RAVI MATHS TUITION CENTRE, CHENNAI - 82. WHATSAPP - 8056206308

MINIMUM STUDY MATERIALS FOR QUARTERLY EXAM

12th Standard

Maths

50 x 2 = 100

1) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

2) If adj(A) = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

3) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

4) Find the inverse of each of the following by Gauss-Jordan method:

$\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

5) Solve the following system of linear equations, using matrix inversion method:

$5x + 2y = 3$, $3x + 2y = 5$.

6) Find the rank of the following matrices which are in row-echelon form :

$\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

7) Simplify the following

$i^{1947} + i^{1950}$

8) Which one of the points i, −2 + i, and 3 is farthest from the origin?

9) If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$.

10) Simplify the following

$\sum_{n=1}^{12} i^n$

11) Simplify the following

$i^{59} + \frac{1}{i^{59}}$

12) If $z = x + iy$, find the following in rectangular form.

$\text{Im}(3z + 4\bar{z} - 4i)$

13) Find the modulus of the following complex number $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$

14) Obtain the Cartesian form of the locus of z = x + iy in each of the following cases:
$|z + i| = |z - 1|$

15) Find the modulus and principal argument of the following complex numbers:
$-\sqrt{3}-i$

16) If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

17) Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}i$ as a root.

18) Find a polynomial equation of minimum degree with rational coefficients, having 2i+3 as a root.

19) If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

20) Construct a cubic equation with roots $2, \frac{1}{2}$ and 1

21) Find the period and amplitude of
y = sin 7x

22) Find $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

23) Find all values of x such that
$-6\pi \leq x \leq 6\pi$ and cos x = 0

24) Find the value of

$$2cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}\left(\frac{1}{2}\right)$$

25) For what value of x, the inequality $\frac{\pi}{2} < cos^{-1}(3x-1) < \pi$ holds?

26) Show that $cot(sin^{-1}x) = \frac{\sqrt{1-x^2}}{x}$ $-1 \leq x \leq 1$ and $x \neq 0$

27) Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$

28) Solve $tan^{-1}\left(\frac{x-1}{x-2}\right)+tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

29) Find the value of $cos^{-1}\left(cos\frac{\pi}{7}cos\frac{\pi}{17}-sin\frac{\pi}{7}sin\frac{\pi}{17}\right)$.

30) Prove that

$$2\tan^{-1}\frac{1}{2}+tan^{-1}\frac{1}{7}=tan^{-1}\frac{31}{17}$$

31) Find the value of $\sin\left(\frac{\pi}{3} + \cos^{-1}\left(-\frac{1}{2}\right)\right)$

32) Find the general equation of a circle with centre (-3, -4) and radius 3 units.

33) Find the general equation of the circle whose diameter is the line segment joining the points (−4, −2)and (1, 1).

34) If y = 4x + c is a tangent to the circle $x^2 + y^2 = 9$, find c

35) Obtain the equation of the circle for which (3, 4) and (2, -7) are the ends of a diameter.

36) Find the equation of the parabola with focus $\left(-\sqrt{2}, 0\right)$ and directrix x =$\sqrt{2}$.

37) Find the equation of the parabola whose vertex is (5, -2) and focus(2, -2)

38) Find the equation of the parabola with vertex (-1, -2), axis parallel to y-axis and passing through (3, 6)

39) Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2-4x-5y-1 = 0$.

40) Find the equation of the ellipse with foci (±2, 0), vertices (±3, 0)

41) Find the vertices, foci for the hyperbola $9x^2-16y^2 = 144$.

42) Identify the type of the conic for the following equations:

(1) $16y^2=-4x^2+64$

(2) $x^2+y^2=-4x-y+4$

(3) $x^2-2y=x+3$

(4) $4x^2-9y^2-16x+18y-29 = 0$

43) Find centre and radius of the following circles.

$2x^2+2y^2-6x+4y+2 = 0$

44) Prove by vector method that an angle in a semi-circle is a right angle.

45) If $\vec{a} = -3\hat{i} - \hat{j} + 5\hat{k},\ \vec{b} = \hat{i} - 2\hat{j} + \hat{k},\ \vec{c} = 4\hat{i} - 4\hat{k}$ and find $\vec{a}.(\vec{b} \times \vec{c})$

46) Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\hat{i} + 14\hat{j} + 10\hat{k}$, $14\hat{i} - 10\hat{j} - 6\hat{k}$ and $2\hat{i} + 4\hat{j} - 2\hat{k}$

47) For any vector $\vec{a}$, prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times \vec{a} \times \hat{k} = 2\vec{a}$.

