Ravi home tutions PH - 8056206308

Differentials and Partial Derivatives 1 MARK Date : 22-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 00:15:00 Hrs Total Marks : 15

15 x 1 = 15

1) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

2) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{\rightarrow}{\rightarrow}$ (b) $\rightarrow$ (c) 5 (d) 31

3) If u (x, y) = e^{x2+y2}, then — is equal to

(a) e^{x2+y2} (b) 2xu (c) x^2u (d) y^2u

4) If v (x, y) = log (ex + ev), then — 0 — is equal to

(a) $e^x + e^y$ (b) $\frac{\rightarrow}{0}$ (c) 2 (d) 1

5) If w (x, y) = xy, x > 0, then — is equal to

(a) x^y log x (b) y log x (c) yx^{y-1} (d) x log y

6) If f (x, y) = e^{xy} then $\frac{\surd}{\quad}$ is equal to

(a) xye^{xy} (b) $(1 +xy)e^{xy}$ (c) $(1 +y)e^{xy}$ (d) $(1 + x)e^{xy}$

7) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

8) The change in the surface area S = $6x^2$ of a cube when the edge length varies from x_o to x_o+ dx is

(a) 12 x_o+dx (b) 12x_o dx (c) 6x_o dx (d) 6x_o+ dx

9) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(a) 0.3xdx m^3 (b) 0.03 xm^3 (c) 0.03.x^2 m^3 (d) 0.03x^3m^3

10) If g(x, y) = $3x^2$ - 5y + 2y, x(t) = e^t and y(t) = cos t, then — is equal to

(a) $6e^{2t}$+5 sin t - 4 cos t sin t (b) $6e^{2t}$- 5 sin t + 4 cos t sin t (c) $3e^{2t}$+ 5 sin t + 4 cos t sin t (d) $3e^{2t}$ - 5 sin t + 4 cos t sin t

11) If f(x) = $\frac{\quad}{0 \rightarrow}$ then its differential is given by

(a) $\frac{\rightarrow}{-0 \rightarrow \surd}$ (b) $\frac{\rightarrow}{-0 \rightarrow \surd}$ (c) $\frac{\rightarrow}{-0}$ (d) $\frac{\rightarrow}{-0}$

12) If u(x, y) = x^2+ 3xy + y - 2019, then — $_{(4, -5)}$ is equal to

(a) -4 (b) -3 (c) -7 (d) 13

13) Linear approximation for g(x) = cos x at x= $\frac{\quad}{\surd}$ is

(a) x + $\frac{\quad}{\surd}$ (b) - x + $\frac{\quad}{\surd}$ (c) x - $\frac{\quad}{\surd}$ (d) - x + $\frac{\quad}{\surd}$

14) If w (x, y, z) = x^2 (v - z) + y^2 (z - x) + z^2(x - y), then — 0 — 0 — is

(a) xy + yz + zx (b) x(y + z) (c) y(z + x) (d) 0

15) If (x,y, z) = xy +yz +zx, then f_x - f_z is equal to

(a) z - x (b) y - z (c) x - z (d) y - x

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Differentials and Partial Derivatives 2 MARKS Date : 22-Oct-19

12th Standard

Maths

Reg.No. :

Exam Time : 01:15:00 Hrs Total Marks : 50

25 x 2 = 50

1) Let – , ‾ . Find the linear approximation at x = 27. Use the linear approximation to approximate √7√

2) Use the linear approximation to find approximate values of

→

3) Find a linear approximation for the following functions at the indicated points.

$f(x) = x^3 - 5x + 12, x_0 = 2$

4) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.find the following in calculating the area of the circular plate:

Absolute error

5) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.find the following in calculating the area of the circular plate:

Percentage error

6) A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9-8 cm. Find approximations for the following: change in the volume

7) Show that the percentage error in the nth root of a number is approximately ⇀ times the percentage error in the number.

8) Find df for $f(x) = x^2 + x\ 3$ and evaluate it for

x = 2 and dx = 0.1

9) Find Δf and df for the function f for the indicated values of x, Δx and compare

$f(x) = x^2 + 2x + 3$; x = -0.5, Δx = dx = 0.1

10) Assuming $\log_{10}e = 0.4343$, find an approximate value of $\log^{10}$ 1003

11) An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

12) The relation between the number of words y a person learns in x hours is given by y = 52 ‾ , 0, ≤ x ≤ 9. What is the approximate number of words learned when x changes from

4 to 4.1 hour?

13) A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

14) Evaluate – 4 , –4√, , if the limit exists, where – 4 , √ / √0 √0

15) Let – 4 , √ / – – for (x, y) ≠ (0, 0). Show that – 4 , –4–, f(x, y) = 0

16) Show that f(x, y) = √ √ / √0 → is continuous at every (x, y) ∈ R^2

17) Find the partial derivatives of the following functions at the indicated point.

