

Complex Numbers TEST 1

12th Standard

Maths

Exam Time : 01:15:00 Hrs

Total Marks : 75

55 x 1 = 55

Multiple Choice Question

- 1) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is
 (a) 0 (b) 1 (c) -1 (d) i
- 2) The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is
 (a) $1+i$ (b) i (c) 1 (d) 0
- 3) The area of the triangle formed by the complex numbers z, iz , and $z+iz$ in the Argand's diagram is
 (a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$
- 4) The conjugate of a complex number is $\frac{1}{i-2}$ /Then the complex number is
 (a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$
- 5) If $z = \frac{(\sqrt{3} + i)^3 (3i + 4)^2}{(8 + 6i)^2}$, then $|z|$ is equal to
 (a) 0 (b) 1 (c) 2 (d) 3
- 6) If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
- 7) If $|z-2+i| \leq 2$, then the greatest value of $|z|$ is
 (a) $\sqrt{3}-2$ (b) $\sqrt{3}+2$ (c) $\sqrt{5}-2$ (d) $\sqrt{5}+2$
- 8) If $\left| z - \frac{3}{z} \right| = 2$ then the least value $|z|$ is
 (a) 1 (b) 2 (c) 3 (d) 5
- 9) If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is
 (a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1
- 10) The solution of the equation $|z|-z=1+2i$ is
 (a) $\frac{3}{2} - 2i$ (b) $-\frac{3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$
- 11) If $|z_1|=1, |z_2|=2, |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is
 (a) 1 (b) 2 (c) 3 (d) 4

- 12) If z is a complex number such that $z \in C/R$ and $z + \frac{1}{z} \in R$ then $|z|$ is
 (a) 0 (b) 1 (c) 2 (d) 3
- 13) z_1, z_2 and z_3 are complex number such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^3$ is
 (a) 3 (b) 2 (c) 1 (d) 0
- 14) If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
- 15) If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is
 (a) real axis (b) imaginary axis (c) ellipse (d) circle
- 16) The principal argument of $\frac{3}{-1+i}$
 (a) $-\frac{5\pi}{6}$ (b) $-\frac{2\pi}{3}$ (c) $-\frac{3\pi}{4}$ (d) $-\frac{\pi}{2}$
- 17) The principal argument of $(\sin 40^\circ + i \cos 40^\circ)^5$ is
 (a) -110° (b) -70° (c) 70° (d) 110°
- 18) If $(1+i)(1+2i)(1+3i)\dots(1+ni) = x + iy$, then $2 \cdot 5 \cdot 10 \dots (1+n^2)$ is
 (a) 1 (b) i (c) $x^2 + y^2$ (d) $1 + n^2$
- 19) If $\omega \neq 1$ is a cubic root of unity and $(1 + \omega)^7 = A + B\omega$, then (A, B) equals
 (a) (1,0) (b) (-1,1) (c) (0,1) (d) (1,1)
- 20) The principal argument of the complex number $\frac{(1 + i\sqrt{3})^2}{4i(1 - i\sqrt{3})}$ is
 (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$
- 21) If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is
 (a) -2 (b) -1 (c) 1 (d) 2
- 22) The product of all four values of $\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^{\frac{3}{4}}$ is
 (a) -2 (b) -1 (c) 1 (d) 2
- 23) If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^2 \end{vmatrix} = 3k$, then k is equal to
 (a) 1 (b) -1 (c) $\sqrt{3i}$ (d) $-\sqrt{3i}$
- 24) The value of $\left(\frac{1 + 3\sqrt{i}}{1 - \sqrt{3i}}\right)^{10}$
 (a) $\text{cis} \frac{2\pi}{3}$ (b) $\text{cis} \frac{4\pi}{3}$ (c) $-\text{cis} \frac{2\pi}{3}$ (d) $-\text{cis} \frac{4\pi}{3}$