48) Prove that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$

49) Find the angle between the planes $\vec{r}.(\hat{i} + \hat{j} - 2\hat{k}) = 3$ and 2x - 2y + z =2

50) For any vector $\vec{a}$ prove that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.

50 x 3 = 150

51) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify thatA(adj A)=(adj A)A = $|A|\ I_3$.

52) If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

53) If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

54) If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|I_2$.

55) If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$

56) Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

57) Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

58) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

59) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions

60) Find the value of k for which the equations

$kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have

(i) no solution

(ii) unique solution

(iii) infinitely many solution

61) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0$, $3x + (3\lambda - 8)y + 3z = 0$, $3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution.

62) Solve the following systems of linear equations by Gaussian elimination method:

$2x + 4y + 6z = 22$, $3x + 8y + 5z = 27$, $-x + y + 2z = 2$

63) Find the value of the real numbers x and y, if the complex number (2+i)x+(1−i)y+2i −3 and x+(−1+2i)y+1+i are equal

64) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form

65) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form

66) The complex numbers u, v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If v = 3−4i and w = 4+3i, find u in rectangular form.

67) Show that the points 1, $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

68) If $(x_1 + iy_1)(x_2 + iy_2)(x3 + iy_3)\ldots(x_n + iy_n) = a + ib$, show that $(x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2)\ldots(x_n^2 + y_n^2) = a^2 + b^2$

69) If $\omega \neq 1$ is a cube root of unity, then the show that $\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} + \frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = -1$

70) Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$

71) If $2cos\alpha = x + \frac{1}{x}$ and $2cos\,\beta = y + \frac{1}{y}$, show that $x^m y^n + \frac{1}{x^m y^n} = 2cos(m\alpha + n\beta)$

72) If $z = (cos\,\theta + isin\,\theta)$, show that $z^n + \frac{1}{z^n} = 2cos\,n\theta$ and $z^n - \frac{1}{z^n} = 2i\,sin\,n\theta$

73) If $2cos\,\alpha = x + \frac{1}{x}$ and $2cos\,\beta = y + \frac{1}{y}$, show that $xy - \frac{1}{xy} = 2isin(\alpha + \beta)$

74) If $2cos\,\alpha = x + \frac{1}{x}$ and $2cos\,\beta = y + \frac{1}{y}$, show that $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2isin(m\alpha - n\beta)$

75) If α and β are the roots of the quadratic equation $2x^2-7x+13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

76) Find the condition that the roots of cubic $x^3+ ax^2+ bx + c = 0$ are in the ratio p : q : r.

77) Solve the equation $2x^3+11x^2-9x-18=0$.

78) Solve the cubic equation : $2x^3-x^2-18x + 9 = 0$ if sum of two of its roots vanishes.

79) Determine k and solve the equation $2x^3$-$6x^2$+3x+k = 0 if one of its roots is twice the sum of the other two roots.

80) Solve: $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$

81) Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

82) Find the exact number of real zeros and imaginary of the polynomial $x^9+9x^7+7x^5+5x^3+3x$.

83) Find the domain of $\sin^{-1}(2-3x^2)$

84) Find the domain of the following

$$f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$$

85) Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

86) Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

87) Find the value of

$$\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$$

88) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

89) Prove that

$$\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

90) Find the value of $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right) + \cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

91) Find the value of

$$\sin\left(\tan^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{4}{5}\right)\right)$$

92) Find the equation of the hyperbola with vertices $(0, \pm 4)$ and foci$(0, \pm 6)$.

93) Find the equation of the ellipse in each of the cases given below:
foci $(0, \pm 4)$ and end points of major axis are $(0, \pm 5)$.

94) A particle acted upon by constant forces $2\hat{j} + 5\hat{j} + 6\hat{k}$ and $-\hat{i} - 2\hat{j} - \hat{k}$ is displaced from the point $(4, -3, -2)$ to the point $(6, 1, -3)$. Find the total work done by the forces.

95) Find the magnitude and direction cosines of the torque of a force represented by $3\hat{i} + 4\hat{j} - 5\hat{k}$ about the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ acting through a point whose position vector is $4\hat{i} + 2\hat{j} - 3\hat{k}$.

96) Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$

97) Find the shortest distance between the two given straight lines
$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + t(-2\hat{i} + \hat{j} - 2\hat{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$

98) Find the intercepts cut off by the plane $\vec{r}.(6\hat{i} + 4\hat{j} - 3\hat{k}) = 12$ on the coordinate axes.

