

- 1) Let X be random variable with probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & 0 < x \leq l \\ 0 & 1 \leq x < 2l \end{cases}$$

Which of the following statement is correct

- (a) both mean and variance exist (b) mean exists but variance does not exist (c) both mean and variance do not exist (d) variance exists but Mean does not exist

- 2) A rod of length 2l is broken into two pieces at random. The probability density function of the shorter of the two pieces is

$$f(x) = \begin{cases} \frac{2}{x^3} & 0 < x < l \\ 0 & 1 \leq x < 2l \end{cases}$$

- (a) $\frac{l}{2}, \frac{l^2}{3}$ (b) $\frac{l}{2}, \frac{l^2}{6}$ (c) $1, \frac{l^2}{12}$ (d) $\frac{1}{2}, \frac{l^2}{12}$

- 3) Consider a game where the player tosses a sixsided fair die. H the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs. k^2 , where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$.

The expected amount to win at this game in Rs is

- (a) $\frac{19}{6}$ (b) $-\frac{19}{6}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

- 4) A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is roUed and the sum is determined.

Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is

- (a) 1 (b) 2 (c) 3 (d) 4

- 5) A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is

- (a) 6 (b) 4 (c) 3 (d) 2

- 6) Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times.

Then the possible values of X are

- (a) $i + 2n, i = 0, 1, 2, \dots, n$ (b) $2i - n, i = 0, 1, 2, \dots, n$ (c) $n - i, i = 0, 1, 2, \dots, n$ (d) $2i + 2n, i = 0, 1, 2, \dots, n$

- 7) If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then

which of the following cannot be the value of a and b?

- (a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

- 8) Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus. Then $E[X]$ and $E[Y]$ respectively are

- (a) 50,40 (b) 40,50 (c) 40.75,40 (d) 41,41

- 9) Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result The value of $E[X]$ is

- (a) 0.11 (b) 1.1 (c) 1.1 (d) 1

- 10) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

- (a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

11) If $P\{X = 0\} = 1 - P\{X = 1\}$. If $E[X] = 3\text{Var}(X)$, then $P\{X = 0\}$.

- (a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

12) If X is a binomial random variable with expected value 6 and variance 2.4, then $P(X=5)$ is

- (a) $\left(\frac{10}{5}\right)^2 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^4$ (b) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5$ (c) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^6$ (d) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5$

13) The random variable X has the probability density function $f(x) = \begin{cases} ax + b & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ and $E(X) = \frac{7}{12}$ then a and b are respectively.

- (a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d) 1 and 2

14) Suppose that X takes on one of the values 0, 1, and 2. If for some constant k , $P(X = i) = k P(X = i-1)$ $i = 1, 2$ and $P(X = 0) = \frac{1}{7}$ then the value of k is

- (a) 1 (b) 2 (c) 3 (d) 4

15) Which of the following is a discrete random variable?

- I. The number of cars crossing a particular signal in a day
II. The number of customers in a queue to buy train tickets at a moment.
III. The time taken to complete a telephone call.

- (a) I and II (b) II only (c) III only (d) II and III

16) If $f(x)$ is a probability density function of a random variable, then the value of a is

- (a) 1 (b) 2 (c) 3 (d) 4

17) The probability mass function of a random variable is defined as:

x	-2	-1	0	1	2
$f(x)$	k	$2k$	$3k$	$4k$	$5k$

- (a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

18) Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X - 3)$ is

- (a) 0.24 (b) 0.48 (c) 0.6 (d) 0.96

19) If in 6 trials, X is a binomial variate which follows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is

- (a) 0.125 (b) 0.25 (c) 0.375 (d) 0.75

20) A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

- (a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

- 1) Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.
- 2) In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.
- 3) An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.
- 4) A six-sided die is marked '2' on one face, '3' on two of its faces, and '4' on the remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.
- 5) Three fair coins are tossed simultaneously. Find the probability mass function for the number of heads occurred.
- 6) Find the probability mass function and cumulative distribution function of the number of girl child in families with 4 children, assuming equal probabilities for boys and girls.
- 7) Suppose a discrete random variable can only take the values 0, 1, and 2. The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k} & \text{for } x = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Find

- (i) the value of k
- (ii) cumulative distribution function
- (iii) $P(X \leq 1)$.
- 8) The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$ Find the value of k.
- 9) The probability density function of X is

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

find $P(0.5 \leq X < 1.5)$

- 10) If X is the random variable with probability density function $f(x)$ given by,

$$f(x) = \begin{cases} x+1 & -1 \leq x < 0 \\ -x+1 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

then find

- (i) the distribution function F(x)
- (ii) $P(-0.5 \leq X \leq 0.5)$
- 11) For the random variable X with the given probability mass function as below, find the mean and variance $f(x) = \begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$
- 12) For the random variable X with the given probability mass function as below, find the mean and variance.

$$f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

13)

Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.