$f(x, y) = 3x^2 - 2xy + y^2 + 5x + 2$, (2,-5)

18) Find the partial derivatives of the following functions at the indicated point

$h(x, y, z) = x \sin(xy) + z^2x$,) √4–4→√

19) For each of the following functions find the f_x, f_y, and show that $f_{xy} = f_{yx}$

$f(x,y) = \cos(x^2 - 3xy)$

20) If $U(x, y, z) = \log (x^3 + y^3 + z^3)$, find — 0 — 0 —

21) For each of the following functions find the g_{xy}, g_{xx}, g_{yy} and g_{yx}.

$g(x, y) = \log (5x + 3y)$

22) If $v(x, y, z) = x^3 + y^3 + z^3 + xyz^3$, show that $\frac{\surd}{\ }$ $\frac{\surd}{\ }$

23) A firm produces two types of calculators each week, x number of type A and y number of type B. The weekly revenue and cost functions (in rupees) are $R(x, y) = 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2$ and $C(x, y) = 8x + 6y + 2000$ respectively

Find the profit function $P(x, y)$

24) If $w(x, y) = x^3 - xy + y^2$, $x, y \in R$, find the linear approximation for w at $(1,-1)$

25) If $v(x,y) = x^2 - xy + \overset{\rightarrow}{-} y + 7$, $x,y \in R$, find the differential dv.

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Differentials and Partial Derivatives

12th Standard

Maths

Exam Time : 01:00:00 Hrs — Total Marks : 40

20 x 2 = 40

1) Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

2) The time T, taken for a complete oscillation of a single pendulum with length I, is given by the equation T = $2\pi\sqrt{\frac{1}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of 1

3) Let f , g : (a,b)→R be differentiable functions. Show that d(fg) = fdg + gdf

4) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

5) Find differential dy for each of the following function
y = $(3 + \sin(2x))^{2/3}$

6) Find df for f(x) = x^2 + x 3 and evaluate it for
x = 3 and dx = 0.02

7) Evaluate $\begin{matrix} lim \\ (x,y) \to (0,0) \end{matrix}\ cos = \left(\frac{x^3+y^3}{x+y+2}\right)$. If the unlimit exists.

8) Find the partial derivatives of the following functions at the indicated point
g(x,y) =$3x^2 + y^2 + 5x + 2$, (1,-2)

9) If U(x, y, z) = $\frac{x^2+y^2}{xy} + 3z^2y$, find $\frac{\partial U}{\partial x}$; $\frac{\partial U}{\partial y}$ and $\frac{\partial U}{\partial z}$

10) Determine whether the following function is homogeneous or not. If it is so, find the degree.$h(x,y) = \frac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$

11) Show that F(x,y) = $\frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

12) Use differentials to find $\sqrt{25.2}$

13) If f (x, y) = $2x^3 - 11x^2y + 3y^3$, prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 3f$

14) If of f(x, y) = $x^2 + y^3 + 2xy^2$ find f_{xx}, f_{yy}, f_{xy} and f_{yx}.

15) If u=$x^2+y^2+z^2-3xyz$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$

16) If u=$x^2+3xy^2+y^2$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

17) If $w = e^{x^2+y^2}$,x=cosθ,y=sinθ, find $\frac{dw}{d\theta}$

18) If w=xye^{xy} find $\frac{\partial^2 u}{\partial x \partial y}$

19) The pressure P and the volume V of a gas are connected by the relation $PV^{1.4}$=constant. Find the % error in P corresponding to a devreased of $\frac{1}{2}\%$ in V.

20) Calculate df for $f = \sqrt{2x+5}$ when x = 22 and dx = 3.

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Differentials and Partial Derivatives 3 MARKS Date : 22-Oct-19

12th Standard

Maths Reg.No. : | | | | | | |

Exam Time : 00:45:00 Hrs Total Marks : 30

1 x 2 = 2

1) A firm produces two types of calculators each week, x number of type A and y number of type B. The weekly revenue and cost functions (in rupees) are R(x, y) = 80x + 90y + 0.04xy − $0.05x^2$ − $0.05y^2$ and C(x, y) = 8x + 6y + 2000 respectively
Find $\frac{\partial P}{\partial x}$ (1200, 1800) and $\frac{\partial v}{\partial y}$ (1200, 1800)

9 x 3 = 27

2) Find the linear approximation for f(x) = Ã$\overline{1+x}$ $, x \geq -1$ at x_0 = 3. Use the linear approximation to estimate f(3.2)

3) Use linear approximation to find an approximate value of Ã$\overline{9.2}$ without using a calculator.

4) Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

5) A right circular cylinder has radius r =10 cm. and height h = 20 cm. Suppose that the radius of the cylinder is increased from 10 cm to 10. 1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.