- 25) If $\omega = cis \frac{2\pi}{3}$, then the number of distinct roots of
- $$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$$
- (a) 1 (b) 2 (c) 3 (d) 4
- 26) The value of $(1+i)(1+i^2)(1+i^3)(1+i^4)$ is
- (a) 2 (b) 0 (c) 1 (d) i
- 27) If $\sqrt{a+ib} = x+iy$, then possible value of $\sqrt{a-ib}$ is
- (a) x^2+y^2 (b) $\sqrt{x^2+y^2}$ (c) $x+iy$ (d) $x-iy$
- 28) If, $i^2 = -1$, then $i^1 + i^2 + i^3 + \dots$ up to 1000 terms is equal to
- (a) 1 (b) -1 (c) i (d) 0
- 29) If $z = \cos \frac{\pi}{4} + i \sin \frac{\pi}{6}$, then
- (a) $|z| = 1, \arg(z) = \frac{\pi}{4}$ (b) $|z| = 1, \arg(z) = \frac{\pi}{6}$ (c) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \frac{5\pi}{24}$ (d) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \tan^{-1} \left(\frac{1}{\sqrt{2}} \right)$
- 30) If $a = \cos \theta + i \sin \theta$, then $\frac{1+a}{1-a} =$
- (a) $\cot \frac{\theta}{2}$ (b) $\cot \theta$ (c) $i \cot \frac{\theta}{2}$ (d) $i \tan \frac{\theta}{2}$
- 31) The principal value of the amplitude of $(1+i)$ is
- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{12}$ (c) $\frac{3\pi}{4}$ (d) π
- 32) The least positive integer n such that $\left(\frac{2i}{1+i} \right)^n$ is a positive integer is
- (a) 16 (b) 8 (c) 4 (d) 2
- 33) If $a = 1+i$, then a^2 equals
- (a) $1-i$ (b) $2i$ (c) $(1+i)(1-i)$ (d) $i-1$
- 34) If $z = \frac{1}{(2+3i)^2}$ then $|z| =$
- (a) $\frac{1}{13}$ (b) $\frac{1}{5}$ (c) $\frac{1}{12}$ (d) none of these
- 35) If $z = 1 - \cos \theta + i \sin \theta$, then $|z| =$
- (a) $2 \sin \frac{1}{3}$ (b) $2 \cos \frac{\theta}{2}$ (c) $2 \left| \sin \frac{\theta}{2} \right|$ (d) $2 \left| \cos \frac{\theta}{2} \right|$
- 36) If $z = \frac{1}{1 - \cos \theta - i \sin \theta}$, the $\operatorname{Re}(z) =$
- (a) 0 (b) $\frac{1}{2}$ (c) $\cot \frac{\theta}{2}$ (d) $\frac{1}{2} \cot \frac{\theta}{2}$
- 37) If $x+iy = \frac{3+5i}{7-6i}$, they $y =$
- (a) $\frac{9}{85}$ (b) $-\frac{9}{85}$ (c) $\frac{53}{85}$ (d) none of these
- 38) The amplitude of $\frac{1}{i}$ is equal to
- (a) 0 (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{2}$ (d) π
- 39) The value of $(1+i)^4 + (1-i)^4$ is
- (a) 8 (b) 4 (c) -8 (d) -4
- 40) The complex number z which satisfies the condition $\left| \frac{1+z}{1-z} \right| = 1$ lies on
- (a) circle $x^2+y^2=1$ (b) x-axis (c) y-axis (d) the lines $x+y=1$
- 41) If $z = a + ib$ lies in quadrant then $\bar{z}$ also lies in the III quadrant if
- (a) $a > b > 0$ (b) $a < b < 0$ (c) $b < a < 0$ (d) $b > a > 0$

