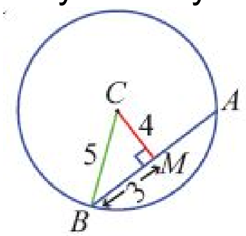
Maths

Time : 01:30:00 Hrs

Total Marks : 75

25 x 3 = 75

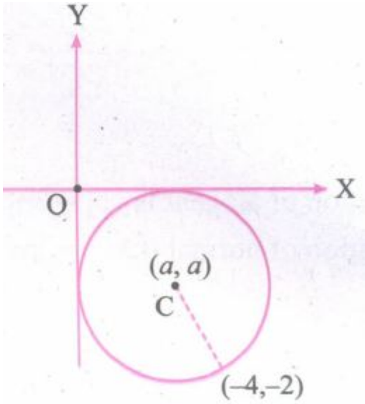
1) A line 3x+4y+10 = 0 cuts a chord of length 6 units on a circle with centre of the circle (2,1) . Find the equation of the circle in general form.
Answer : C(2,1) is the centre and 3x+4y+10 = 0 cuts a chord AB on the circle. Let M be the midpoint of AB , then
AM = BM = 3 . Now BMC is a right triangle.
CM = $\frac{|3(2)+4(1)+10|}{\sqrt{3^2+4^2}} = 4$
$BC^2=BM^2+MC^2=3^2+4^2=25$
BC = 5 = radius
Equation of the required circle is
$(x-2)^2+(y-1) = 5^2$
$x^2+y^2-4x-2y-20 = 0$.



2) Find the centre and radius of the circle$3x^2+(a+1)y^2+6x-9y+a+4=0$.
Answer : Coefficient of x^2 = Coefficient of y^2 (characteristic (ii) for a second degree equation to represent a circle).
That is, 3=a+1 and a = 2 .
Therefore the equation of the circle is
$3x^2+3y^2+6x-9y+6 = 0$
$x^2+y^2+2x-3y+2 = 0$
centre is $\left(-1\frac{3}{2}\right)$ and radius r = $\sqrt{1+\frac{9}{4}-2}$
= $\sqrt{\frac{5}{2}}$

3) Find the equation of circles that touch both the axes and pass through (-4,-2) in general form.
Answer : Since the circle touch both the axis. Its equation will be
$(x - a)^2 + (y - a)^2 = a^2$ .... (1)
It passes through (-4, -2)
$\therefore (-4 - a)^2 + (-2 - a)^2 = a^2$
$16 + a^2 + 8a + 4 + a^2 + 4a = a^2$
$\Rightarrow a^2 + 12a + 20 = 0$
$\Rightarrow (a + 10)(a + 2) = 0$

a =-10 or -2



Case (i)

When a = -10, (1) becomes

$(x + 10)^2 + (y + 10)2 = 10^2$

$\Rightarrow x^2 + 100 + 20x + y^2 + 100 + 20y = 100$

$\Rightarrow x^2 + y^2 + 20x + 20y + 100 = 0$

Case (ii)

When a = -2, (1) becomes

$\Rightarrow (x + 2)^2 + (y + 2)^2 = 2^2$

$x^2 + 4x + 4 + y^2 + 4y + 4 = 4$

$x^2 + y^2 + 4x + 4y + 4 = 0$

Hence, equation of the circles are

$x^2 + y^2 + 4x + 4y + 4 = 0$

or $x^2 + y^2 + 20x + 20y + 100 = 0$

4) A circle of area 9π square units has two of its diameters along the lines x+y=5 and x−y=1.
Find the equation of the circle.

Answer : Area of the circle = 9π sq.units

$\pi r^2 = 9\pi \Rightarrow r^2 \Rightarrow 9 \Rightarrow r \Rightarrow 3$

Diameters are x + y = 5 (1) and x - y = 1 (2)

We know that centre is the point of intersection of diameters.