- 14) If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X + 3) = 10$ and $E(X + 3)^2 = 116$, find μ and σ^2 .

- 15) The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density

$$f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected life of this electronic equipment.

- 16) Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where

$$n = 6, p = \frac{1}{3}, k = 3$$

- 17) Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where

$$P(X = 10) = \binom{10}{4} \left(\frac{1}{5}\right)^4 \left(1 - \frac{1}{5}\right)^{10-4}$$

- 18) The mean and standard deviation of a binomial variate X are respectively 6 and 2.

Find

(i) the probability mass function

(ii) $P(X = 3)$

(iii) $P(X \leq 2)$.

- 19) If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$. Find the distribution, mean and standard deviation of X.

- 20) In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively.

Find the mean and variance of the random variables.

Probability Distributions

12th Standard

Maths

Exam Time : 00:45:00 Hrs

Total Marks : 30

15 x 2 = 30

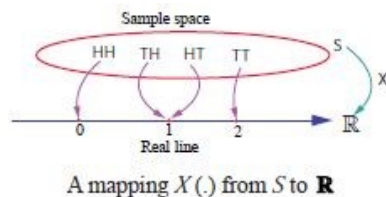
- 1) Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.
- 2) Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred
- 3) The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$ Find the value of k.
- 4) Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where $n = 6, p = \frac{1}{3}, k = 3$
- 5) Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where $n = 9, p = \frac{1}{2}, k = 7$
- 6) Define discrete random variable
- 7) Define Probability Density function
- 8) Define Variance of a random variable X?
- 9) Prove that $\text{Var}(X) = E(X^2) - [E(X)]^2$
- 10) When do we say that a discrete random variable X is a binomial random variable.
- 11) A coin is tossed twice. If X is a random variable defined as the number of heads minus the number of tails, then obtain its probability distribution.
- 12) Is it possible that the mean of a binomial distribution is 15 and its standard deviation is 5?
- 13) In an investment a man can make a profit of Rs.5000 with a probability of 0.62 or a loss of Rs.8000 with a probability of 0.38. Find the expected gain or loss %
- 14) Prove that $\text{Var}(X) = E(X^2)$ if $E(X) = 0$
- 15) Prove that $\text{Var}(X + b) = \text{Var}(X)$

- 1) The probability density function of X is given

$$f(x) = \begin{cases} Ke^{\frac{-x}{3}} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

Find

- (i) the value of k
 - (ii) the distribution function.
 - (iii) $P(X < 3)$
 - (iv) $P(5 \leq X)$
 - (v) $P(X \leq 4)$
- 2) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
- (i) exactly 10 will have a useful life of at least 600 hours;
 - (ii) at least 11 will have a useful life of at least 600 hours;
 - (iii) at least 2 will not have a useful life of at least 600 hours.
- 3) The probability that Mr.Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target
- (i) exactly 4 times
 - (ii) at least one time.
- 4) Using binomial distribution find the mean and variance of X for the following experiments
- (i) A fair coin is tossed 100 times, and X denote the number of heads.
 - (ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.
- 5) A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be
- (i) at least one defective item
 - (ii) exactly two defective items.
- 6) Suppose two coins are tossed once. If X denotes the number of tails,
- (i) write down the sample space
 - (ii) find the inverse image of 1
 - (iii) the values of the random variable and number of elements in its inverse images



7)

Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down

- (i) the sample space
- (ii) the values taken by the random variable X ,
- (iii) the inverse image of 10, and
- (iv) the number of elements in inverse image of X .

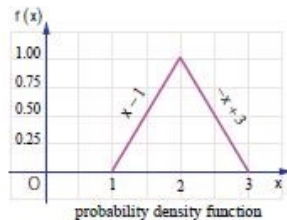
8) An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images

9) Find the constant C such that the function $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$ is a density function, and compute (i) $P(1.5 < X < 3.5)$

(ii) $P(X \leq 2)$

(iii) $P(3 < X)$.

