Subject: Mathematics

Instructions

1. JOIN MY 9TH CBSE PAID WHATSAPP TEST GROUP WITH ANSWERS. ONE TIME FEES RS.1500 TILL 2026 FINAL EXAM. WHATSAPP - 8056206308
2. THIS PAPER PDF ANSWERS AVAILABLE IN MY WEBSITE www.ravitestpapers.com

Q1. Verify that:

3 Marks

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$$

Ans: We know that,

$$\begin{aligned} x^3 + y^3 + z^3 - 3xyz &= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2] \\ x^3 + y^3 + z^3 - 3xyz &= \frac{1}{2}(x + y + z)2(x^2 + y^2 + z^2 - xy - yz - xz) \\ &= \frac{1}{2}(x + y + z)(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz) \\ &= \frac{1}{2}(x + y + z)[(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2xz)] \\ &= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2] \end{aligned}$$

Q2. Factorize the following expressions:

3 Marks

$$a^3 + 3a^2b + 3ab^2 + b^3 - 8$$

Ans: $= (a + b)^3 - 8$

$$\begin{aligned} &[\because a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3] \\ &= (a + b)^3 - 2^3 \\ &= (a + b - 2)((a + b)^2 + (a + b) \times 2 + 2^2) \\ &= (a + b - 2)(a^2 + 2ab + b^2 + 2a + 2b + 4) \\ &\therefore a^3 + 3a^2b + 3ab^2 + b^3 - 8 = (a + b - 2)(a^2 + 2ab + b^2 + 2a + 2b + 4) \end{aligned}$$

Q3. Write the following cubes in expanded form:

3 Marks

$$(2a - 3b)^3$$

Ans: $(2a - 3b)^3$

$$\begin{aligned} &\text{Using identity, } (a - b)^3 = a^3 - b^3 - 3ab(a - b) \\ (2a - 3b)^3 &= (2a)^3 - (3b)^3 - (3 \times 2a \times 3b)(2a - 3b) \\ &= 8a^3 - 27b^3 - 18ab(2a - 3b) \\ &= 8a^3 - 27b^3 - 36a^2b + 54ab^2 \end{aligned}$$

Q4. If $a - b = 6$ and $ab = 20$, find the value of $a^3 - b^3$.

3 Marks

Ans: We have,

$$\begin{aligned} a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ &= (a - b)(a^2 + ab + b^2 - 2ab + 2ab) \text{ [Adding and subtracting } 2ab \text{ in the second break]} \\ &= (a - b)[(a^2 + b^2 - 2ab) + 3ab] \\ &= (a - b)[(a - b)^2 + 3ab] \text{ [}\because (a - b)^2 = a^2 + b^2 - 2ab\text{]} \\ &= 6 \times [(6)^2 + 3 \times 20] \text{ [}\because a - b = 6 \text{ and } ab = 20\text{]} \\ &= 6 \times [36 + 60] \\ &= 6 \times 96 \\ &= 576 \\ \therefore a^3 - b^3 &= 576 \end{aligned}$$

Q5. Factorise:

3 Marks

$$(5a - 7b)^3 + (7b - 9c)^3 + (9c - 5a)^3$$

Ans: Put $(5a - 7b) = x$, $(7b - 9c) = y$, $(9c - 5a) = z$.

Here,

$$x + y + z = 5a - 7b + 9c - 5a + 7b - 9c = 0$$

Thus,

We have:

$$(5a - 7b)^3 + (9c - 5a)^3 + (7b - 9c)^3 = x^3 + z^3 + y^3$$

$$= 3xyz \text{ [When } x + y + z = 0, x^3 + y^3 + z^3 = 3xyz]$$

$$= 3(5a - 7b)(9c - 5a)(7b - 9c)$$

Q6. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

3 Marks

Ans: We know that,

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$$

Now put $(x + y + z) = 0$,

$$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$$

$$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$$

Q7. Factorise the following:

$$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}$$

3 Marks

Ans: $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}$

Using identity, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2$$

$$= \left(3p - \frac{1}{6}\right)^3$$

$$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$

Q8. If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$.

3 Marks

Ans: We have that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + bc + 2ca$

$$\Rightarrow (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$$

$$\Rightarrow 9^2 = (a^2 + b^2 + c^2) + 2(26)$$

[Putting the value of $a + b + c$ and $ab + bc + ca$]

$$\Rightarrow 81 = (a^2 + b^2 + c^2) + 52$$

$$\Rightarrow (a^2 + b^2 + c^2) = 81 - 52 = 29$$

Q9. Evaluate the following:

$$(99)^3$$

3 Marks

Ans: We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$$\Rightarrow (99)^3 \text{ can be written as } (100 - 1)^3$$

