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Time : 02:30:00 Hrs

Total Marks : 90

20 x 1 = 20

1) The area of the triangle formed by the complex numbers z , iz , and $z+iz$ in the Argand's diagram is

- (a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

2) If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is then $|z|$ is

- (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

3) If $\left|z - \frac{3}{z}\right| = 2$ then the least value $|z|$ is

- (a) 1 (b) 2 (c) 3 (d) 5

4) The solution of the equation $|z| - z = 1 + 2i$ is

- (a) $\frac{3}{2} - 2i$ (b) $-\frac{3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$

5) z_1, z_3 and z_3 are complex number such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is

- (a) 3 (b) 2 (c) 1 (d) 0

6) If $z = \cos \frac{\pi}{4} + i \sin \frac{\pi}{6}$, then

- (a) $|z| = 1, \arg(z) = \frac{\pi}{4}$ (b) $|z| = 1, \arg(z) = \frac{\pi}{6}$ (c) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \frac{5\pi}{24}$ (d) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

7) The least positive integer n such that $\left(\frac{2i}{1+i}\right)^n$ is a positive integer is

- (a) 16 (b) 8 (c) 4 (d) 2

8) If $z = 1 - \cos \theta + i \sin \theta$, then $|z| =$

- (a) $2 \sin \frac{1}{3}$ (b) $2 \cos \frac{\theta}{2}$ (c) $2 \left| \sin \frac{\theta}{2} \right|$ (d) $2 \left| \cos \frac{\theta}{2} \right|$

9) If $x + iy = \frac{3+5i}{7-6i}$, they $y =$

- (a) $\frac{9}{85}$ (b) $-\frac{9}{85}$ (c) $\frac{53}{85}$ (d) none of these

10) If ω is the cube root of unity, then the value of $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$ is

- (a) 9 (b) -9 (c) 16 (d) 32

11) The modular of $\frac{(-1+i)(1-i)}{1+i\sqrt{3}}$ is _____

- (a) $\sqrt{2}$ (b) 2 (c) 1 (d) $\frac{1}{2}$

12) A polynomial equation in x of degree n always has

- (a) n distinct roots (b) n real roots (c) n imaginary roots (d) at most one root

13) If α, β and γ are the roots of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

- (a) $-\frac{q}{r}$ (b) $\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

14) According to the rational root theorem, which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$?

- (a) -1 (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

15) The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

- (a) 2 (b) 4 (c) 1 (d) $^\circ$

- 16) The polynomial x^3+2x+3 has
 (a) one negative and two real roots (b) one positive and two imaginary roots (c) three real roots (d) no solution
- 17) The number of positive roots of the polynomial $\sum_{j=0}^n n_{C_r} (-1)^r x^r$ is
 (a) 0 (b) n (c) $< n$ (d) r
- 18) If $f(x) = 0$ has n roots, then $f'(x) = 0$ has _____ roots
 (a) n (b) n-1 (c) n+1 (d) (n-r)
- 19) If $(2+\sqrt{3})x^2-2x+1+(2-\sqrt{3})x^2-2x-1=\frac{2}{2-\sqrt{3}}$ then x=
 (a) 0,2 (b) 0,1 (c) 0,3 (d) 0, $\sqrt{3}$
- 20) If $p(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$ where $ac \neq 0$ then $p(x) \cdot Q(x) = 0$ has at least _____ real roots.
 (a) no (b) 1 (c) 2 (d) infinite

7 x 2 = 14

- 21) Given the complex number $z=2+3i$, represent the complex numbers in Argand diagram
 z , iz , and $z+iz$
- 22) If $z_1 z_2 = 1-3i$, $z_1 = -4$, and $z_3 = 5$, show that $(z_1+z_2)+z_3=z_1+(z_2+z_3)$
- 23) Find the modulus of the following complex number
 $\frac{2-i}{1+i} + \frac{1-2i}{1-i}$
- 24) Obtain the Cartesian form of the locus of $z=x+iy$ in each of the following cases
 $\text{Im}[(1-i)z+1]=0$
- 25) Show that the following equations represent a circle, and, find its centre and radius
 $|3x-6+12i|=8$
- 26) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.
- 27) Solve: $(2x-1)(x+3)(x-2)(2x+3)+20=0$
- 28) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that $1+2i$ and $\sqrt{3}$ are two of its zeros.
- 29) Find all real numbers satisfying $4^x-3(2^{x+2})+2^5=0$
- 30) Examine for the rational roots of $x^8-3x+1=0$

7 x 3 = 21

- 31) Show that $|z+2-i| < 2$ represents interior points of a circle. Find its centre and radius.
- 32) Find the fourth roots of unity.
- 33) Simplify the following:
 $i^{2^2} i^3 \dots i^{40}$
- 34) Obtain the Cartesian form of the locus of z in the cases.
 $|2z-3-i|=3$
- 35) Find the locus of z if $\text{Re}\left(\frac{z+1}{z-i}\right) = 0$ where $z=x+iy$.
- 36) If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$ in terms of p .
- 37) Solve the equation $2x^3+11x^2-9x-18=0$.
- 38) If the roots of $x^3+px^2+qx+r=0$ are in H.P. prove that $9pqr = 27r^3+2p$.
- 39) Solve the equation $x^3-5x^2-4x+20=0$
- 40) Find solution, if any, of the equation $2\cos^2x-9\cos x+4=0$

14 x 5 = 70

- 41) a)

Find the value of the real numbers x and y, if the complex number $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal

(OR)

b) Let z_1, z_2 , and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$ prove that

$$\left| \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3} \right| = r$$

42) a) If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2+y^2=1$.

(OR)

b) If $1, \omega, \omega^2$ are the cube roots of unity then show that $(1+5\omega^2+\omega^4)(1+5\omega+\omega^2)(5+\omega+\omega^5)=64$

43) a) Find all the roots $(2-2i)^{\frac{1}{3}}$ and also find the product of its roots.

(OR)

b) Show that, if p,q,r are rational, the roots of the equation $x^2-2px+p^2-q^2+2qr-r^2=0$ are rational.

44) a) Discuss the nature of the roots of the following polynomials:

$$x^{2018}+1947x^{1950}+15x^8+26x^6+2019$$

(OR)

b) If a, b, c, d and p are distinct non-zero real numbers such that $(a^2+b^2+c^2)p^2-2(ab+bc+cd)p+(b^2+c^2+d^2) \leq 0$ then prove that a,b,c,d are in G.P and $ad=bc$

45) a) Find the product $\frac{3}{2} \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \cdot 6 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$ in rectangular form

(OR)

b) Represent the complex number $1 + i\sqrt{3}$ in polar form.

46) a) Show that the equation $2x^2-6x+7=0$ cannot be satisfied by any real values of x.

(OR)

b) Solve the equation $(x-2)(x-7)(x-3)(x+2)+19=0$

47) a) Find the principal argument $\text{Arg } z$, when $z = \frac{-2}{1+i\sqrt{3}}$

(OR)

b) Show that the polynomial $9x^9+2x^5-x^4-7x^2+2$ has at least six imaginary roots.