6) Let f (x,y) = $\frac{3x-5y+8}{x^2+y^2+1}$ for all (x, y) ∈R^2 Show that f is continuous on R^2

7) f(x,y) = $\frac{xy}{x^2+y^2}$, (x,y) ≠ (0,0) and f (0,0) = 0 Show that f is not continuous at f, -(0,0) and continuous at all other points of R^2

8) Consider g(x,y) = $\frac{2x^2y}{x^2+y^2}$, if (x,y) ≠ (0,0) and g(0,0) = 0 Show that g is continuous on R^2

9) If w(x, y, z) = x^2 y + y^2z + z^2x, x, y, z∈R, 6Tfind the differential dw .

10) Let U(x, y, z) = x^2 − xy + 3 sin z, x, y, z ∈ R Find the linear approximation for U at (2,−1,0).

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Differentials and Partial Derivatives

12th Standard

Maths

Exam Time : 02:00:00 Hrs Total Marks : 90

30 x 3 = 90

1) Find the linear approximation for f(x) = $\sqrt{1+x}, x \geq -1$ at x_0 = 3. Use the linear approximation to estimate f(3.2)
2) A right circular cylinder has radius r =10 cm. and height h = 20 cm. Suppose that the radius of the cylinder is increased from 10 cm to 10. 1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.
3) Use the linear approximation to find approximate values of $(123)^{\frac{2}{3}}$
4) Use the linear approximation to find approximate values of $\sqrt[3]{26}$
5) Find a linear approximation for the following functions at the indicated points. g(x) = $g(x) = \sqrt{x^2+9}, x_0 = -4$
6) Find a linear approximation for the following functions at the indicated points. $h(x) = \frac{x}{1+x} = \frac{1}{2}$
7) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.find the following in calculating the area of the circular plate:
Relative error
8) Find Δf and df for the function f for the indicated values of x, Δx and compare
$f(x) = x^3 - 2x^2$; x = 2, Δ x = dx = 0.5
9) Assuming $\log_{10}e$ = 0.4343, find an approximate value of $\log^{10}$ 1003
10) The trunk of a tree has diameter 30 cm. During the following year, the circumference grew 6cm.
What is the percentage increase in area of the tree's cross-section?
11) Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?
12) The relation between the number of words y a person learns in x hours is given by y = 52 $\sqrt{x}$, 0, ≤ x ≤ 9. What is the approximate number of words learned when x changes from
1 to 1.1 hour?
13) A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.
14) Let f (x,y) = $\frac{3x-5y+8}{x^2+y^2+1}$ for all (x, y) ∈R^2 Show that f is continuous on R^2
15) Consider g(x,y) = $\frac{2x^2y}{x^2+y^2}$, if (x,y) ≠ (0,0) and g(0,0) = 0 Show that g is continuous on R^2
16) Let F(x, y) = x^3 y + y^2x + 7 for all (x, y)∈ R^2. Calculate $\frac{\partial F}{\partial x}$ (-1,3) and $\frac{\partial F}{\partial y}$ (-2,1).

17) Let $f(x,y) = \frac{y^2-xy}{\sqrt{x}-\sqrt{y}}$ for (x, y) ≠ (0, 0). Show that $\lim_{(x,y)\to(0,0)}$ f(x, y) = 0

18) Let g(x, y) = $\frac{x^2y}{x^4+y^2}$ for (x, y) ≠ (0, 0) and f(0, 0) = 0

Show that $\lim_{(x,y)\to(1,2)}$ g(x, y) = 0 along every line y = mx, m ∈ R

19) Let g(x, y) = $\frac{x^2y}{x^4+y^2}$ for (x, y) ≠ (0, 0) and f(0, 0) = 0

Show that $\lim_{(x,y)\to(0,0)}$ g(x, y) = $\frac{k}{1+k^2}$ along every parabola y = kx^2, k ∈ R \ {0}.

20) Let g(x, y) = $\frac{e^y sinx}{x}$, for x ≠ 0 and g(0, 0) = 1. Show that g is continuous at (0,0).

21) Find the partial derivatives of the following functions at the indicated point

G(x, y) = e^{x+3y} log (x^2 + y^2)

22) For each of the following functions find the g_{xy}, g_{xx}, g_{yy} and g_{yx}.

g(x, y) = xe^y + $3x^2y$

23) Let w(x, y, z) = $\frac{1}{\sqrt{x^2+y^2+z^2}}, (x,y,z) \neq (0,0,0)$. Show that

$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$

24) If w(x, y) = x^3 − xy + y^2, x, y ∊ R, find the linear approximation for w at (1,−1)

25) Let g(x, y) = 2y + x^2, x = 2r -s, y = r^2+ 2s, r, s ∊ R. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

26) Let U(x, y, z) = xyz, x = e^{-t}, y = e^{-t} cos t, z = sin t, t ∈ R. Find $\frac{dU}{dt}$

27) Let z(x, y) = x^3 - $3x^2y^3$, where x = set, y = se^{-t}, s, t ∈ R. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

28) If f = $\frac{x}{x^2+y^2}$ then show that = $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ = -f

29) If u=log(x^2+y^2+z^2), then prove that $x\frac{\partial^2 u}{\partial z \partial x} = y\frac{\partial^2 u}{\partial z \partial x} = z\frac{\partial^2 u}{\partial x \partial y}$

30) If w= log(x^2+y^2) and x=rcosθ and y=rsinθ then, find $\frac{\partial w}{\partial r} and \frac{\partial w}{\partial \theta}$

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Differentials and Partial Derivatives 5 MARKS Date : 22-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 01:30:00 Hrs Total Marks : 75

15 X 5 = 75

1) Let g(x, y) = $\frac{e^y sinx}{x}$, for x ≠ 0 and g(0, 0) = 1. Show that g is continuous at (0,0).

