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2ND MID TERM IMPORTANT 5 MARKS Date : 23-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 02:00:00 Hrs Total Marks : 200

40X5=200

1) Find intervals of concavity and points of inflexion for the following functions
$f(x) = x(x - 4)^3$

2) Find two positive numbers whose sum is 12 and their product is maximum.

3) Find two positive numbers whose product is 20 and their sum is minimum.

4) A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.

5) A rectangular page is to contain 24 cm^2 of print. The margins at the top and bettemof the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

6) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain1,80,000 sq. mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the I minimum needed fencing material

7) A manufacturer wants to design an open box having a square base and a surface area of I 108 sq. em. Determine the dimensions of the box for the maximum volume.

8) Write the Maclaurin series expansion of the following function
$\tan^{-1}(x); -1 \le x \le 1$

9) Evaluate the following limit, if necessary use l'Hôpital Rule $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$

10) Evaluate the following limit, if necessary use l'Hôpital Rule
$\lim_{x \to 0^+} (\cos x)^{\frac{1}{x^2}}$

11) If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as n →∞), show that the amount after t years is $A=A_o e^{rt}$.

12) Let $f(x, y) = \frac{y^2 - xy}{\sqrt{x} - \sqrt{y}}$ for $(x, y) \neq (0, 0)$. Show that $\lim_{(x, y) \to (0, 0)} f(x, y) = 0$

13) Evaluate $\lim_{(x, y) \to (0, 0)} \cos = \left(\frac{e^x \sin y}{y}\right)$, if the limit exists.

14) Let $g(x, y) = \frac{x^2 y}{x^4 + y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$
Show that $\lim_{(x, y) \to (0, 0)} g(x, y) = \frac{k}{1 + k^2}$ along every parabola $y = kx^2$, $k \in R \setminus \{0\}$.

15) Let $g(x, y) = \frac{e^y \sin x}{x}$, for $x \neq 0$ and $g(0, 0) = 1$. Show that g is continuous at (0,0).

16) If $U(x, y, z) = \frac{x^2 + y^2}{xy} + 3z^2 y$, find $\frac{\partial U}{\partial x}; \frac{\partial U}{\partial y}$ and $\frac{\partial U}{\partial z}$

17) If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$

18) Let $w(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, (x, y, z) \neq (0, 0, 0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$

19) If $V(x,y) = e^x(x \cos y - y \sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial y^2} = 0$

20) If w(x, y) = xy + sin (xy), then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$

21) If v(x, y, z) = $x^3 + y^3 + z^3 + xyz^3$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

22) If w (x,y, z) =$x^2 + y^2 + y^2$, x= e^t, y = e^t sin t, z = e^t cos t, find $\frac{dw}{dt}$

23) Let U(x, y, z) = xyz, x = e^{-t}, y = e^{-t} cos t, z = sin t, t ∈ R. Find $\frac{dU}{dt}$

24) Evaluate the limit $lim x \to 0 (\frac{sinmx}{x})$

25) $lim \theta \to 0 (\frac{1 - cos\ m\theta}{1 - cos\ n\theta})$ =1, then prove that, $m = \pm n$

26) Find the local extrema of the function f (x)= $4x^6 - 6x^4$

27) Find the local maximum and minimum of the function $x^2\ y^2$ on the line x + y =10

28) f(x,y) = $\frac{xy}{x^2 + y^2}$, (x,y) ≠ (0,0) and f (0,0) = 0 Show that f is not continuous at f, -(0,0) and continuous at all other points of R^2

29) Consider g(x,y) = $\frac{2x^2 y}{x^2 + y^2}$, if (x,y) ≠ (0,0) and g(0,0) = 0 Show that g is continuous on R^2

30) Find the equation of the tangent and normal to the Lissajous curve given by x = 2cos3t and y = 3sin 2t, t ∈ R

31) Find the acute angle between y = x^2 and y = $(x - 3)^2$.

32) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

33) Discuss the monotonicity and local extrema of the function $f(x) = log(1 + x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, $log(1 + x) > \frac{x}{1+x}$

34) We have a 12 square unit piece of thin material and want to make an open box by cutting small squares from the corners of our material and folding the sides up. The question is, which cut produces the box of maximum volume?

35) Prove that among all the rectangles of the given area square has the least perimeter.

36) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

37) Let f(x, y) = $sin(xy^2) + e^{x3+5y}$ for all ∈ R^2. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$

38) Let w(x, y) = xy+$\frac{e^y}{y^2 + 1}$ for all (x, y) ∈ R^2. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

39) Let (x, y) = e^{-2y} cos(2x) for all (x, y) ∈ R^2. Prove that u is a harmonic function in R^2.

40) Let g(x, y) = 2y + x^2, x = 2r -s, y = r^2+ 2s, r, s ∈ R. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

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