10) If X is the random variable with probability density function $f(x)$ given by,



$$f(x) = \begin{cases} x-1 & 1 \leq x < 2 \\ -x+3 & 2 \leq x < 3 \\ 0 & \text{Otherwise} \end{cases}$$

find (i) the distribution function $F(x)$

(ii) $P(1.5 \leq X \leq 2.5)$

11) If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2 \leq X \leq 0.7)$

12) Let X be a random variable denoting the life time of an electrical equipment having probability density function

$$f(x) = \begin{cases} ke^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

Find

- (i) the value of k
- (ii) Distribution function
- (iii) $P(X < 2)$
- (iv) calculate the probability that X is at least for four unit of time
- (v) $P(X = 3)$

13) Suppose that $f(x)$ given below represents a probability mass function

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find

- (i) the value of c
- (ii) Mean and variance.

- 14) Find the binomial distribution function for each of the following.
- (i) Five fair coins are tossed once and X denotes the number of heads.
 - (ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared.
- 15) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

Probability Distributions

12th Standard

Maths

Exam Time : 02:00:00 Hrs

Total Marks : 75

25 x 3 = 75

- 1) In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.
- 2) A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

- 3) The probability density function of X is

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(0.2 \leq X < 0.6)$

- 4) The probability density function of X is

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

find $P(0.5 \leq X < 1.5)$

- 5) For the random variable X with the given probability mass function as below, find the mean and variance $f(x) = \begin{cases} 2(x-1) & \end{cases}$ & 1
- 6) Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.
- 7) Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.
- 8) A lottery with 600 tickets gives one prize of Rs.200, four prizes of Rs.100, and six prizes of Rs. 50. If the ticket costs is Rs.2, find the expected winning amount of a ticket
- 9) Using binomial distribution find the mean and variance of X for the following experiments
 - (i) A fair coin is tossed 100 times, and X denote the number of heads.
 - (ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.
- 10) If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$ • Find the distribution, mean and standard deviation of X.
- 11) Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down
 - (i) the sample space
 - (ii) the values taken by the random variable X,
 - (iii) the inverse image of 10, and
 - (iv) the number of elements in inverse image of X.
- 12) Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

- 13) Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$

- 14) Find the mean and variance of a random variable X , whose probability density function is $f(x) = \begin{cases} \lambda e^{-2x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
- 15) Give any three properties on expectation and variance.
- 16) Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of spades.
- 17) Four defective oranges are accidentally mixed with sixteen good ones. Three oranges are drawn from the mixed lot. Find the probability distribution of X , the number of defective oranges.
- 18) Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the probability distribution of number of jacks.
- 19) Find the mean, variance and standard deviation of the number of heads in two tosses of a coin
- 20) In 3 trials of a binomial distribution, the probability of 2 success is 9 times the probability of 3 success. Find the parameter of p of the distribution.
- 21) How many times must a man toss a coin so that the probability of having atleast one head is more than 80%?
- 22) If the mean and variance of a binomial distribution are respectively 9 and 6, find the distribution.
- 23) Consider a random variable X with p.d. $f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ Find $\text{Var}(3X - 2)$.
- 24) A person tosses a coin and is to receive Rs.4 for a head and has to pay Rs.2 for a tail. Find the variance of the game.
- 25) Let X be a continuous random variable with $f(x) = \begin{cases} \frac{2}{x^4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$ Find the mean and the variance of X .

- 1) Find the binomial distribution function for each of the following.
 - (i) Five fair coins are tossed once and X denotes the number of heads.
 - (ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared.
- 2) Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

- 3) A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.
- 4) If the probability mass function f(x) of a random variable X is

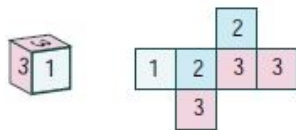
x	1	2	3	4
f(x)	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find

(ii) $P(X \leq 3)$ and,

(iii) $P(X \geq 2)$

- 5) A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.
 - (i) Find the probability mass function.
 - (ii) Find the cumulative distribution function.
 - (iii) Find $P(3 \leq X < 6)$ (iv) Find $P(X \geq 4)$.