- 42) $\frac{1+e^{-i\theta}}{1+e^{i\theta}} =$
 (a) $\cos\theta + i \sin\theta$ (b) $\cos\theta - i \sin\theta$ (c) $\sin\theta - i \cos\theta$ (d) $\sin\theta + i \cos\theta$
- 43) If $z^n = \cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3}$, then $z_1, z_2, \dots, z_6$ is
 (a) 1 (b) -1 (c) i (d) -i
- 44) If $x = \cos\theta + i \sin\theta$, then the value of $x^n + \frac{1}{x^n}$ is
 (a) $2 \cos\theta$ (b) $2i \sin n\theta$ (c) $2i \sin n\theta$ (d) $2i \cos n\theta$
- 45) If ω is the cube root of unity, then the value of $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$ is
 (a) 9 (b) -9 (c) 16 (d) 32
- 46) The points represented by $3 - 3i, 4 - 2i, 3 - i$ and $2 - 2i$ form _____ in the argand plane.
 (a) collinear points (b) Vertices of a parallelogram (c) Vertices of a rectangle (d) Vertices of a square
- 47) $(1+i)^3 =$ _____
 (a) $3 + 3i$ (b) $1 + 3i$ (c) $3 - 3i$ (d) $2i - 2$
- 48) $\frac{(\cos\theta + i \sin\theta)^6}{(\cos\theta - i \sin\theta)^5} =$ _____
 (a) $\cos 11\theta - i \sin 11\theta$ (b) $\cos 11\theta + i \sin 11\theta$ (c) $\cos\theta + i \sin\theta$ (d) $\cos \frac{6\theta}{5} + i \sin \frac{6\theta}{5}$
- 49) If $a = \cos\alpha + i \sin\alpha$, $b = -\cos\beta + i \sin\beta$ then $\left(ab - \frac{1}{ab}\right)$ is _____
 (a) $-2i \sin(\alpha - \beta)$ (b) $2i \sin(\alpha - \beta)$ (c) $2 \cos(\alpha - \beta)$ (d) $-2 \cos(\alpha - \beta)$
- 50) The conjugate of $\frac{1+2i}{1-(1-i)^2}$ is _____
 (a) $\frac{1+2i}{1-(1-i)^2}$ (b) $\frac{5}{1-(1-i)^2}$ (c) $\frac{1-2i}{1+(1+i)^2}$ (d) $\frac{1+2i}{1+(1-i)^2}$
- 51) The modular of $\frac{(-1+i)(1-i)}{1+i\sqrt{3}}$ is _____
 (a) $\sqrt{2}$ (b) 2 (c) 1 (d) $\frac{1}{2}$
- 52) The value of $\frac{(\cos 45^\circ + i \sin 45^\circ)^2 (\cos 30^\circ - i \sin 30^\circ)}{\cos 30^\circ + i \sin 30^\circ}$ is _____
 (a) $\frac{1}{2} + i \frac{\sqrt{3}}{2}$ (b) $\frac{1}{2} - i \frac{\sqrt{3}}{2}$ (c) $-\frac{\sqrt{3}}{2} + \frac{1}{2}$ (d) $\frac{\sqrt{3}}{2} + \frac{1}{2}$
- 53) If $x = \cos\theta + i \sin\theta$, then $x^n + \frac{1}{x^n}$ is _____
 (a) $2 \cos n\theta$ (b) $2i \sin n\theta$ (c) $2^n \cos\theta$ (d) $2^n i \sin\theta$
- 54) If z_1, z_2, z_3 are the vertices of a parallelogram, then the fourth vertex z_4 opposite to z_2 is _____
 (a) $z_1 + z_2 - z_3$ (b) $z_1 + z_2 - z_3$ (c) $z_1 + z_2 - z_3$ (d) $z_1 - z_2 - z_3$
- 55) If $x_r = \cos\left(\frac{\pi}{2^r}\right) + i \sin\left(\frac{\pi}{2^r}\right)$ then $x_1, x_2 \dots x_\infty$ is _____
 (a) $-\infty$ (b) -2 (c) -1 (d) 0

Match the following

- 56) $\operatorname{Re}(z)$ (1) $z = -\bar{z}$
 57) $\operatorname{Im}(z)$ (2) 2
 58) z is real (3) 2
 59) z is imaginary (4) $\frac{z - \bar{z}}{2}$
 60) $|z|$ (5) $|z_1| |z_2|$
 61) $|z_1 + z_2|$ (6) $2n\pi - \frac{\pi}{2}$
 62) $|z_1 - z_2|$ (7) $\geq |z_1| + |z_2|$

