VERY IMPORTANT 3 MARKS FOR SLOW LEARNERS

12th Standard

Maths

Exam Time : 02:00:00 Hrs — Total Marks : 210

70 x 3 = 210

1) Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

2) If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

3) Find adj(adj A) if adj $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

4) Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that AXB = C.

5) Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

6) Solve the following system of linear equations by matrix inversion method:
$2x + 5y = -2$, $x + 2y = -3$

7) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

8) Test for consistency of the following system of linear equations and if possible solve:
$x - y + z = -9$, $2x - 2y + 2z = -18$, $3x - 3y + 3z + 27 = 0$.

9) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z

10) The complex numbers u,v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If $v=3-4i$ and $w=4+3i$, find u in rectangular form.

11) If $|z|=2$ show that $3 \leq |z+3+4i| \leq 7$

12) Which one of the points $10 - 8i$, $11+ 6i$ is closest to $1+ i$.

13) If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\ldots(x_n+iy_n) = a+ib$, show that

i) $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\ldots(x_n^2+y_n^2)=a^2+b^2$

ii) $\sum_{r=1}^{n} tan^{-1}\left(\frac{y_r}{x_r}\right) = tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k\epsilon Z$

14) If $cos\alpha + cos\beta + cos\gamma = sin\alpha + sin\beta + sin\gamma = 0$ then show that

(i) $cos3\alpha + cos3\beta + cos3\gamma = 3cos(\alpha+\beta+\gamma)$

(ii) $sin3\alpha + sin3\beta + sin3\gamma + sin3\gamma = 3sin\,(\alpha+\beta+\gamma)$

15) If $\omega \neq 1$ is a cube root of unity, show that

$\left(1-\omega+\omega^2\right)^6 + \left(1+\omega-\omega^2\right)^6 = 128$

16) If $z = (cos\theta + isin\theta)$, show that $z^n + \frac{1}{z^n} = 2cosn\theta$ and $z^n - \frac{1}{z^n} = 2isinn\theta$

17) If $\omega \neq 1$ is a cube root of unity, show that

$(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)\ldots\left(1+\omega^{2^{11}}\right) = 11.$

18) If p and q are the roots of the equation $lx^2+nx+n = 0$, show that

$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}=0.$

19) Solve the equation $x^3-3x^2-33x+35 = 0$.

20) It is known that the roots of the equation $x^3-6x^2-4x+24=0$ are in arithmetic progression. Find its roots.

21) Solve the following equations,

$\sin^2x-5\sin x+4=0$

22) Find all real numbers satisfying $4^x-3(2^{x+2})+2^5=0$

23) Solve the cubic equations:

$8x^3-2x^2-7x+3=0$

24) Prove that

$\tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

25) Solve $\tan^{-1} 2x+\tan^{-1} 3x=\frac{\pi}{4}$, if $6x^2< 1$

26) Prove that $tan^{-1}x + tan^{-1}\frac{2x}{1-x^2} = tan^{-1}\frac{3x-x^2}{1-3x^2}, |x| < \frac{1}{\sqrt{3}}$

27) Simplify: $tan^{-1}\frac{x}{y} - tan^{-1}\frac{x-y}{x+y}$

28) Solve

$sin^{-1}\frac{5}{x} + sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

29) Prove that

$2\tan^{-1}\frac{1}{2} + tan^{-1}\frac{1}{7} = tan^{-1}\frac{31}{17}$

30) Find the equations of tangent and normal to the parabola $x^2+6x+4y+5=0$ at (1,-3) .

31) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

$y^2 = -8x$

32) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following:

$\frac{x^2}{3}+\frac{y^2}{10}=1$

33) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$\frac{(x+1)^2}{100}+\frac{(y-2)^2}{64}=1$

34) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :

$\frac{(x+3)^2}{225}-\frac{(y-4)^2}{64}=1$

35) A particle acted upon by constant forces $2\hat{j}+5\hat{j}+6\hat{k}$ and $-\hat{i}-2\hat{j}-\hat{k}$ is displaced from the point (4, −3, −2) to the point (6, 1, −3) . Find the total work done by the forces.

36) Find the magnitude and the direction cosines of the torque about the point (2, 0, -1) of a force $(2\hat{i}+\hat{j}-\hat{k})$, whose line of action passes through the origin

37) A particle acted on by constant forces $8\vec{i}+2\vec{j}-6\vec{k}$ and $6\vec{i}+2\vec{j}-2\vec{k}$ is displaced from the point (1, 2, 3) to the point (5, 4, 1). Find the total work done by the forces.