- 6) Find the probability mass function f(x) of the discrete random variable X whose cumulative distribution function F(x) is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$

- 7) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find

(i) $P(2 < X < 6)$

(ii) $P(2 \leq X < 5)$

(iii) $P(X \leq 4)$

(iv) $P(3 < X)$

- 8) The probability density function of random variable X is given by $f(x) = \begin{cases} k & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$ Find

(i) Distribution function

(ii) $P(X < 3)$

(iii) $P(2 < X < 4)$

(iv) $P(3 \leq X)$

- 9) Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance

- 10) Find the mean and variance of a random variable X, whose probability density function is $f(x) = \begin{cases} \lambda e^{-2x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Probability Distributions

12th Standard

Maths

Exam Time : 02:00:00 Hrs

Total Marks : 75

25 x 3 = 75

- 1) In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.
- 2) A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

- 3) The probability density function of X is

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(0.2 \leq X < 0.6)$

- 4) The probability density function of X is

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$$

find $P(0.5 \leq X < 1.5)$

- 5) For the random variable X with the given probability mass function as below, find the mean and variance $f(x) = \begin{cases} 2(x-1) & \end{cases}$ & 1
- 6) Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.
- 7) Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.
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- 10) If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$ • Find the distribution, mean and standard deviation of X.
- 11) Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down
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Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$

- 14) Find the mean and variance of a random variable X , whose probability density function is $f(x) = \begin{cases} \lambda e^{-2x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$
- 15) Give any three properties on expectation and variance.
- 16) Two cards are drawn successively without replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of spades.
- 17) Four defective oranges are accidentally mixed with sixteen good ones. Three oranges are drawn from the mixed lot. Find the probability distribution of X , the number of defective oranges.
- 18) Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the probability distribution of number of jacks.
- 19) Find the mean, variance and standard deviation of the number of heads in two tosses of a coin
- 20) In 3 trials of a binomial distribution, the probability of 2 success is 9 times the probability of 3 success. Find the parameter of p of the distribution.
- 21) How many times must a man toss a coin so that the probability of having atleast one head is more than 80%?
- 22) If the mean and variance of a binomial distribution are respectively 9 and 6, find the distribution.
- 23) Consider a random variable X with p.d. $f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ Find $\text{Var}(3X - 2)$.
- 24) A person tosses a coin and is to receive Rs.4 for a head and has to pay Rs.2 for a tail. Find the variance of the game.
- 25) Let X be a continuous random variable with $f(x) = \begin{cases} \frac{2}{x^4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$ Find the mean and the variance of X .

- 1) Let X be random variable with probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & 0 < x \leq l \\ 0 & 1 \leq x < 2l \end{cases}$$

Which of the following statement is correct

- (a) both mean and variance exist (b) mean exists but variance does not exist (c) both mean and variance do not exist (d) variance exists but Mean does not exist
- 2) A rod of length 2l is broken into two pieces at random. The probability density function of the shorter of the two pieces is

$$f(x) = \begin{cases} \frac{2}{x^3} & 0 < x > l \\ 0 & 1 \leq x < 2l \end{cases}$$

- (a) $\frac{1}{2}, \frac{l^2}{3}$ (b) $\frac{1}{2}, \frac{l^2}{6}$ (c) $1, \frac{l^2}{12}$ (d) $\frac{1}{2}, \frac{l^2}{12}$
- 3) Consider a game where the player tosses a sixsided fair die. If the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs. k^2 , where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$.

The expected amount to win at this game in Rs is

- (a) $\frac{19}{6}$ (b) $\frac{19}{6}$ (c) $\frac{3}{2}$ (d) $\frac{3}{2}$
- 4) A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is
- (a) 1 (b) 2 (c) 3 (d) 4
- 5) A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is
- (a) 6 (b) 4 (c) 3 (d) 2
- 6) Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

- (a) $i + 2n, i = 0, 1, 2, \dots, n$ (b) $2i - n, i = 0, 1, 2, \dots, n$ (c) $n - i, i = 0, 1, 2, \dots, n$ (d) $2i + 2n, i = 0, 1, 2, \dots, n$

- 7) If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then

which of the following cannot be the value of a and b?

- (a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24
- 8) Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus. Then $E[X]$ and $E[Y]$ respectively are
- (a) 50,40 (b) 40,50 (c) 40.75,40 (d) 41,41

9)

Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E[X]$ is

- (a) 0.11 (b) 1.1 (c) 1.1 (d) 1

10) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

- (a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

11) If $P\{X = 0\} = 1 - P\{X = 1\}$. If $E[X] = 3\text{Var}(X)$, then $P\{X = 0\}$.

