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Time : 02:30:00 Hrs

- 1) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then $\frac{|\text{adj } B|}{|C|} =$
 (a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1
- 2) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|\text{adj } (AB)| =$
 (a) -40 (b) -80 (c) -60 (d) -20
- 3) If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is
 (a) 15 (b) 12 (c) 14 (d) 11
- 4) If $A^T A^{-1}$ is symmetric, then $A^2 =$
 (a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$
- 5) If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is
 (a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$
- 6) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is
 (a) 1 (b) 2 (c) 4 (d) 3
- 7) If $\rho(A) = \rho([A \mid B])$, then the system $AX = B$ of linear equations is
 (a) consistent and has a unique solution (b) consistent (c) consistent and has infinitely many solution (d) inconsistent
- 8) Let A be a 3×3 matrix and B its adjoint matrix If $|B| = 64$, then $|A| =$
 (a) ± 2 (b) ± 4 (c) ± 8 (d) ± 12
- 9) The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz =$ has a unique solution if
 (a) $k \neq 0$ (b) $-1 < k < 1$ (c) $-2 < k < 2$ (d) $k = 0$
- 10) If $A = \begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ and $A(\text{adj } A) = \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then λ is
 (a) $\sin x \cos x$ (b) 1 (c) 2 (d) none
- 11) In the non - homogeneous system of equations with 3 unknowns if $\rho(A) = \rho([A|B]) = 2$, then the system has _____
 (a) unique solution (b) one parameter family of solution (c) two parameter family of solutions (d) inconsistent
- 12) Cramer's rule is applicable only when _____
 (a) $\Delta \neq 0$ (b) $\Delta = 0$ (c) $\Delta = 0, \Delta_x = 0$ (d) $\Delta_x = \Delta_y = \Delta_z = 0$
- 13) The area of the triangle formed by the complex numbers z, iz , and $z+iz$ in the Argand's diagram is
 (a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$
- 14) If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is then $|z|$ is

- (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

15) If $z - 2 + i \leq 2$ then the greatest value of $|z|$ is

- (a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$ (c) $\sqrt{5} - 2$ (d) $\sqrt{5} + 2$

16) If $|z_1|=1, |z_2|=2, |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is

- (a) 1 (b) 2 (c) 3 (d) 4

17) If z is a complex number such that $z \in C/R$ and $z + \frac{1}{z} \in R$ then $|z|$ is

- (a) 0 (b) 1 (c) 2 (d) 3

18) If $\omega = cis \frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$

- (a) 1 (b) 2 (c) 3 (d) 4

19) The complex number z which satisfies the condition $\left| \frac{1+z}{1-z} \right| = 1$ lies on

- (a) circle $x^2+y^2=1$ (b) x-axis (c) y-axis (d) the lines $x+y=1$

20) If $x = \cos \theta + i \sin \theta$, then $x^n + \frac{1}{x^n}$ is _____

- (a) $2 \cos n\theta$ (b) $2i \sin n\theta$ (c) $2^n \cos \theta$ (d) $2^n i \sin \theta$

10 x 2 = 20

21) If $F(\alpha) = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

22) If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$

23) Find the matrix A for which $A \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

24) Find the rank of the following matrices by row reduction method:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$$

25) Solve the following system of linear equations by matrix inversion method:

$$2x + 5y = -2, x + 2y = -3$$

26) Write the following in the rectangular form:

$$\overline{(5 + 9i)} + (2 - 4i)$$

27) If $z = x + iy$, find the following in rectangular form.

$$Re \left(\frac{1}{z} \right)$$

28) If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$.

29) Find the square roots of $4 + 3i$

30) Show that the following equations represent a circle, and, find its centre and radius

$$|2z + 2 - 4i| = 2$$

10 x 3 = 30

31)

Find a matrix A if $\text{adj}(A) = \begin{bmatrix} 7 & 7 & -7 \end{bmatrix}$.

32) Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 7 & 7 \end{bmatrix}$.

33) Prove that $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ is orthogonal

34) Solve $2x - 3y = 7$, $4x - 6y = 14$ by Gaussian Jordan method.

35) If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then find λ .

36) If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$, find $\frac{z_1}{z_2}$

37) Find the cube roots of unity.

38) Simplify the following:

$$\sum_{n=1}^{102} i^n$$

39) Show that the complex numbers $3 + 2i$, $5i$, $-3 + 2i$ and $-i$ form a square.

40) Find the locus of Z if $|3z - 5| = 3|z + 1|$ where $z = x + iy$.

7 x 5 = 35

41) a) Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a$, $x + 2y + 3z = b$, $3x + 5y + 7z = c$.

(OR)

b) If $z_1 = 3 + 4i$, $z_2 = 5 - 12i$, and $z_3 = 6 + 8i$, find $|z_1|$, $|z_2|$, $|z_3|$, $|z_1 + z_2|$, $|z_2 - z_3|$, and $|z_1 + z_3|$

42) a) Find the modulus and principal argument of the following complex numbers:

$$-\sqrt{3} - i$$

(OR)

b) Find the rank of the following matrices which are in row-echelon form :

$$\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

43) a)

$$\text{Find the quotient } \frac{2 \left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)}{4 \left(\cos \left(\frac{-3\pi}{2} \right) + i \sin \left(\frac{-3\pi}{2} \right) \right)} \text{ in rectangular form}$$

(OR)

b) Find the following $\left| \frac{i(2+i)^3}{(1+i)^2} \right|$

44) a) Find the radius and centre of the circle $z\bar{z} - (2+3i)z - (2-3i)\bar{z} + 9 = 0$ where z is a complex number.

(OR)

b) If $z = x + iy$ and $\arg \left(\frac{z-1}{z+1} \right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$.

45) a) Test for consistency of the following system of linear equations and if possible solve:

$$4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21.$$

(OR)

b) Find the rank of each of the following matrices:

$$\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$$

- 46) a) Using Gaussian Jordan method, find the values of λ and μ so that the system of equations $2x - 3y + 5z = 12$, $3x + y + \lambda z = \mu$, $x - 7y + 8z = 17$ has (i) unique solution (ii) infinite solutions and (iii) no solution.

(OR)

- b) Let z_1, z_2 , and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$ prove that

$$\left| \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3} \right| = r$$

- 47) a) Test for consistency of the following system of linear equations and if possible solve:

$$x - y + z = -9, 2x - 2y + 2z = -18, 3x - 3y + 3z + 27 = 0.$$

(OR)

- b) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (30, 18) can you conclude that Chennai Super Kings won the match?

Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)