2) Find the partial derivatives of the following functions at the indicated point

h (x, y, z) = x sin (xy) + z^2x, $\left(2, \frac{\pi}{4}, 1\right)$

3) Find the partial derivatives of the following functions at the indicated point

G(x, y) = e^{x+3y} log (x^2 + y^2)

4) If U(x, y, z) = $\frac{x^2+y^2}{xy} + 3z^2y$, find $\frac{\partial U}{\partial x}$; $\frac{\partial U}{\partial y}$ and $\frac{\partial U}{\partial z}$

5) Let w(x, y, z) = $\frac{1}{\sqrt{x^2+y^2+z^2}}$, $(x, y, z) \neq (0, 0, 0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$

6) If w(x, y) = xy + sin (xy), then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$

7) If v(x, y, z) = x^3 + y^3 + z^3 + xyz^3, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

8) Let f , g : (a,b)→R be differentiable functions. Show that d(fg) = fdg + gdf

9) Let g(x) = x^2 + sin x. Calculate the differential dg.

10) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

11) Let f (x, y) = 0 if xy ≠ 0 and f (x, y) =1 if xy = 0.

(i) Calculate: $\frac{\partial f}{\partial x}(0, 0)$, $\frac{\partial f}{\partial y}(0, 0)$.

(ii) Show that f is not continuous at (0,0)

12) Let F(x, y) = x^3 y + y^2x + 7 for all (x, y)∈ R^2. Calculate $\frac{\partial F}{\partial x}$ (-1,3) and $\frac{\partial F}{\partial y}$ (-2,1).

13) Let f(x, y) = sin(xy^2) + e^{x3+5y} for all ∈ R^2. Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$

14) Let w(x, y) = xy+$\frac{e^y}{y^2+1}$ for all (x, y) ∈ R^2. Calculate $\frac{\partial^2 w}{\partial y \partial x}$ and $\frac{\partial^2 w}{\partial x \partial y}$

15) Let (x, y) = e^{-2y} cos(2x) for all (x, y) ∈ R^2. Prove that u is a harmonic function in R^2.

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Differentials and Partial Derivatives

12th Standard

Maths

Exam Time : 02:00:00 Hrs | Total Marks : 75

15 x 5 = 75

1) Let f(x, y) = sin(xy^2) + e^{x3+5y} for all ∈ R^2. Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$and $\frac{\partial^2 f}{\partial x \partial y}$

2) For each of the following functions find the f_x, f_y, and show that f_{xy} =f_{yx}
f(x,y) = $\frac{3x}{y+sinx}$

3) If U(x, y, z) = log (x^3 + y^3 + z^3), find $\frac{\partial U}{\partial x}+\frac{\partial U}{\partial y}+\frac{\partial U}{\partial z}$

4) If w(x, y) = xy + sin (xy), then prove that $\frac{\partial^2 w}{\partial y \partial x}=\frac{\partial^2 w}{\partial x \partial y}$

5) Let U(x, y) = ex sin y, where x = st^2, y = s^2 t, s, t ∈ R. Find $\frac{\partial U}{\partial s}$, $\frac{\partial U}{\partial t}$ and evaluate them at s = t = 1.

6) Prove that f(x, y) = x^3 - $2x^2y$ + $3xy^2$ + y^3 is taomogeneous; what is the degree? Verify fuler's Theorem for f.

7) If u=$\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that x $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}tanu$

8) If u = $\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$
Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ sin 2u.

9) Using differential find the approximate value of cos 61; if it is given that sin 60° = 0.86603 and 1^0 = 0.01745 radians.

10) If z =f(x - cy) + F (x + cy) where f and Fare any two functions and c is a constant, show that $\frac{\partial^2 z}{\partial x^2}=\frac{\partial^2 z}{\partial y^2}$

11) If P=CL^ak^B,c > 0,$\alpha+\beta=1$, then prove that $k\frac{\partial P}{\partial k}+L\frac{\partial P}{\partial K}=P$.(Without using Euler's theorem)

12) If w=u^2e^v where $u=\frac{x}{y}$ and v=logx. Find $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$

13) The height of a cone is increased by k%, its semi vertical angle remaining the same. What is the approximate percentage increases in (i) T.S.A (ii) Volume assuming k is small.