- (a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

12) If X is a binomial random variable with expected value 6 and variance 2.4, then $P(X=5)$ is

- (a) $\left(\frac{10}{5}\right)^2 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^4$ (b) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5$ (c) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^6$ (d) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5$

13) The random variable X has the probability density function $f(x) = \begin{cases} ax + b & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ and $E(X) = \frac{7}{12}$ then a and b are respectively.

- (a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d) 1 and 2

14) Suppose that X takes on one of the values 0, 1, and 2. If for some constant k , $P(X = i) = k P(X = i-1)$ $i = 1, 2$ and $P(X = 0) = \frac{1}{7}$ then the value of k is

- (a) 1 (b) 2 (c) 3 (d) 4

15) Which of the following is a discrete random variable?

- I. The number of cars crossing a particular signal in a day
II. The number of customers in a queue to buy train tickets at a moment.
III. The time taken to complete a telephone call.

- (a) I and II (b) II only (c) III only (d) II and III

16) If $f(x)$ is a probability density function of a random variable, then the value of a is

- (a) 1 (b) 2 (c) 3 (d) 4

17) The probability mass function of a random variable is defined as:

x	-2	-1	0	1	2
$f(x)$	k	$2k$	$3k$	$4k$	$5k$

- (a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

18) Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X - 3)$ is

- (a) 0.24 (b) 0.48 (c) 0.6 (d) 0.96

19) If in 6 trials, X is a binomial variate which follows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is

- (a) 0.125 (b) 0.25 (c) 0.375 (d) 0.75

20) A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

$$(a) \frac{57}{20^3}$$

$$(b) \frac{57}{20^2}$$

$$(c) \frac{19^3}{20^3}$$

$$(d) \frac{57}{20}$$

$$7 \times 2 = 14$$

- 21) An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.
- 22) A six-sided die is marked '2' on one face, '3' on two opposite faces, and '4' on the remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.
- 23) The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -10 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$

Find

- (i) the probability mass function
(ii) $P(X < 1)$ and
(iii) $P(X \sim 2)$
- 24) If X is the random variable with probability density function $f(x)$ given by,

$$f(x) = \begin{cases} x+1 & -1 \leq x < 0 \\ -x+1 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

then find

- (i) the distribution function $F(x)$
(ii) $P(-0.5 \leq X \leq 0.5)$
- 25) For the random variable X with the given probability mass function as below, find the mean and variance.

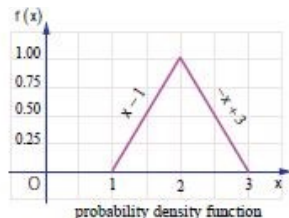
$$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

- 26) If μ and σ^2 are the mean and variance of the discrete random variable X , and $E(X+3)=10$ and $E(X+3)^2=116$, find μ and σ^2 .
- 27) If $X \sim B(n, p)$ such that $4P(X=4) = P(X=2)$ and $n=6$. Find the distribution, mean and standard deviation of X .

$$7 \times 3 = 21$$

- 28) An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images.
- 29) Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win Rs.30 for each black ball selected and we lose Rs.20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.
- 30) Find the constant C such that the function $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$ is a density function, and compute (i) $P(1.5 < X < 3.5)$
(ii) $P(X \leq 2)$
(iii) $P(3 < X)$.

- 31) If X is the random variable with probability density function $f(x)$ given by,



$$f(x) = \begin{cases} x-1 & 1 \leq x < 2 \\ -x+3 & 2 \leq x < 3 \\ 0 & \text{Otherwise} \end{cases}$$

find (i) the distribution function $F(x)$

(ii) $P(1.5 \leq X \leq 2.5)$

- 32) If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2 \leq X \leq 0.7)$

- 33) Suppose that $f(x)$ given below represents a probability mass function

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find

(i) the value of c

(ii) Mean and variance.

- 34) The mean and variance of a binomial variate X are respectively 2 and 1.5. Find

(i) $P(X = 0)$

(ii) $P(X = 1)$

(iii) $P(X \geq 1)$

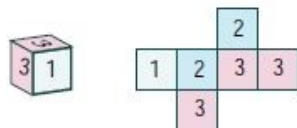
$$4 \times 5 = 20$$

- 35) A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function.

(ii) Find the cumulative distribution function.