- 63) $|z_1 z_2|$
 64) $\arg(0)$
 65) $\arg(i)$
 66) $\arg(z^n)$
 67) $\arg(z)$
 68) $|\sqrt{3} + i|$
 69) $|\sqrt{3} - i|$
- (8) $n \arg(z)$
 (9) $\arg z + 2n\pi, n \in \mathbb{Z}$
 (10) $|\bar{z}|$
 (11) $\frac{z+\bar{z}}{2}$
 (12) $2n\pi$
 (13) $z = \bar{z}$
 (14) $|z_1| + |z_2|$

Odd one out

70) $i^{-1} =$

- (i) $\frac{1}{i}$
 (ii) i
 (iii) $-i$
 (4) $\frac{1}{i^2}$

71) When $z = x+iy$, then iz is

- (1) $x-iy$
 (2) $i(x+iy)$
 (3) $-y+ix$
 (4) Rotation of z by 90° in the counter clockwise direction

72) $(1+3i)(1-3i)$

- (i) $(1)^2 - (3i)^2$
 (2) $1 + 9$
 (3) 10
 (4) -8

73) If $z = x+iy$, then $z \bar{z} =$

- (i) $(x+iy)(x-iy)$
 (2) $|z|^2$
 (3) $x^2 + y^2$
 (4) $|z|$

74) The principle argument of a complex number.

- (1) $\theta = \alpha$
 (2) $\theta = -\alpha$
 (3) $\frac{\pi}{2} - \alpha$
 (4) $\theta = \alpha - \pi$

75) Application of De Moivre's theorem.

- (1) $(\sin \theta + i \cos \theta)^n = \sin n\theta + i \cos n\theta$
 (2) $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$
 (3) $(\cos \theta + i \sin \theta)^{-n} = \cos n\theta - i \sin n\theta$
 (4) $(\cos \theta - i \sin \theta)^{-n} = \cos n\theta + i \sin n\theta$

Complex Numbers TEST 2

12th Standard

Maths

Exam Time : 01:00:00 Hrs

Total Marks : 60

30 x 2 = 60

- 1) Simplify the following
 $i^{1947} + i^{1950}$
- 2) If $z_1 = 1 - 3i$, $z_2 = 4i$, and $z_3 = 5$, show that $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$
- 3) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$
- 4) Find z^{-1} , if $z = (2+3i)(1-i)$.
- 5) If $z = x + iy$, find the following in rectangular form.
 $Re\left(\frac{1}{z}\right)$
- 6) Which one of the points i , $-2+i$, and 3 is farthest from the origin?
- 7) Find the modulus of the following complex numbers
 $\frac{2i}{3+4i}$
- 8) Find the square roots of $4+3i$
- 9) Show that the following equations represent a circle, and, find its centre and radius
 $|z - 2 - i| = 3$
- 10) Write in polar form of the following complex numbers
 $2 + i2\sqrt{3}$
- 11) Find the value of $\sum_{k=1}^8 \left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$.
- 12) Show that $|z+2-i| < 2$ represents interior points of a circle. Find its centre and radius.
- 13) Represent the complex number $-1-i$
- 14) Simplify the following
 $i^{1948} - i^{1869}$
- 15) Simplify the following
 $i^{59} + \frac{1}{i^{59}}$
- 16) Simplify the following
 $\sum_{n=1}^{10} i^{n+50}$
- 17) Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$
 $2z + 3w$
- 18) If $z_1 = 1 - 3i$, $z_2 = 4i$, and $z_3 = 5$, show that $(z_1 + z_2)z_3 = z_1(z_2 z_3)$
- 19) Write the following in the rectangular form:
 $\overline{3i} + \frac{1}{2-i}$

- 20) If $z=x+iy$, find the following in rectangular form.
 $\text{Im}(3z+4\bar{z}-4i)$
- 21) Find the modulus of the following complex numbers
 $(1-i)^{10}$
- 22) Find the square roots of $-6+8i$
- 23) Obtain the Cartesian form of the locus of $z=x+iy$ in
 $\bar{z} = 2^{-1}$
- 24) Show that the following equations represent a circle, and, find its centre and radius
 $|3z-6+12i|=8$
- 25) Write in polar form of the following complex numbers
 $-2-i2$
- 26) Simplify the following:
 $\sum_{n=1}^{102} i^n$
- 27) Find the following $\left| \overline{(1+i)}(2+3i)(4i-3) \right|$
- 28) Find the modulus and principal argument of the following complex numbers.
 $-\sqrt{3} + i$
- 29) Find the modulus and principal argument of the following complex numbers.
 $\sqrt{3}-i$
- 30) If z_1 and z_2 are $1-i$, $-2+4i$ then find $\text{Im}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$.