14) If $w=x^2sin\left(\frac{x}{y}\right)+y^2cos\left(\frac{x}{y}\right)+xytan\left(\frac{x}{y}\right)$,then prove that
$x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}=2w$

15) Find the approximate value of $\sqrt[3]{1.02}+\sqrt{1.02}$

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Differentials and Partial Derivatives FULL TEST Date : 22-Oct-19

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 02:30:00 Hrs Total Marks : 90

15 x 1 = 15

1) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

2) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{\rightarrow}{\rightarrow}$ (b) $\rightarrow$ (c) 5 (d) 31

3) If u (x, y) = e^{x2+y2}, then — is equal to

(a) e^{x2+y2} (b) 2xu (c) x^2u (d) y^2u

4) If v (x, y) = log (ex + ev), then — 0 — is equal to

(a) $e^x + e^y$ (b) $\frac{\rightarrow}{0}$ (c) 2 (d) 1

5) If w (x, y) = xy, x > 0, then — is equal to

(a) x^y log x (b) y log x (c) yx^{y-1} (d) x log y

6) If f (x, y) = e^{xy} then $\frac{\surd}{\quad}$ is equal to

(a) xye^{xy} (b) $(1 +xy)e^{xy}$ (c) $(1 +y)e^{xy}$ (d) $(1 + x)e^{xy}$

7) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

8) The change in the surface area S = $6x^2$ of a cube when the edge length varies from x_o to x_o+ dx is

(a) 12 x_o+dx (b) 12x_o dx (c) 6x_o dx (d) 6x_o+ dx

9) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(a) 0.3xdx m^3 (b) 0.03 xm^3 (c) 0.03.x^2 m^3 (d) 0.03x^3m^3

10) If g(x, y) = $3x^2$ - 5y + 2y, x(t) = e^t and y(t) = cos t, then — is equal to

(a) $6e^{2t}$+5 sin t - 4 cos t sin t (b) $6e^{2t}$- 5 sin t + 4 cos t sin t (c) $3e^{2t}$+ 5 sin t + 4 cos t sin t (d) $3e^{2t}$ - 5 sin t + 4 cos t sin t

11) If f(x) = $\frac{\quad}{0\rightarrow}$ then its differential is given by

(a) $\frac{\rightarrow}{-0\rightarrow\surd}$ (b) $\frac{\rightarrow}{-0\rightarrow\surd}$ (c) $\frac{\rightarrow}{-0}$ (d) $\frac{\rightarrow}{-0}$

12) If u(x, y) = x^2+ 3xy + y - 2019, then — $_{(4, -5)}$ is equal to

(a) -4 (b) -3 (c) -7 (d) 13

13) Linear approximation for g(x) = cos x at x= $\frac{\quad}{\surd}$ is

(a) x + $\frac{\quad}{\surd}$ (b) - x + $\frac{\quad}{\surd}$ (c) x - $\frac{\quad}{\surd}$ (d) - x + $\frac{\quad}{\surd}$

14) If w (x, y, z) = x^2 (v - z) + y^2 (z - x) + z^2(x - y), then — 0 — 0 — is

(a) xy + yz + zx (b) x(y + z) (c) y(z + x) (d) 0

15) If (x,y, z) = xy +yz +zx, then f_x - f_z is equal to

(a) z - x (b) y - z (c) x - z (d) y - x

10 x 2 = 20

16) Use the linear approximation to find approximate values of

$\surd$

17) A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9-8 cm. Find approximations for the following: change in the volume

18)

The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation T = 2π $\frac{}{}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l

19) Find df for $f(x) = x^2 + x\ 3$ and evaluate it for x = 3 and dx = 0.02

20) An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

21) Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?

22) Evaluate − 4 , −4√, , if the limit exists, where − 4 , √ / √0 √0

23) Let g(x, y) = ——, for x ≠ 0 and g(0, 0) = 1. Show that g is continuous at (0,0).

24) Find the partial derivatives of the following functions at the indicated point h (x, y, z) = x sin (xy) + z^2x,) √4−4→√

25) If U(x, y, z) = log ($x^3 + y^3 + z^3$), find — 0 — 0 —

5 x 3 = 15

26) Use linear approximation to find an approximate value of 7√ without using a calculator.

27) Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

28) Consider g(x,y) = √√ / √0 √ , if (x,y) ≠ (0,0) and g(0,0) = 0 Show that g is continuous on R^2

29) If w(x, y, z) = x^2 y + y^2z + z^2x, x, y, z∈R, 6T find the differential dw .

30) Let U(x, y, z) = x^2 − xy + 3 sin z, x, y, z ∈ R Find the linear approximation for U at (2,−1,0).

8 x 5 = 40

31) Let f , g : (a,b)→R be differentiable functions. Show that d(fg) = fdg + gdf

32) Let g(x) = x^2 + sin x. Calculate the differential dg.

33) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

34) Let f (x, y) = 0 if xy ≠ 0 and f (x, y) =1 if xy = 0.