(iii) Find $P(3 \leq X < 6)$ (iv) Find $P(X \geq 4)$.



- 36) A random variable X has the following probability mass function

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find

(i) $P(2 < X < 6)$

(ii) $P(2 \leq X < 5)$

(iii) $P(X \leq 4)$

(iv) $P(3 < X)$

37) The probability density function of random variable X is given by $f(x) = \begin{cases} k & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$ Find

(i) Distribution function

(ii) $P(X < 3)$

(iii) $P(2 < X < 4)$

(iv) $P(3 \leq X)$

38) Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance

Probability Distributions FULL TEST

12th Standard

Maths

Reg.No. :

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Exam Time : 03:00:00 Hrs

Total Marks : 90

Answer ALL

20 x 1 = 20

- 1) Let X be random variable with probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & 0 < x \leq l \\ 0 & l \leq x < 2l \end{cases}$$

Which of the following statement is correct

- (a) both mean and variance exist (b) mean exists but variance does not exist (c) both mean and variance do not exist (d) variance exists but Mean does not exist
- 2) A rod of length 2l is broken into two pieces at random. The probability density function of the shorter of the two pieces is

$$f(x) = \begin{cases} \frac{2}{x^3} & 0 < x < l \\ 0 & l \leq x < 2l \end{cases}$$

- (a) $\frac{1}{2}, \frac{l^2}{3}$ (b) $\frac{1}{2}, \frac{l^2}{6}$ (c) $1, \frac{l^2}{12}$ (d) $\frac{1}{2}, \frac{l^2}{12}$
- 3) Consider a game where the player tosses a sixsided fair die. If the face that comes up is 6, the player wins Rs.36, otherwise he loses Rs. k^2 , where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$. The expected amount to win at this game in Rs is
- (a) $\frac{19}{6}$ (b) $\frac{19}{6}$ (c) $\frac{3}{2}$ (d) $\frac{3}{2}$
- 4) A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is
- (a) 1 (b) 2 (c) 3 (d) 4
- 5) A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is
- (a) 6 (b) 4 (c) 3 (d) 2
- 6) Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are
- (a) $i + 2n, i = 0, 1, 2, \dots, n$ (b) $2i - n, i = 0, 1, 2, \dots, n$ (c) $n - i, i = 0, 1, 2, \dots, n$ (d) $2i + 2n, i = 0, 1, 2, \dots, n$
- 7) If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random

variable X, then which of the following cannot be the value of a and b?

- (a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

- 8) Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus. Then $E[X]$ and $E[Y]$ respectively are
- (a) 50,40 (b) 40,50 (c) 40.75,40 (d) 41,41
- 9) Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E[X]$ is
- (a) 0.11 (b) 1.1 (c) 1.1 (d) 1
- 10) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is
- (a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$
- 11) If $P\{X = 0\} = 1 - P\{X = 1\}$. If $E[X] = 3\text{Var}(X)$, then $P\{X = 0\}$.
- (a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$
- 12) If X is a binomial random variable with expected value 6 and variance 2.4, then $P(X=5)$ is
- (a) $\left(\frac{10}{5}\right)^2 \left(\frac{3}{5}\right)^6 \left(\frac{2}{5}\right)^4$ (b) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5$ (c) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^6$ (d) $\left(\frac{10}{5}\right) \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^5$
- 13) The random variable X has the probability density function $f(x) = \begin{cases} ax + b & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ and $E(X) = \frac{7}{12}$
- then a and b are respectively.
- (a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d) 1 and 2
- 14) Suppose that X takes on one of the values 0, 1, and 2. If for some constant k , $P(X = i) = k P(X = i-1)$ $i = 1, 2$ and $P(X = 0) = \frac{1}{7}$ then the value of k is
- (a) 1 (b) 2 (c) 3 (d) 4
- 15) Which of the following is a discrete random variable?
- I. The number of cars crossing a particular signal in a day
 II. The number of customers in a queue to buy train tickets at a moment.
 III. The time taken to complete a telephone call.
- (a) I and II (b) II only (c) III only (d) II and III
- 16) If $f(x)$ is a probability density function of a random variable, then the value of a is
- (a) 1 (b) 2 (c) 3 (d) 4
- 17) The probability mass function of a random variable is defined as:

x	-2	-1	0	1	2
f(x)	k	2k	3k	4k	5k

(a) $\frac{1}{15}$

(b) $\frac{1}{10}$

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

18) Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X - 3)$ is

(a) 0.24

(b) 0.48

(c) 0.6

(d) 0.96

19) If in 6 trials, X is a binomial variate which follows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is

(a) 0.125

(b) 0.25

(c) 0.375

(d) 0.75

20) A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?