38) Find the torque of the resultant of the three forces represented by $-3\hat{i}+6\hat{j}+3\hat{k}$, $4\hat{i}-10\hat{j}+12\hat{k}$ and $4\hat{i}+7\hat{j}$ acting at the point with position vector $8\hat{i}-6\hat{j}-4\hat{k}$, about the point with position vector $18\hat{i}+3\hat{j}-9\hat{k}$

39) Show that the four points (6, -7, 0), (16, -19, -4), (0, 3, -6), (2, -5, 10) lie on a same plane.

40) Prove that $[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]$ = $[\vec{a},\vec{b},\vec{c}]^2$

41) Show that the straight lines x + 1= 2y = −12z and x = y + 2 = 6z − 6 are skew and hence find the shortest distance between them.

42) Find the parametric form of vector equation of the straight line passing through (−1, 2,1) and parallel to the straight line $\vec{r}=(2\hat{i}+3\hat{j}-\hat{k})+t(\hat{i}-2\hat{j}+\hat{k})$ and hence find the shortest distance between the lines.

43) Find the equation of the plane passing through the intersection of the planes 2x+3y−z+7=0 and andx+y−2z+5=0 and is perpendicular to the planex+y−3z−5=0.

44) A particle is fired straight up from the ground to reach a height of s feet in t seconds, where s(t) =128t −16t^2.

(1) Compute the maximum height of the particle reached.

(2) What is the velocity when the particle hits the ground?

45) Find the equations of tangent and normal to the curve y = x^2 + 3x − 2 at the point (1, 2)

46) Using the Lagrange’s mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval: f(x) = x^3 − 3x + 2, x ∈ [-2, 2]

47) Write the Maclaurin series expansion of the following function

e^x

48) Expand sin x in ascending powers x - $\frac{\pi}{4}$ upto three non-zero terms.

49) Determine the intervals of concavity of the curve y = 3+ sin x .

50) Find the local extremum of the function f (x) = x^4 + x 32x

51) Use the linear approximation to find approximate values of $\sqrt[4]{15}$

52) Use the linear approximation to find approximate values of $\sqrt[3]{26}$

53) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.find the following in calculating the area of the circular plate: Percentage error

54) If v(x, y, z) = $x^3 + y^3 + z^3 + xyz^3$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

55) If w (x,y, z) =$x^2 + y^2 + y^2$, x= e^t, y = e^t sin t, z = e^t cos t, find $\frac{dw}{dt}$

56) If u(x, y) = $\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

57) Evaluate$\int_0^1 xdx$, as the limit of a sum.

58) Evaluate$\int_0^1 x^3 dx$, as the limit of a sum.

59) In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.

60) Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win Rs. 15 for each red ball selected and we lose Rs. 10 for each black ball selected. X denotes the winning amount, then find the values of X and number of points in its inverse images.

61) Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

62) A lottery with 600 tickets gives one prize of Rs.200, four prizes of noo, and six prizes of Rs. 50. If the ticket costs is Rs.2, find the expected winning amount of a ticket

63) Find the mean and variance of a random variable X , whose probability density function is $f(x) = \begin{cases} \lambda e^{-2x} & for \geq 0 \\ 0 & otherwise \end{cases}$

64) Write down the

(i) conditional statement

(ii) converse statement

(iii) inverse statement, and

(iv) contrapositive statement for the two statements p and q given below.

p: The number of primes is infinite.

q: Ooty is in Kerala.

65) Construct the truth table for $(p \overline{\vee} q) \wedge (p \overline{\vee} \neg q)$

66) Let $A = \begin{pmatrix} 1 & 0\ 1 & 0 \\ 0 & 1\ 0 & 1 \\ 1 & 0\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1\ 0 & 1 \\ 1 & 0\ 1 & 0 \\ 1 & 0\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1\ 0 & 1 \\ 0 & 1\ 1 & 0 \\ 1 & 1\ 1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type.

Find AVB

67) Let $A=\begin{pmatrix}1 & 0\,1 & 0\\ 0 & 1\,0 & 1\\ 1 & 0\,0 & 1\end{pmatrix}, B=\begin{pmatrix}0 & 1\,0 & 1\\ 1 & 0\,1 & 0\\ 1 & 0\,0 & 1\end{pmatrix}, C=\begin{pmatrix}1 & 1\,0 & 1\\ 0 & 1\,1 & 0\\ 1 & 1\,1 & 1\end{pmatrix}$ be any three boolean matrices of the same type.

Find (A∧B)∨C

68) Show that

¬(p ∧ q) ≡ ¬p V ¬q

69) Show that q → p ≡ ¬p → ¬q

70) Show that ¬(p↔q) ≡ p↔¬q

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