(i) Calculate: — −4−, 4— −4−, 7

(ii) Show that f is not continuous at (0,0)

35) Let F(x, y) = x^3 y + y^2x + 7 for all (x, y)∈ R^2. Calculate $\frac{w}{}$ (-1,3) and $\frac{w}{}$ (-2,1).

36) Let f(x, y) = sin(xy^2) + e^{x3+5y} for all ∈ R^2. Calculate —4—4— √ and √ ——

37) Let w(x, y) = xy+ —— / √0 → for all (x, y) ∈ R^2. Calculate √ —— and √ ——

38) Let (x, y) = e^{-2y} cos(2x) for all (x, y) ∈ R^2. Prove that u is a harmonic function in R^2.

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RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
WHATSAPP - 8056206308

Differentials and Partial Derivatives

12th Standard

Maths

Exam Time : 02:30:00 Hrs | Total Marks : 100

20 x 1 = 20

1) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is
(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

2) If u (x, y) = e^{x2+y2}, then $\frac{\partial u}{\partial x}$ is equal to
(a) e^{x2+y2} (b) 2xu (c) x^2u (d) y^2u

3) If w (x, y) = x^y, x > 0, then $\frac{\partial w}{\partial x}$ is equal to
(a) $x^y \log x$ (b) y log x (c) yx^{y-1} (d) x log y

4) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

5) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
(a) 0.3xdx m^3 (b) 0.03 xm^3 (c) 0.03.x^2 m^3 (d) 0.03x^3m^3

6) If f(x) = $\frac{x}{x+1}$ then its differential is given by
(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{1+x}dx$ (d) $\frac{-1}{1+x}dx$

7) Linear approximation for g(x) = cos x at x=$\frac{-\pi}{2}$ is
(a) x + $\frac{-\pi}{2}$ (b) - x + $\frac{\pi}{2}$ (c) x - $\frac{\pi}{2}$ (d) - x + $\frac{\pi}{2}$

8) If (x,y, z) = xy +yz +zx, then $f_x - f_z$ is equal to
(a) z - x (b) y - z (c) x - z (d) y - x

9) If y = x^4 - 10 and if x changes from 2 to 1.99, the approximate change in y is
(a) -32 (b) -0.32 (c) - 10 (d) 10

10) If $\log_e 4$ = 1.3868, then $\log_e 4.01$ =
(a) 1.3968 (b) 1.3898 (c) 1.3893 (d) none

11) If u = $x^y + y^x$ then $u_x + u_y$ at x = y = 1 is
(a) 0 (b) 2 (c) 1 (d) ∞

12) If f(x, y, z) = sin (xy) + sin (yz) + sin (zx) then f_{xx} is
(a) -y sin (xy) + z^2 cos (xz) (b) y sin (xy) - z^2 cos (xz) (c) y sin (xy) + z^2 cos (xz) (d) -y sin (xy) - z^2 cos (xz)

13) If f (x, y) = $x^3 + y^3 - 3xy^2$ then $\frac{\partial f}{\partial x}$ at x = 2,
(a) -15 (b) 15 (c) -9 (d) 16

14) The approximate value of $(627)^{\frac{1}{4}}$ is ....
(a) 5.002 (b) 5.003 (c) 5.005 (d) 5.004

15) If y = sin x and x changes from $\frac{\pi}{2}$ to π the approximate change in y is ....
(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{22}{14}$

16) If u = sin-1 $\left(\frac{x^4+y^4}{x^2+y^2}\right)$ and f= sin u then f is a homogeneous function of degree ....

(a) 0 (b) 1 (c) 2 (d) 4

17) The percentage error in the 11^{th} root of the number 28 is approximately .... times the percentage error in 28.

(a) $\frac{1}{28}$ (b) $\frac{1}{11}$ (c) 11 (d) 28

18) If u = $(\frac{y}{x})$ then x $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ = ...

(a) 0 (b) 1 (c) 2u (d) u

19) If u = y sin x then $\frac{\partial^2 u}{\partial x \partial y}$ = ...

(a) cos x (b) cos y (c) sin x (d) 0

20) If is a homogeneous function of x and y of degree n, then $x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x \partial y}$ = ... $\frac{\partial u}{\partial x}$

(a) n (b) 0 (c) 1 (d) n - 1

8 x 2 = 16

21) A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9-8 cm. Find approximations for the following:
change in the volume

22) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation T = 2π$\sqrt{\frac{1}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of 1

23) Let f , g : (a,b)→R be differentiable functions. Show that d(fg) = fdg + gdf

24) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

25) If U(x, y, z) = $\frac{x^2+y^2}{xy}+3z^2y$, find $\frac{\partial U}{\partial x}$; $\frac{\partial U}{\partial y}$ and $\frac{\partial U}{\partial z}$

26) In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.
f(x, y) = $x^2y + 6x^3 + 7$

27) Show that F(x,y) = $\frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

28) If f (x, y) = $2x^3 - 11x^2y + 3y^3$, prove that $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=3f$

8 x 3 = 24

29) A right circular cylinder has radius r =10 cm. and height h = 20 cm. Suppose that the radius of the cylinder is increased from 10 cm to 10. 1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.