(a) $\frac{57}{20^3}$

(b) $\frac{57}{20^2}$

(c) $\frac{19^3}{20^3}$

(d) $\frac{57}{20}$

Answer any 7 questions in which question no. 30 is compulsory

$7 \times 2 = 14$

21) An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.

22) A six-sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

23) The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -10 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$

Find

(i) the probability mass function

(ii) $P(X < 1)$ and

(iii) $P(X \sim 2)$

24) If X is the random variable with probability density function $f(x)$ given by,

$$f(x) = \begin{cases} x+1 & -1 \leq x < 0 \\ -x+1 & 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

then find

(i) the distribution function $F(x)$

(ii) $P(-0.5 \leq X \leq 0.5)$

25)

For the random variable X with the given probability mass function as below, find the mean and variance.

$$f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

26) If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X + 3) = 10$ and $E(X + 3)^2 = 116$, find μ and σ^2 .

27) If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$. Find the distribution, mean and standard deviation of X.

28) For the random variable X with the given probability mass function as below, find the mean and variance

$$f(x) = \begin{cases} 2(x - 1) & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

29) Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

30) Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where

$$n=9, p = \frac{1}{2}, k=7$$

Answer any 7 questions in which question no. 40 is compulsory

7 x 3 = 21

31) An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images

32) Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win Rs.30 for each black ball selected and we lose Rs.20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.

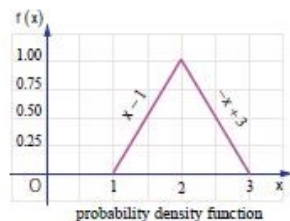
33) Find the constant C such that the function $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$ is a density function, and compute

(i) $P(1.5 < X < 3.5)$

(ii) $P(X \leq 2)$

(iii) $P(3 < X)$.

34) If X is the random variable with probability density function $f(x)$ given by,



$$f(x) = \begin{cases} x - 1 & 1 \leq x < 2 \\ -x + 3 & 2 \leq x < 3 \\ 0 & \text{Otherwise} \end{cases}$$

find (i) the distribution function $F(x)$

(ii) $P(1.5 \leq X \leq 2.5)$

35) If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2 \leq X \leq 0.7)$

36) Suppose that $f(x)$ given below represents a probability mass function

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find

(i) the value of c

(ii) Mean and variance.

37) The mean and variance of a binomial variate X are respectively 2 and 1.5. Find

(i) $P(X = 0)$

(ii) $P(X = 1)$

(iii) $P(X \geq 1)$

38) A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer

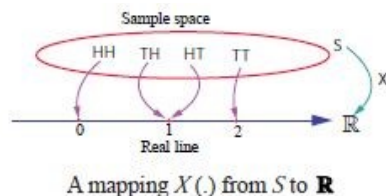
39) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

40) Suppose two coins are tossed once. If X denotes the number of tails,

(i) write down the sample space

(ii) find the inverse image of 1

(iii) the values of the random variable and number of elements in its inverse images



ANSWER ANY 7

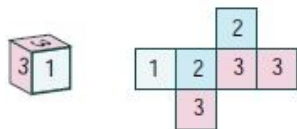
7 x 5 = 35

41) A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function.

(ii) Find the cumulative distribution function.

(iii) Find $P(3 \leq X < 6)$ (iv) Find $P(X \geq 4)$.



42) A random variable X has the following probability mass function

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find

(i) $P(2 < X < 6)$

(ii) $P(2 \leq X < 5)$

(iii) $P(X \leq 4)$

(iv) $P(3 < X)$

43)

The probability density function of random variable X is given by $f(x) = \begin{cases} k & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$ Find

(i) Distribution function

(ii) $P(X < 3)$

(iii) $P(2 < X < 4)$

(iv) $P(3 \leq X)$

44) Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance

45) Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$

46) Find the mean and variance of a random variable X , whose probability density function is

$$f(x) = \begin{cases} \lambda e^{-2x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

47) Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

48) A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

49) If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find

(ii) $P(X \leq 3)$ and,

(iii) $P(X \geq 2)$