99) Find the angle between the straight line $\vec{r} = (2\hat{i} + \hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$ and the plane $2x-y+z = 5$

100) Find the equation of the plane passing through the intersection of the planes 2x + 3y −z + 7 = 0 and and x +y −2z + 5 = 0 and is perpendicular to the plane x +y −3z −5 = 0.

50 x 5 = 250

101) If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

102) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.

103) Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

104) In a T20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (40, 22) can you conclude that the team won the match?
Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

105) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

106) An amount of Rs. 65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is Rs. 4,800. The income from the third bond is Rs. 600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

107) By using Gaussian elimination method, balance the chemical reaction equation:
$C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$.
(The above is the reaction that is taking place in the burning of organic compound called isoprene.)

108) Determine the values of λ for which the following system of equations x + y + 3z = 0, 4x + 3y + λz = 0, 2x + y + 2z = 0 has
(i) a unique solution
(ii) a non-trivial solution

109) Solve the following systems of linear equations by Cramer's rule:

$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$

110) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

111) If z_1, z_2, and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_2 + z_2z_3| = 6$

112) If z = x + iy is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

113) Find the fourth roots of unity.

114) Find the cube roots of unity.

115) If 2+i and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140 = 0$, find all roots.

116) Solve the equation (x-2) (x-7) (x-3) (x+2)+19 = 0

117) Solve: (2x-1) (x+3) (x-2) (2x+3)+20 = 0

118) Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that 1+2i and $\sqrt{3}$ are two of its zeros.

119) Solve the equations:

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

120) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

121) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

122) Prove that $\tan^{-1} x + \tan^{-1} z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

123) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that x + y + z = xyz

124) Find the number of solution of the equation $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}(3x)$

125) Find the equation of the circle passing through the points(1, 1), (2, -1), and (3, 2) .

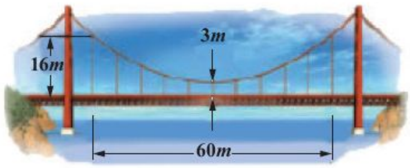
126) For the ellipse $4x^2+y^2+24x-2y+21 = 0$, find the centre, vertices and the foci. Also prove that the length of latus rectum is 2

127) Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

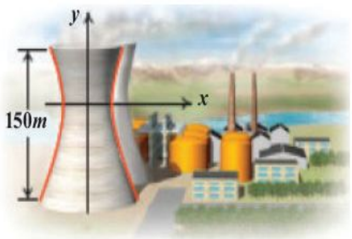
128) A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre, on either sides.

129) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this

portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



130) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150 m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



131) Assume that water issuing from the end of a horizontal pipe, 7. 5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2. 5 m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

132) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

133) Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
$x^2-2x+8y+17=0$

134) Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2-4y-8x+12 = 0$

135) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
$18x^2+12y^2-144x+48y+120 = 0$

136) Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
$9x^2-y^2-36x-6y+18 = 0$

137) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

138) Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

139) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$

140) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

141) If $\vec{a} = \vec{i} - \vec{j}, \vec{b} = \hat{i} - \hat{j} - 4\hat{k}, \vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$

(i) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

(ii) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$

142) If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{j} + 2\hat{k}, \vec{c} = -\hat{i} - 2\hat{j} + 3\hat{k}$, verify that

(i) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a}.\vec{c}) \times \vec{b} - (\vec{b}.\vec{c})\vec{a}$

(ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c}) \times \vec{b} - (\vec{a}.\vec{b})\vec{c}$

143) Determine whether the pair of straight lines $\vec{r}(2\hat{i} + 3\hat{j} - \hat{k}) + t(2\hat{i} + 3\hat{j} + 2\hat{k})$, $\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$are parallel. Find the shortest distance between them.

144) Find the parametric form of vector equation and Cartesian equations of a straight line passing through (5, 2,8) and is perpendicular to the straight lines

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + s(2\hat{i} - 2\hat{j} + \hat{k})$

$\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + t(\hat{i} + 2\hat{j} + 2\hat{k})$.

145) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points (-1, 2, 0), (2, 2, -1)and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

146) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$

147) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1), (1, −2, 3) and parallel to the straight line passing through the points (2, 1, −3) and (−1, 5, −8)

148) Find the non-parametric form of vector equation of the plane passing through the point (1, −2, 4) and perpendicular to the plane x + 2y −3z = 11 and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

149) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points (3, 6, −2), (−1,−2, 6) , and (6, −4, −2).

150) If the straight lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

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