30) Use the linear approximation to find approximate values of
$(123)^{\frac{2}{3}}$

31) The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.find the following in calculating the area of the circular plate:
Percentage error

32) f(x,y) = $\frac{xy}{x^2+y^2}$, (x,y) ≠ (0,0) and f (0,0) = 0 Show that f is not continuous at f, -(0,0) and continuous at all other points of R^2

33) Find the partial derivatives of the following functions at the indicated point
G(x, y) = e^{x+3y} log (x^2 + y^2)

34) Let w(x, y, z) = $\frac{1}{\sqrt{x^2+y^2+z^2}}, (x, y, z) \neq (0, 0, 0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$

35) Let g(x, y) = 2y + x^2, x = 2r -s, y = r^2+ 2s, r, s ∈ R. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$

36) If u(x, y) = $\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

8 x 5 = 40

37) Let f(x, y) = sin(xy^2) + e^{x3+5y} for all ∈ R^2. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial x \partial y}$

38) For each of the following functions find the f_x, f_y, and show that f_{xy} =f_{yx}
f(x,y) = $\frac{3x}{y+sinx}$

39) If V(x,y) = e^x(x cos y - y siny), then prove that $\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial y^2}$ = 0

40) If w(x, y) = xy + sin (xy), then prove that $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$

41) Prove that f(x, y) = x^3 - $2x^2y$ + $3xy^2$ + y^3 is taomogeneous; what is the degree? Verify fuler's Theorem for f.

42) If u=$\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that x $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}tanu$

43) If $u = tan^{-1}(\frac{x^2+y^2}{x-y})$,then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}sin2u$

44) If $u = log\sqrt{x^2+y^2+z^2}$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{x^2+y^2+z^2}$

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Ravi home tutions PH - 8056206308

Differentials and Partial Derivatives FULL TEST

12th Standard

Maths

Reg.No. : | | | | | | |

Exam Time : 03:00:00 Hrs

Total Marks : 90

ANSWER ALL

20 x 1 = 20

1) A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

2) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{\rightarrow}{\rightarrow}$ (b) $\frac{\rightarrow}{}$ (c) 5 (d) 31

3) If u (x, y) = e^{x2+y2}, then — is equal to

(a) e^{x2+y2} (b) 2xu (c) x^2u (d) y^2u

4) If v (x, y) = log (ex + ev), then — 0 — is equal to

(a) $e^x + e^y$ (b) $\frac{\rightarrow}{0}$ (c) 2 (d) 1

5) If w (x, y) = xy, x > 0, then — is equal to

(a) x^y log x (b) y log x (c) yx^{y-1} (d) x log y

6) If f (x, y) = e^{xy} then $\frac{\surd}{}$ is equal to

(a) xye^{xy} (b) $(1 +xy)e^{xy}$ (c) $(1 +y)e^{xy}$ (d) $(1 + x)e^{xy}$

7) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

8) The change in the surface area S = $6x^2$ of a cube when the edge length varies from x_o to x_o+ dx is

(a) 12 x_o+dx (b) $12x_o$ dx (c) $6x_o$ dx (d) $6x_o$+ dx

9) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(a) 0.3xdx m^3 (b) 0.03 xm^3 (c) 0.03.x^2 m^3 (d) $0.03x^3m^3$

10) If g(x, y) = $3x^2$ - 5y + 2y, x(t) = e^t and y(t) = cos t, then — is equal to

(a) $6e^{2t}$+5 sin t - 4 cos t sin t (b) $6e^{2t}$- 5 sin t + 4 cos t sin t (c) $3e^{2t}$+ 5 sin t + 4 cos t sin t (d) $3e^{2t}$ - 5 sin t + 4 cos t sin t

11) If f(x) = $\frac{}{0 \rightarrow}$ then its differential is given by

(a) $\frac{\rightarrow}{-0 \rightarrow^{\surd}}$ (b) $\frac{\rightarrow}{-0 \rightarrow^{\surd}}$ (c) $\frac{\rightarrow}{-0}$ (d) $\frac{\rightarrow}{-0}$

12) If u(x, y) = x^2+ 3xy + y - 2019, then $—_{(4, -5)}$ is equal to

(a) -4 (b) -3 (c) -7 (d) 13

13) Linear approximation for g(x) = cos x at x=$\frac{}{\surd}$ is

(a) x + $\frac{}{\surd}$ (b) - x + $\frac{-}{\surd}$ (c) x - $\frac{-}{\surd}$ (d) - x + $\frac{-}{\surd}$