∴ To find the centre, solve (1) and (2).

$$\Rightarrow \begin{array}{rcl} x + y & = & 5 \\ x - y & = & 1 \\ \hline \end{array}$$

2x = 6

⇒ x = 3

∴ (1) ⇒ 3 + y = 5

⇒ y = 5 - 3 = 2

∴ Centre is (3, 2)

Hence, equation of the circle is

(x - h)2 + (y - k)2 = r2

⇒ (x - 3)2 + (y - 2)2 = 32

$\Rightarrow x^2 - 6x + 9 + y^2 - 4y + 4 = 9$

$\Rightarrow x^2 + y^2 - 6x - 4y + 4 = 0$

5) Determine whether the points(-2,1),(0,0) and (-4,-3) lie outside, on or inside the circle
$x^2+y^2-5x+2y-5=0$.

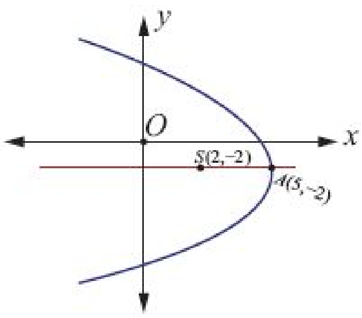
Answer : Given qu tion of the circle is

$x^2 + y^2 - 5x + 2y - 5 = 0$

At (-2, 1), (1) becomes
$(-2)^2 + 1^2 - 5(-2) + 2(1) - 5$
$= 4 + 1 + 10 + 2 - 5$
$= 17 - 5 = 12 > 0$.
∴ (-2, 1) lies outside the circle.
At (0, 0), (I) becomes $-5 < 0$
∴ (0,0) lies inside the circle.
At (-4, -3), (1) becomes
$(-4)^2 + (-3)^2 - 5(-4) + 2(-3) - 5$
$= 16 + 9 + 20 - 6 - 5$
$= 45 - 11 = 34 > 0$
∴ (-4, -3) lies outside the circle.

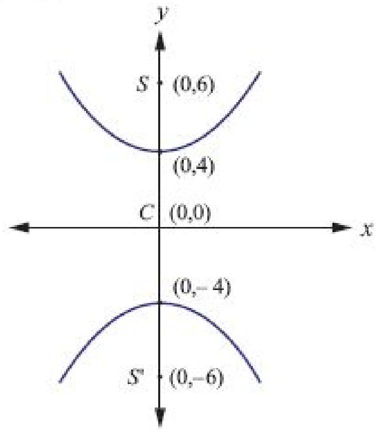
6) Find the length of Latus rectum of the parabola $y^2 = 4ax$.
Answer : Equation of the parabola is $y^2 = 4ax$.
Latus rectum LL′ passes through the focus (a,0).
Hence the point L is (a,y_1).
Therefore $y_1^2=4a^2$
Hence $y_1= \pm 2a$.
The end points of latus rectum are (a,2a) and (a,-2a) .
Therefore length of the latus rectum LL' = 4a .

7) Find the equation of the parabola whose vertex is (5,-2) and focus(2,-2) .
Answer : Given vertex A(5,-2) and focus S(2,-2) and the focal distance AS = a = 3 .
Parabola is open left and symmetric about the line parallel to x -axis.
Then, the equation of the required parabola is
$(y + 2)^2 = -4(3)(x - 5)$
$y^2 + 4y + 4 = -12x + 60$
$y^2 + 4y + 12x - 56 = 0$.



8) Find the equation of the hyperbola with vertices (0,±4) and foci(0,±6).
Answer : the midpoint of line joining foci is the centre C(0,0).
Transverse axis is y -axis AA′ = 2a⇒2a=8,
SS′ = 2c=12,c=6
a = 4
$b^2 = c^2 - a^2 = 36 - 16 = 20$

Hence the equation of the required hyperbola is $\frac{y^2}{16}\frac{x^2}{20}=1$



9) Find the equation of the hyperbola in each of the cases given below:
(i) foci(±2,0), eccentricity = $\frac{3}{2}$
(ii) Centre (2,1) , one of the foci (8,1) and corresponding directrix x = 4.
(iii) passing through(5,−2)and length of the transverse axis along x axis and of length 8 units.

