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Time : 01:30:00 Hrs

Total Marks : 50

5 x 1 = 5

1) If  $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$  then  $|\text{adj}(AB)| =$

- (a) -40 (b) -80 (c) -60 (d) -20

2) If  $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$  is the adjoint of  $3 \times 3$  matrix A and  $|A| = 4$ , then x is

- (a) 15 (b) 12 (c) 14 (d) 11

3) If  $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$  and  $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$  then the value of  $a_{23}$  is

- (a) 0 (b) -2 (c) -3 (d) -1

4) If A, B and C are invertible matrices of some order, then which one of the following is not true?

- (a)  $\text{adj } A = |A|A^{-1}$  (b)  $\text{adj}(AB) = (\text{adj } A)(\text{adj } B)$  (c)  $\det A^{-1} = (\det A)^{-1}$  (d)  $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

5) If  $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$  and  $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ , then  $B^{-1} =$

- (a)  $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$  (b)  $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$  (c)  $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$  (d)  $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

10 x 2 = 20

6) Find the matrix A for which  $A \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$ .

7) Given  $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ , find a matrix X such that  $AXB = C$ .

8) Decrypt the received encoded message  $\begin{bmatrix} 2 & -3 \end{bmatrix} \begin{bmatrix} 20 & 4 \end{bmatrix}$  with the encryption matrix  $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$

and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

9) Find the rank of the following matrices by row reduction method:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$$

10)

4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

- 11) In a competitive examination, one mark is awarded for every correct answer while  $\frac{1}{4}$  mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).
- 12) If  $ax^2 + bx + c$  is divided by  $x + 3$ ,  $x - 5$ , and  $x - 1$ , the remainders are 21, 61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)
- 13) Find the value of k for which the equations  $kx - 2y + z = 1$ ,  $x - 2ky + z = -2$ ,  $x - 2y + kz = 1$  have  
 (i) no solution  
 (ii) unique solution  
 (iii) infinitely many solution
- 14) Find the adjoint of the following:

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

- 15) Solve the following system of linear equations by matrix inversion method:

$$2x - y = 8, 3x + 2y = -2$$

$$5 \times 3 = 15$$

- 16) The upward speed  $v(t)$  of a rocket at time  $t$  is approximated by  $v(t) = at^2 + bt + c$ ,  $0 \leq t \leq 100$  where a, b and c are constants. It has been found that the speed at times  $t = 3$ ,  $t = 6$ , and  $t = 9$  seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time  $t = 15$  seconds. (Use Gaussian elimination method.)
- 17) Solve the system:  $x + y - 2z = 0$ ,  $2x - 3y + z = 0$ ,  $3x - 7y + 10z = 0$ ,  $6x - 9y + 10z = 0$ .
- 18) Determine the values of  $\lambda$  for which the following system of equations  $(3\lambda - 8)x + 3y + 3z = 0$ ,  $3x + (3\lambda - 8)y + 3z = 0$ ,  $3x + 3y + (3\lambda - 8)z = 0$  has a non-trivial solution.
- 19) By using Gaussian elimination method, balance the chemical reaction equation:  $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$ . (The above is the reaction that is taking place in the burning of organic compound called isoprene.)
- 20) If the system of equations  $px + by + cz = 0$ ,  $ax + qy + cz = 0$ ,  $ax + by + rz = 0$  has a non-trivial solution and  $p - a, q - b, r - c$ , prove that  $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$ .

$$2 \times 5 = 10$$

- 21) Test for consistency of the following system of linear equations and if possible solve:

$$x + 2y - z = 3, 3x - y + 2z = 1, x - 2y + 3z = 3, x - y + z + 1 = 0$$

- 22) Investigate for what values of  $\lambda$  and  $\mu$  the system of linear equations

$$x + 2y + z = 7, x + y + \lambda z = \mu, x + 3y - 5z = 5$$
 has

- (i) no solution  
 (ii) a unique solution  
 (iii) an infinite number of solutions

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