Here, $a = 100$ and $b = 1$

$$(99)^3 = (100 - 1)^3$$

$$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$$

$$= 1000000 - 1 - (300 \times 99)$$

$$= 1000000 - 1 - 29700$$

$$= 1000000 - 29701$$

$$= 970299$$

$$\text{The value of } (99)^3 = 970299$$

Q10. Factorise:

$$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8zx$$

3 Marks

Ans: $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Using identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$$

$$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2 \times -\sqrt{2}x \times y)$$

$$+ (2 \times y \times 2\sqrt{2}z) + (2 \times 2\sqrt{2}z \times -\sqrt{2}x)$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$$

$$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$$

Q11. Factorise:

$$8a^3 - b^3 - 12a^2b + 6ab^2$$

3 Marks

Ans: $8a^3 - b^3 - 12a^2b + 6ab^2$

Using identity, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$$8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - b^3 - 3(2a)^2b + 3(2a)(b)^2$$

$$= (2a - b)^3$$

$$= (2a - b)(2a - b)(2a - b)$$

Q12. Evaluate the following using suitable identities:

$$(102)^3$$

3 Marks

Ans: $(102)^3 = (100 + 2)^3$

Using identity, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$(100 + 2)^3 = (100)^3 + 2^3 + (3 \times 100 \times 2)(100 + 2)$$

$$= 1000000 + 8 + 600(100 + 2)$$

$$= 1000000 + 8 + 60000 + 1200$$

$$= 1061208$$

Q13. Simplify:

$$\frac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$$

3 Marks

Ans: $\frac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$

$$= \frac{173^3 + 127^3}{173^2 - 173 \times 127 + 127^2}$$

$$= \frac{(173 + 127)(173^2 - 173 \times 127 + 127^2)}{173^2 - 173 \times 127 + 127^2}$$

$$[\therefore a^3 + b^3 = (a + b)(a^2 - ab + b^2)]$$

$$= (173 + 127)$$

$$= 300$$

Q14. Factorize the following expressions:

$$8x^3 - 125y^3 + 180xy + 216$$

3 Marks

Ans: $8x^3 - 125y^3 + 180xy + 216$

or, $8x^3 - 125y^3 + 216 + 180xy$

$$= (2x)^3 + (-5y)^3 + 6^3 - 3 \times (2x)(-5y)(6)$$

$$= (2x + (-5y) + 6)((2x)^2 + (-5y)^2 + 6^2 - 2x(-5y) - (-5y)6 - 6(2x))$$

$$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

$$\therefore 8x^3 - 125y^3 + 180xy + 216$$

$$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

Q15. Factorise:

$$x^3 + x^2 - 4x - 4$$

3 Marks

Ans: Let $p(x) = x^3 + x^2 - 4x - 4$

Constant term of $p(x) = -4$

$\therefore$ Factors of -4 are $\pm 1, \pm 2, \pm 4$

By trial, we find that $p(-1) = 0$, so $(x + 1)$ is a factor of $p(x)$.

Now, we see that $x^3 + x^2 - 4x - 4$

$$= x^2(x + 1) - 4(x + 1)$$

$$= (x + 1)(x^2 - 4) \text{ [taking } (x + 1) \text{ common factor]}$$

Now, $x^2 - 4 = x^2 - 2^2$

$$= (x + 2)(x - 2) \text{ [Using identity, } a^2 - b^2 = (a - b)(a + b)]$$

$$\therefore x^3 + x^2 - 4x - 4 = (x + 1)(x - 2)(x + 2)$$

Q16. Factorise:

$$6x^2 - 11x - 35$$

2 Marks

Ans: $6x^2 - 11x - 35$
 $= 6x^2 - 21x + 10x - 35$
 $= 3x(2x - 7) + 5(2x - 7)$
 $= (2x - 7)(3x + 5)$

Q17. Evaluate the following products without multiplying directly:
 103×107

2 Marks

Ans: $103 \times 107 = (100 + 3)(100 + 7)$
Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 100$, $a = 3$ and $b = 7$
 $103 \times 107 = (100 + 3)(100 + 7) = (100)^2 + (3 + 7)100 + (3 \times 7)$
 $= 10000 + 1000 + 21$
 $= 11021$

Q18. Find the following product:

2 Marks

$$\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$$

Ans: $\left(\frac{x}{2} + 2y\right)\left(\frac{x^2}{4} - xy + 4y^2\right)$
 $= \left(\frac{x}{2} + 2y\right)\left\{\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right)(2y) + (2y)^2\right\}$
 $= \left(\frac{x}{2}\right)^3 + (2y)^3 \left[\because (a + b)(a^2 - ab + b^2) = a^3 + b^3\right]$
 $= \frac{x^3}{8} + 8y^3$

Q19. Factorise:

2 Marks

$$x^2 - x - 156$$

Ans: $x^2 - x - 156$
 $= x^2 - 13x + 12x - 156$
 $= x(x - 13) + 12(x - 13)$
 $= (x - 13)(x + 12)$

Q20. Write the following in the expanded form:

2 Marks

$$(-3x + y + z)^2$$

Ans: We have,

$$(-3x + y + z)^2 = [(-3x)^2 + y^2 + z^2 + 2(-3x)y + 2yz + 2(-3x)z]$$

$$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$$

$$9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$$

$$(-3x + y + z)^2 = 9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$$

Q21. Factorise:

2 Marks

$$x^2 + 5\sqrt{5}x + 30$$

Ans: $x^2 + 5\sqrt{5}x + 30$
 $= x^2 + 2\sqrt{5}x + 3\sqrt{5}x + 30$
 $= x(x + 2\sqrt{5}) + 3\sqrt{5}(x + 2\sqrt{5})$
 $= (x + 2\sqrt{5})(x + 3\sqrt{5})$

Q22. Factorise:

2 Marks

$$x^2 - 32x - 105$$

Ans: $x^2 - 32x - 105$
 $= x^2 - 35x + 3x - 105$
 $= x(x - 35) + 3(x - 35)$
 $= (x - 35)(x + 3)$

Q23. Factorise:

2 Marks

$$81x^4 - y^4$$

Ans: $81x^4 - y^4$

$$= (9x^2)^2 - (y^2)^2$$

$$= (9x^2 - y^2)(9x^2 + y^2)$$

$$= [(3x)^2 - y^2](9x^2 + y^2)$$

$$= (3x - y)(3x + y)(9x^2 + y^2)$$

Q24. Evaluate the following products without multiplying directly:

$$95 \times 96$$

2 Marks

Ans: $95 \times 96 = (90 + 5)(90 + 4)$

Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here, $x = 90$, $a = 5$ and $b = 4$

$$95 \times 96 = (90 + 5)(90 + 4) = 90^2 + 90(5 + 4) + (5 \times 4)$$

$$= 8100 + (11 \times 90) + 20$$

$$= 8100 + 990 + 20$$

$$= 9110$$

Q25. Factorise:

$$x^2 + 11x + 30$$

2 Marks

Ans: $x^2 + 11x + 30$

$$= x^2 + 6x + 5x + 30$$

$$= x(x + 6) + 5(x + 6)$$

$$= (x + 6)(x + 5)$$

Q26. Without actually calculating the cubes, find the value of:

$$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$$

2 Marks

Ans: Given, $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$ or $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 + \left(-\frac{5}{6}\right)^3$

Here, we see that, $\frac{1}{2} + \frac{1}{3} - \frac{5}{6} = \frac{3+2-5}{6} = \frac{5-5}{6} = 0$

$$\therefore \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = 3 \times \frac{1}{2} \times \frac{1}{3} \times \left(-\frac{5}{6}\right) = -\frac{5}{12}$$

[Using identity, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]

Q27. Expand:

$$\left(\frac{1}{2}a - \frac{1}{4}b + 2\right)^2$$

2 Marks

Ans: $\left(\frac{1}{2}a - \frac{1}{4}b + 2\right)^2 = \left[\left(\frac{a}{2}\right) + \left(-\frac{b}{4}\right) + (2)\right]^2$

$$= \left(\frac{a}{2}\right)^2 + \left(-\frac{b}{4}\right)^2 + (2)^2$$

$$+ 2\left(\frac{a}{2}\right) \times \left(-\frac{b}{4}\right)(2) + 2\left(\frac{a}{2}\right)(2)$$

$$= \frac{a^2}{4} + \frac{b^2}{16} + 4 - \frac{ab}{4} - b + 2a$$

Q28. Factorize the following expressions:

$$64a^3 - b^3$$

2 Marks

Ans: $64a^3 - b^3$

$$= (4a)^3 - b^3$$

$$= (4a - b)((4a)^2 + 4a \times b + b^2)$$

$$\therefore [a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= (4a - b)(16a^2 + 4ab + b^2)$$

$$\therefore 64a^3 - b^3 = (4a - b)(16a^2 + 4ab + b^2)$$

Q29. Expand the following:

$$\left(\frac{1}{x} + \frac{y}{3}\right)^3$$

2 Marks

Ans: $\left(\frac{1}{x} + \frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3\left(\frac{1}{x}\right)\left(\frac{y}{3}\right)\left(\frac{1}{x} + \frac{y}{3}\right)$

[Using identity, $(a - b)^3 = a^3 - b^3 + 3a(-b)(a - b)$]

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x} = \left(\frac{1}{x} + \frac{y}{3}\right)$$

$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

Q30. Factorise the following:

$$\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$$

2 Marks

Ans: $\left(2x + \frac{1}{3}\right)^2 - \left(x - \frac{1}{2}\right)^2$

$$= \left[\left(2x + \frac{1}{3}\right) - \left(x - \frac{1}{2}\right)\right] \left[\left(2x + \frac{1}{3}\right) + \left(x - \frac{1}{2}\right)\right]$$

[Using identity, $a^2 - b^2 = (a - b)(a + b)$]

$$= \left(2x - x + \frac{1}{3} + \frac{1}{2}\right) \left(2x + x + \frac{1}{3} - \frac{1}{2}\right)$$

$$= \left(x + \frac{2+3}{6}\right) \left(3x + \frac{2-3}{6}\right)$$

$$= \left(x + \frac{5}{6}\right) \left(3x - \frac{1}{6}\right)$$