Answer : (i) $a\left(\frac{3}{2}\right)=2 \Rightarrow a\frac{4}{3}$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow b^2 = \frac{16}{9}\left(\frac{9}{4}-1\right) = \frac{16}{9}\left(\frac{9-4}{4}\right) = \frac{\overset{4}{\cancel{16}}}{9}\times\frac{5}{4}$

$\Rightarrow b^2 = \frac{4\times5}{9} = \frac{20}{9}$

∴ Equation of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow \frac{x^2}{\frac{16}{9}}-\frac{y^2}{\frac{20}{9}}=1$

$\Rightarrow \frac{9x^2}{16}-\frac{9y^2}{20}=1$

(ii) ae = distance between centre and focus

ae = $\sqrt{(8-2)^2-(1-1)^2}=\sqrt{6^2}=6$ ...(1)

Also $\frac{a}{e}=\sqrt{(4-2)^2+(1-1)^2}=\sqrt{2^2}=2$

[∴ (4, 1) is a point on the directrix]

$(1)\times(2)\rightarrow a\cancel{e}\times\frac{a}{\cancel{e}} = 6\times2$

$\Rightarrow a^2 = 12$

(1) → $a^2e^2 = 36$

12(e2) = 36

⇒ e2 = 3

⇒ e = $\sqrt{3}$

Also, $b^2 = a^2(e^2 - 1)$= 12(3- 1)= 12(2)= 24

∴ Equation of the hyperbola is

$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

$\Rightarrow \frac{(x-2)^2}{12}-\frac{(y-1)^2}{24}=1$

(iii) Passing through (5, -2) length of the transverse axis is a long x-axis and of length 8 units.

2a = 8 ⇒ a=4

Since the transverse axis is along x-axis, centre is (0, 0)
Equation of the hyperbola is
$\frac{x^2}{16} - \frac{y^2}{b^2} = 1$
Since (5, -2)passes through the parabola,
$\frac{25}{16} - \frac{4}{b^2} \Rightarrow \frac{4}{b^2} = \frac{25}{16} - 1 = \frac{25-16}{16} = \frac{9}{16}$
$\therefore b^2 = \frac{16\times4}{9} = \frac{64}{9}$
∴ Equation of the hyperbola is
$\frac{x^2}{16} - \frac{y^2}{\frac{64}{9}} = 1 \Rightarrow \frac{x^2}{16} - \frac{9y^2}{64} = 1$

10) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
$\frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$
Answer : $\frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$
Given equation is $\frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$
This is an equation of the ellipse a^2 = 289, b^2 = 225 and $c^2 = a^2 + b^2$ ⇒ 289 - 225 = 64 ⇒ c = 8.
$e = \sqrt{\frac{b^2}{a^2}} = \sqrt{1 - \frac{225}{289}} = \sqrt{\frac{289-225}{289}}$
$= \sqrt{\frac{64}{289}} = \frac{8}{17}$
(a) Center is (3, 4) ⇒ h = 3, k= 4
(b) fociare (h,k+c), (h,k-c)
⇒ (3, 4 + 8), (3, 4 - 8) ⇒ (3, 12), (3,-4)
(c) Verticesare (h, k - a), (h, k + a)
⇒ (3, 4 -17), (3, 4 + 17) ⇒ (3, -13), (3, 21)
(d) Equations of directrices are y - 4 = $\pm\frac{a}{e}$
$\Rightarrow y - 4 = \pm\frac{17}{\frac{8}{17}} + 4 \Rightarrow y - 4 = \pm\frac{-289}{8} + 4$
$\Rightarrow y = \frac{289}{8} + 4$ and $y = \frac{-289}{8} + 4$
$\Rightarrow y = \frac{289+32}{2}$ and $y = \frac{-289+32}{8}$
$\Rightarrow y = \frac{321}{8}$ and $y = \frac{-257}{8}$

