HALF YEARLY COMMON EXAMINATION - DEC 2019

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs | Total Marks :

PART I | 20 x 1 =

ANSWER ALL

1) If a=cosθ + i sinθ, then $\frac{1+a}{1-a}$ =

(a) cot $\frac{\theta}{2}$ (b) cot θ (c) i cot $\frac{\theta}{2}$ (d) i tan$\frac{\theta}{2}$

2) z_1, z_2 and z_3 are complex number such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$ then $z_1^2+z_2^2+z_3^3$ is

(a) 3 (b) 2 (c) 1 (d) 0

3) If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c = 0$ are (11,2) , the coordinates of the other end are

(a) (-5,2) (b) (2,-5) (c) (5,-2) (d) (-2,5)

4) If a parabolic reflector is 20 em in diameter and 5 em deep, then its focus is

(a) (0,5) (b) (5,0) (c) (10,0) (d) (0, 10)

5) If $\sin^{-1}$ x+$\cot^{-1}(\frac{1}{2}) = \frac{\pi}{2}$, then x is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

6) Consider an ellipse whose centre is of the origin and its major axis is along x-axis. If its eccentrcity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is

(a) 8 (b) 32 (c) 80 (d) 40

7) The area of the parallelogram having diagonals $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is ________________

(a) 4 (b) $2\sqrt{3}$ (c) $4\sqrt{3}$ (d) $5\sqrt{3}$

8) If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$ and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ i equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

9) If f and g are polynomials of degrees m and n respectively, and if h(x) =(f 0 g)(x), then the degree of h i

(a) mn (b) m+n (c) m^n (d) n^m

10) The number of positive zeros of the polynomial $\sum_{j=0}^{n} n_{C_r}$ $(-1)^r x^r$ is

(a) 0 (b) n (c) < n (d) r

11) If P(x, y) be any point on $16x^2+25y^2=400$ with foci F1 (3,0) and F2 (-3,0) then PF_1 PF_2 + is

(a) 8 (b) 6 (c) 10 (d) 12

12) If sin⁻1x = 2$\sin^{-1}\alpha$ has a solution, then

(a) $|\alpha| \leq \frac{1}{\sqrt{2}}$ (b) $|\alpha| \geq \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$

13) Let a > 0, b > 0, c > 0. h n both th root of th quatlon ax^2+b+C= 0 are

(a) real and negative (b) real and positive (c) rational numb rs (d) none

14) If $\vec{d} = \vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times \left(\vec{a} + \vec{b}\right)$, then

(a) $\left|\vec{d}\right|$ (b) $\vec{d} = \vec{a} + \vec{b} + \vec{c}$ (c) $\vec{d} = \vec{0}$ (d) a, b, c are coplanar

15) If the length of the perpendicular from the origin to the plane 2x + 3y + λz =1, λ > 0 is $\frac{1}{5}$ then the valu of λ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

16) The conjugate of a complex number is $\frac{1}{i-2}$ /Then the complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

17) If adj A = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj B = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

(a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

18) If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are any three vectors such that $\vec{a}, \vec{b} \neq 0$ and $\vec{a}.\vec{b} \neq 0$ then $\vec{a}$ and $\vec{c}$ are

(a) perpendicular (b) parallel (c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

19) The number of solutions of the equation $tan^{-1}2x + tan^{-1}3x = \frac{\pi}{4}$

(a) 2 (b) 3 (c) 1 (d) none

20) If A^T is the transpose of a square matrix A, then

(a) $|A| \neq |A^T|$ (b) $|A| = |A^T|$ (c) $|A| + |A^T| = 0$ (d) $|A| = |A^T|$ only

PART II

7 x 2 =

ANSWER ANY SIX QUESTIONS AND QUESTION NUMBER 30 IS COMPULSORY.

21) Obtain the condition that the roots of $x^3+px^2+qx+r=0$ are in A.P.

22) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$

23) If y=4x+c is a tangent to the circle $x^2+y^2=9$, find c .

24) Find the square roots of 4+3i

25) Find the value of

$2cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(\frac{1}{2}\right)$

26) Find the acute angle between the planes $\vec{r}.(2\hat{i} + 2\hat{j} + 2\hat{k})$ and 4x-2y+2z=15.