14) If w (x, y, z) = x^2 (v - z) + y^2 (z - x) + z^2(x - y), then — 0 — 0 — is

(a) xy + yz + zx (b) x(y + z) (c) y(z + x) (d) 0

15) If (x,y, z) = xy +yz +zx, then f_x - f_z is equal to

(a) z - x (b) y - z (c) x - z (d) y - x

16) If u = log $\surd 0$ $\surd$, then $\frac{\surd}{\surd}$ 0 $\frac{\surd}{\surd}$ is

(a) (b) 0 (c) u (d) 2u

17) If u = log√0(x³ + √ y³ + z³ - 3xyz) then — 0 — 0 — =

(a) $\frac{}{0\ 0}$ (b) x+y+z (c) $\frac{}{-0\ \ 0\ \ ,\surd}$ (d) $\frac{}{-0\ \ 0\ \ ,\surd}$

18) If u = y^x then — = ...

(a) xy^{x-1} (b) yx^{y-1} (c) 0 (d) 1

19) If u = —, then x — 0 — = ...

(a) 0 (b) 1 (c) 2u (d) u

20) If u = y sin x then $\frac{\surd}{}$ = ...

(a) cos x (b) cos y (c) sin x (d) 0

ANSWER ANY 7 7 x 2 = 14

21) Use the linear approximation to find approximate values of

√

22) A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9-8 cm. Find approximations for the following:
change in the volume

23) The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation T = 2π —⃗, where g is a constant. Find the approximate percentage error in the calculated value of T

corresponding to an error of 2 percent in the value of 1

24) Find df for f(x) = x^2 + x 3 and evaluate it for
x = 3 and dx = 0.02

25) An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm and radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

26) Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?

27) Evaluate – 4 , —4√, , if the limit exists, where – 4 , $\frac{\surd}{\surd 0\ \ \surd 0}$

28) Let g(x, y) = —— , for x ≠ 0 and g(0, 0) = 1. Show that g is continuous at (0,0).

29) Find the partial derivatives of the following functions at the indicated point
h (x, y, z) = x sin (xy) + z^2x,) √4–4–√

30) If U(x, y, z) = log (x^3 + y^3 + z^3), find — 0 — 0 —

ANSWER ANY 7 7 x 3 = 21

31) Use linear approximation to find an approximate value of 7√ without using a calculator.

32) Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

33) Consider g(x,y) = $\frac{\surd\ \surd}{\surd 0\ \ \surd}$, if (x,y) ≠ (0,0) and g(0,0) = 0 Show that g is continuous on R^2

34) If w(x, y, z) = x^2 y + y^2z + z^2x, x, y, z∈R, бт find the differential dw .

35) Let U(x, y, z) = x^2 – xy + 3 sin z, x, y, z ∈ R Find the linear approximation for U at (2,–1,0).

36)

f(x,y) = $\frac{}{\sqrt{0}\ \sqrt{}}$, (x,y) ≠ (0,0) and f (0,0) = 0 Show that f is not continuous at f, -(0,0) and continuous at all other points of R^2

37) If u=$\sin^{-1}\left(\frac{0}{0}\right)$, Show that x — 0 — $\frac{\rightarrow}{\sqrt{}}$

38) Use differentials to find the value of 7

39) Find the approximate value of f (3.02) where f(x) = $3x^2 + 5x + 3$.

40) If w = xy + z where x = cos t; y = sin t; z = t find —

ANSWER ANY 7 7 x 5 = 35

41) Let f , g : (a,b)→R be differentiable functions. Show that d(fg) = fdg + gdf

42) Let g(x) = x^2 + sin x. Calculate the differential dg.

43) If the radius of a sphere, with radius 10 cm, has to decrease by 0 1. cm, approximately how much will its volume decrease?

44) Let f (x, y) = 0 if xy ≠ 0 and f (x, y) =1 if xy = 0.

(i) Calculate: — 4 , 4 — 4 , 7

(ii) Show that f is not continuous at (0,0)

45) Let F(x, y) = $x^3 y + y^2x + 7$ for all (x, y)∈ R^2. Calculate $\frac{w}{}$ (-1,3) and $\frac{w}{}$ (-2,1).

46) Let f(x, y) = $\sin(xy^2) + e^{x3+5y}$ for all ∈ R^2. Calculate —4—4$\frac{\sqrt{}}{}$ and $\frac{\sqrt{}}{}$

47) Let w(x, y) = xy+$\frac{}{\sqrt{0}\rightarrow}$ for all (x, y) ∈ R^2. Calculate $\frac{\sqrt{}}{}$ and $\frac{\sqrt{}}{}$

48) Let (x, y) = e^{-2y} cos(2x) for all (x, y) ∈ R^2. Prove that u is a harmonic function in R^2.

49) Let g(x,y)= x^3 - yx + sin(x+y), x(t) = e^{3t}, y(t) = t^2, t ∈ R. Find —

50) If u = $\tan^{-1}\left(\frac{0}{}\right)$

Prove that — 0 — sin 2u.

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