11) Find the equations of tangent and normal to the ellipse $x^2+4y^2=32$ when $\theta = \frac{\pi}{4}$
Answer : Equation of ellipse is
$x^2 + 4y^2 = 32$
$\frac{x^2}{32} + 0\frac{y^2}{8} = 1$
$a^2 = 32$, $b^2 = 8$
$a = 4\sqrt{2}, b = 2\sqrt{2}$
Equation of tangent at $\theta = \frac{\pi}{4}$ is $\frac{xcos\frac{\pi}{4}}{4\sqrt{2}} \frac{ysin\frac{\pi}{4}}{2\sqrt{2}} = 1$
$\frac{X}{8} + \frac{Y}{4} = 1$
x+2y−8 = 0.
Equation of normal is $\frac{4\sqrt{2}X}{cos\frac{\pi}{4}} - \frac{2\sqrt{2}Y}{sin\frac{\pi}{4}} = 32 - 8$

That is 8x-4y = 24

2x-y-6 = 0.

Aliter At, $\theta = \frac{\pi}{4}$

$(acos\theta, bsin\theta) = \left(4\sqrt{2}cos\frac{\pi}{4}, 2\sqrt{2}sin\frac{\pi}{4}\right)$

= (4,2)

∴ Equation of tangent at $\theta = \frac{\pi}{4}$ is same at (4,2) .

Equation of tangent in cartesian form is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

x+2y−8 = 0

Slope of tangent is -$\frac{1}{2}$

Slope of normal is 2 Equation of normal is y - 2 = 2(x -4)

y−2x+6 = 0 .

12) Find the equation of the tangent at t = 2 to the parabolay^2 = 8x. (Hint: use parametric form)

Answer : Equation of the parabola is y^2 = 8x

∴ 4a = 8 ⇒ a = 2

Equation of tangent to the parabola in parametric form is yt = x + at^2

When t = 2, the equation of tangent is

y(2) = x + $2(2)^2$ ⇒ 2y = x + 8

⇒ x - 2y + 8 = 0 is the required equation of tangent.

13) If the normal at the point ‘ t_1 ’ on the parabola y^2 = 4ax meets the parabola again at the point‘ t_2 ’, then prove that t_2=-$\left(t_1 \frac{2}{t_1}\right)$

Answer : Equation of nonnal at 't_1' to the parabola y^2 = 4 axis is

$y + xt_1 = at_1^3 + 2at_1$ ...(1)

(1) meets the-parabola y_2 = 4ax at 't_2'.

At 't_2', the point on the parabola is x = $at^2{}_2$, y = $2at_2$ (2) lies on (1)

∴ Substituting (2) in (1) we get,

$2at_2 + (at_2^2)2t_1 = at_1^3 + 2at_1$

⇒ $2at_2 + at_1t_2^2 = at_1^3 + 2at_1$

⇒ $2a(t_2 - t_1) = -at1[t_2^2 - t_1^2]$

⇒ $2a(t_2 - t_1) = -at_1(t_2 + t_1)(t_2 - t_1)$

⇒ $2 = -t_1(t_2 + t_1)$

⇒ $\frac{-2}{t_1} = t_2 + t_1 \Rightarrow t_2 = \frac{-2}{t_1} - t_1$

⇒ t2 = -(t1 + $\frac{-2}{t_1}$)

Hence proved.

14) A semielliptical archway over a one-way road has a height of 3m and a width of 12m. The truck has a width of 3m and a height of 2.7m. Will the truck clear the opening of the archway?