27) Show that the following equations represent a circle, and, find its centre and radius

$|2z + 2 - 4i| = 2$

28) Find the rank of the following matrices which are in row-echelon form :

$\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

29) If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

30) If y=$2\sqrt{2}$ x +c is a tangent to the circle$x^2+y^2=16$, find the value of c.

PART III

7 x 3 =

ANSWER ANY SIX QUESTIONS AND QUESTION NUMBER 40 IS COMPULSORY.

31) Solve the equation $2x^3+11x^2-9x-18=0$.

32) If A = $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A(adj A) = $|A|I_2$.

33) Prove that

$2tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{7} = tan^{-1}\frac{31}{17}$

34)

Find the rank of the following matrices by row reduction method:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$$

35) Prove that $tan^{-1}x + tan^{-1}\frac{2x}{1-x^2} = tan^{-1}\frac{3x-x^2}{1-3x^2}, |x| < \frac{1}{\sqrt{3}}$

36) Forces of magnit $5\sqrt{2}$ and $5\sqrt{2}$ units acting in the directions $3\hat{i} + 4\hat{j} + 5\hat{k}$ and $10\hat{i} + 6\hat{j} - 8\hat{k}$ respectively, act on a particle which is displaced from the point with position vector $4\hat{i} + 3\hat{j} - 2\hat{k}$ to tl point with position vector $6\hat{i} + \hat{j} - 3\hat{k}$. Find the work done by the forces.

37) If $cos\alpha + cos\beta + cos\gamma = sin\alpha + sin\beta + sin\gamma = 0$ then show that

(i) $cos3\alpha + cos3\beta + cos3\gamma = 3cos(\alpha + \beta + \gamma)$

(ii) $sin3\alpha + sin3\beta + sin3\gamma + sin3\gamma = 3sin\,(\alpha + \beta + \gamma)$

38) Find the sum of squares of roots of the equation $2x^4-8x+6x^2-3=0$.

39) Prove that $tan^{-1}\left(\frac{m}{n}\right) - tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}$

40) If |z|=2 show that $3 \leq |z+3+4i| \leq 7$

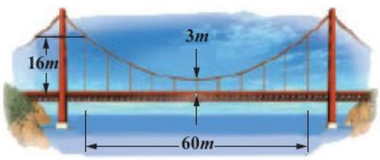
PART IV 7 x 5 =

ANSWER ALL

41) a) Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532 = 0$.

(OR)

b) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the leng of first two of these vertical cables from the vertex.



42) a) Solve the following system of equations, using matrix inversion method:

$2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$.

(OR)

b) Investigate for what values of λ and μ the system of linear equations

$x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions

43) a) If 2+i and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$, find all roots.

(OR)

b) Solve:

(x-5)(x-7)(x+6)(x+4)=504

44) a) An amount of Rs.65,000 is invested in three bonds at the rates of 6%,8% and 9% per annum respectively. The total annual income is Rs.4,800. The income from the third bond is Rs.600 more tha that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

(OR)

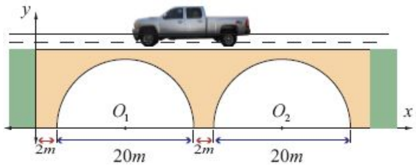
b) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

45) a) Find the parametric form of vector equation, and Cartesian equations of the plane passing through t points (2, 2,1), (9,3,6) and perpendicular to the plane 2x + 6y + 6z = 9

(OR)

b) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points (3,6,−2), (−1,−2,6) , and (6,−4,−2).

46) a) A road bridge over an irrigation canal have two semi circular vents each with a span of 20m and th supporting pillars of width 2m. Use Fig.5.16 to write the equations that model the arches.



(OR)

b) Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256 = 0$

47) a) If z_1,z_2, and z_3 are three complex numbers such that $|z_1|=1$,$|z_2|=2$$|z_3|=3$ and $|z_1+z_2+z_3|=1$,show that $|9z_1z_2+4z_1z_2+z_2z_3|=6$

(OR)

b) If z=x+iy is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$ show that the locus of z is $2x^2+2y^2+$ 2y=0

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