Answer : Since the truck’s width is 3m, to determine the clearance, we must find the height of the archway 1 5 . m from the centre. If this height is 2 7 . m or less the truck will not clear the archway. From the diagram a = 6 and b =1 5 . yielding the equation of ellipse as $\frac{x^2}{6^2} + \frac{y^2}{3^2} = 1$

The edge of the 3m wide truck corresponds tox=1.5m. We will find the height of the archway 1 5 . m from the centre by substituting x =1 5 . and solving for y

$$\frac{\left(\frac{3}{2}\right)^2}{36}\frac{y^2}{9} = 1$$
$$y^2 = 9\left(1 - \frac{9}{144}\right)$$
$$\frac{9(135)}{144} = \frac{135}{16}$$
$$y = \frac{\sqrt{135}}{4}$$
= 2.90

Thus the height of arch way 1.5m from the centre is approximately2.90m . Since the truck's height is 2 7 . m, the truck will clear the archway.

15) The equation y=$\frac{1}{32}$$x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

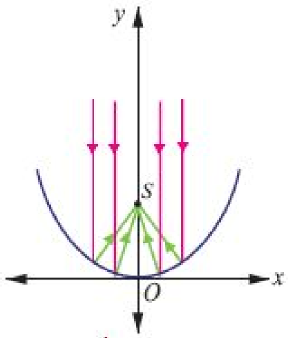
Answer : Equation of the parabola is y=$\frac{1}{32}$$x^2$

That is x^2 =32y ; the vertex is (0,0)

= 4 (8)y

$\Rightarrow a = 8$

So the heating tube needs to be placed at focus (0,a) . Hence the heating tube needs to be placed 8 units above the vertex of the parabola.



16) An equation of the elliptical part of an optical lens system is $\frac{x^2}{16} + \frac{y^2}{9}$ =1. The parabolic part of the system has a focus in common with the right focus of the ellipse .The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola.

Answer : In the given ellipse a^2 = 16, b^2 = 9

then c^2 = a^2 -b^2

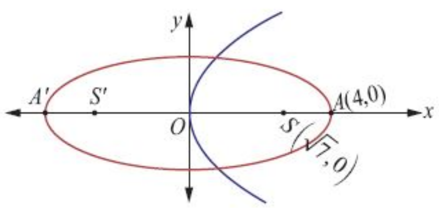
c^2 = 16 - 9

= 7

c = ±$\sqrt{7}$

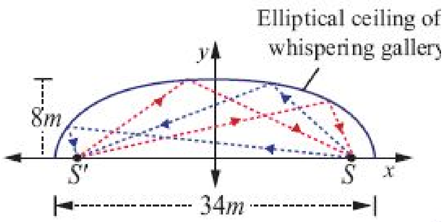
Therefore the foci are F($\sqrt{7}$,0)F^1(-$\sqrt{7}$,0). The focus of the parabola is ($\sqrt{7}$,0) $\Rightarrow$a= $\sqrt{7}$

Equation of the parabola isy^2 =4$\sqrt{7}$ x.



17) A room 34m long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown in Fig. 5.64. If the maximum height of the ceiling is 8m, determine where the foci are located.

Answer : The length a of the semi major axis of the elliptical ceiling is17m . The height b of the semi minor axis is 8m.Thus $c^2 = a^2 - b^2 = 17^2 - 8^2$
then c =$\sqrt{289-64} = \sqrt{225} = 15$
For the elliptical ceiling the foci are located on either side about 15m from the centre, along its major axis.



18) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following:
$\frac{x^2}{3} + \frac{y^2}{10} = 1$
Answer : $\frac{x^2}{3} + \frac{y^2}{10} = 1$
Given equation $\frac{x^2}{3} + \frac{y^2}{10} = 1$
This is an equation of the ellipse with major axis parallel to the y-axis.
∴ $b^2 = 3, a^2 = 10$
⇒ $c^2 = a^2 + b^2 = 10 + 3$
⇒ c = $\sqrt{3}$
(a) Center is (0, 0)
⇒ h = 0, k = 0
(b) vertic are (h, k - a), (h, k + a)
⇒ (0, 0 - $\sqrt{10}$), (0, 0 + $\sqrt{10}$)
⇒ (0 - $\sqrt{10}$), (0 + $\sqrt{10}$)
(c) Foci are (h, k-c)(h, k+c)
⇒ (0, 0 - $\sqrt{13}$), (0, 0 + $\sqrt{13}$)
⇒ (0 - $\sqrt{13}$), (0 + $\sqrt{13}$)
$\sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{3}{10}} = \sqrt{\frac{7}{10}}$
Directrices are y = $\pm\frac{a}{e}$
$y = \pm\frac{\sqrt{10}}{\frac{\sqrt{7}}{\sqrt{10}}} \Rightarrow y = \pm\frac{\sqrt{10}\times\sqrt{10}}{\sqrt{7}}$

19) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following:
$\frac{x^2}{16} + \frac{y^2}{9} = 1$
Answer : $\frac{x^2}{16} + \frac{y^2}{9} = 1$
Given equation is $\frac{x^2}{16} + \frac{y^2}{9} = 1$
This is an equation of the hyperbola where the transverse axis is parallel to the y-axis.
∴ $a^2 = 16, b^2 = 9, c^2 = a^2 + b^2$
⇒ $c^2 = 16 + 9 = 25$ ⇒ c = 5
$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$
(a) Center is (0, 0)

⇒ h = 0, k=0

(b) Vertices are (h, k + a), (h, k-a)

⇒ (0, 0+ 4), (0, 0- 4) ⇒ (0, 4) (0,-4)

(c) Foci are (h, k + c), (h, k- c)

⇒ (0, 0 + 5), (0, 0 - 5) ⇒ (0, 5) (0, -5)

(d) Equations of Directrices are y = $x = \pm\frac{a}{e}$

$\Rightarrow y = \pm\frac{4}{\frac{5}{4}} \Rightarrow y = \pm\frac{16}{5}$

20) Find the circumference and area of the circle $x^2 + y^2 - 2x + 5y + 7 = 0$

Answer : Given equation is $x^2 + y^2 - 2x + 5y + 7 = 0$

Here 2g = -2 ⇒ g = -1 ⇒ 2f = 5 ⇒ f = $\frac{5}{2}$

c = 7

Centre is (-g, -f) = $\left(1, \frac{-5}{2}\right)$

r = $\sqrt{g^2 + f^2 - c} = \sqrt{1^2 + \left(\frac{-5}{2}\right)^2 - 7}$

= $\sqrt{1 + \frac{25}{4} - 7} = \sqrt{\frac{25}{4} - 6} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

∴ Cireumferenee of elrele = 2πr = 2π$\left(\frac{1}{2}\right)$ = π units

Area of the circle = πr2 = π$\left(\frac{1}{2}\right)^2$ = $\frac{\pi}{4}$ sq.units

21) Find the value of p so that 3x + 4y - p = 0 is a tangent to the circle $x^2 + y^2 - 64 = 0$.

Answer : Equation of circle is $x^2 + y^2 = 64$

∴ a2 = 64 ⇒ a = 8

Given line is 3x+ 4y = P

4y = -3x + p

y = $y = \frac{-3}{4}x + \frac{p}{4}$

m = $\frac{-3}{4}$ and c = $\frac{p}{4}$

The condition for y = mx + c to be a tangent to the circle in $c^2 = a^2(1 + m^2)$.

∴ $\left(\frac{p}{4}\right)^2 = 64\left(1 + \frac{9}{16}\right)$

$\Rightarrow \frac{p^2}{16} = 64\left(\frac{16+9}{16}\right) \Rightarrow p^2 = 64(25)$

$p = \pm\sqrt{64(25)} = \pm 8(5)$

∴ p = ±40

22) Find the equation of the ellipse whose latus rectum is 5 and e= $\frac{2}{3}$

Answer : Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given e = $\frac{2}{3}$ and $\frac{2b^2}{a}$ = 5 ⇒ 2b2 = 5a ...(1)

∴ $b^2 = a^2(1 - e^2) = a^2$ $a^2\left(1 - \frac{4}{9}\right) = a^2\left(\frac{5}{9}\right)$

∴ $2b^2 = \frac{10a^2}{9}$ .....(2)

From (1) and (2),

$\frac{10a^2}{9}$ = 5a ⇒ 10a2 = 45a

10a2 - 45a = 0 ⇒ 5a(2 - 9a) = 0

⇒ a = 0 or a = $\frac{9}{2}$ [∵ a = 0 is not possible]

∴ $a^2 = \frac{81}{4}$

$\therefore 2b^2 = \frac{5\times9}{2} = \frac{45}{2} \Rightarrow b^2 = \frac{45}{4}$

$\therefore$ Equation of the ellipse is $\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$

$\Rightarrow \frac{4x^2}{81} + \frac{4y^2}{45} = 1$

23) Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

Answer : Given 2b = 5 and 2ae = 13

$b^2 = a^2(e^2 - 1) - b \Rightarrow a\sqrt{e^2-1}$

$2b = 5 \Rightarrow 2a\sqrt{e^2-1} = 5$

$\Rightarrow 4a^2(e^2 - 1) = 25$ [squaring both sides]

$\Rightarrow 4a^2e^2 - 4a^2 = 25$

$\Rightarrow (2ae)^2 - 4a^2 = 25$

$\Rightarrow 13^2 - 4a^2 = 25$ [$\because$ 2ae=13]

$\Rightarrow 169 - 25 = 4a^2$

$\Rightarrow 4a^2 = 144$

$\Rightarrow a^2 = 36$

$\Rightarrow a = 6$

$\therefore 2b = 5 \Rightarrow b = \frac{5}{2} \Rightarrow b^2 = \frac{25}{4}$

$\therefore$ Equation of the hyperbola is $\frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$

$\frac{x^2}{36} - \frac{4y^2}{25} = 1$

24) Show that the line x + y + 1 = 0 touches the hyperbola $\frac{x^2}{16} - \frac{y^2}{15} = 1$ and find the co-ordinates of the point of contact

Answer : Given line is x + y + 1 = 0

$\Rightarrow y = -x-1$

m = -1, c = -1

Equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{15} = 1$

a2 = 16, b2 = 15

The condition for the line y = mx + c to be a tangent to the hyperbola is $c^2 = a^2m^2 - b^2$

$\therefore (-1)^2 = 16(-1)^2 - 15$

1 = 16 -15

1 = 1

Since the condition is satisfied, x + y + 1 = 0 touches the hyperbola $\frac{x^2}{16} - \frac{y^2}{15} = 1$

The point of contact is $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right) = \left(\frac{-16(-1)}{-1}, \frac{-15}{-1}\right)$ = (-16, 15)

Hence, the point of contact is (-16, 15)

25) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$\frac{(y-2)^2}{25} \frac{(x+1)^2}{16} = 1$

Answer : $\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$

Given equatlon is $\frac{(y-2)^2}{25}-\frac{(x+1)^2}{16}=1$

Thi is an equation of the hyperbola where transverse axis is parallel to y-axis.

∴ a^2 = 25, b^2 = 16

⇒ c^2 - a^2 + b^2=25 +16 = 41

⇒ c = $\sqrt{41}$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1+\frac{16}{25}}=\sqrt{\frac{41}{25}}=\frac{\sqrt{41}}{5}$

(a) Center is (-1,2) ⇒ h = -1, k = 2

(b) Foci are (h, k + c), (h, k- c)

⇒ (-1, 2 + $\sqrt{41}$), (-1, 2 - $\sqrt{41}$)

(c) Vertices are (h, k + a), (h, k-a)

⇒ (-1, 2 + 5), (-1, 2 - 5)

= (-1,7), (-1, -3)

(d) Equation of dir ctrices are

$y-2=\pm\frac{5}{\frac{\sqrt{41}}{5}}$

$\Rightarrow y-2=\pm\frac{25}{\sqrt{41}}$

$\Rightarrow y=2+\frac{25}{\sqrt{41}}$ and

$y=2-\frac{25}{\sqrt{41}}$