Complex Numbers TEST 3

12th Standard

Maths

Exam Time : 01:00:00 Hrs

Total Marks : 60

30 x 2 = 60

- 1) Evaluate the following if $z=5-2i$ and $w= -1+3i$
 $z+w$
- 2) Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find its real and imaginary parts.
- 3) If $z_1=3-2i$ and $z_2=6+4i$, find $\frac{z_1}{z_2}$
- 4) Write the following in the rectangular form:
 $(5+9i) + (2-4i)$
- 5) Find the following $\left| \frac{2+i}{-1+2i} \right|$
- 6) Find the square root of $6-8i$.
- 7) If the area of the triangle formed by the vertices z, iz and $z+iz$ is 50 square units, find the value of $|z|$
- 8) Obtain the Cartesian form of the locus of $z=x+iy$ in
 $[Re(iz)]^2 = 3$
- 9) Obtain the Cartesian equation for the locus of $z=x+iy$ in
 $|z-4|=16$
- 10) If $\omega \neq 1$ is a cube root of unity, then show that
 $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = 1$
- 11) Show that $|3z-5+i|=4$ represents a circle, and, find its centre and radius.
- 12) Find the modulus and principal argument of the following complex numbers.
 $\sqrt{3}+i$.
- 13) Find the principal argument $\text{Arg } z$, when $z = \frac{-2}{1+i\sqrt{3}}$
- 14) Simplify the following
 $\sum_{n=1}^{12} i^n$
- 15) Simplify the following
 $i \cdot i^2 \cdot i^3 \dots i^{2000}$
- 16) Evaluate the following if $z=5-2i$ and $w= -1+3i$
 $z-iw$
- 17) Evaluate the following if $z=5-2i$ and $w= -1+3i$
 $z w$

18) Write the following in the rectangular form:

$$\frac{10 - 5i}{6 + 2i}$$

19) If $z = x + iy$, find the following in rectangular form.

$$\operatorname{Re}(iz)$$

20) Find the modulus of the following complex number $\frac{2 - i}{1 + i} + \frac{1 - 2i}{1 - i}$

21) Find the modulus of the following complex numbers

$$2i(3 - 4i)(4 - 3i).$$

22) Find the square roots of

$$-5 - 12i.$$

23) Show that the following equations represent a circle, and, find its centre and radius

$$|2z + 2 - 4i| = 2$$

24) Write in polar form of the following complex numbers

$$3 - i\sqrt{3}$$

25) Simplify the following:

$$i^{-1924} + i^{2018}$$

26) Simplify the following:

$$i i^2 i^3 \dots i^{40}$$

27) Find the following $\left| \frac{i(2 + i)^3}{(1 + i)^2} \right|$

28) Find the modulus and principal argument of the following complex numbers:

$$-\sqrt{3} - i$$

29) Represent the complex number $1 + i\sqrt{3}$ in polar form.

30) If 1, ω , ω^2 are the cube roots of unity show that $(1 + \omega^2)^3 - (1 + \omega)^3 = 0$

Complex Numbers TEST 4

12th Standard

Maths

Exam Time : 01:00:00 Hrs

Total Marks : 60

20 x 3 = 60

- 1) Simplify the following i^7
- 2) Given the complex number $z=2+3i$, represent the complex numbers in Argand diagram z , iz , and $z+iz$
- 3) If $z_1=3, z_2=-7i$, and $z_3=5+4i$, show that $z_1(z_2+z_3)=z_1z_2+z_1z_3$
- 4) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z
- 5) The complex numbers u, v , and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If $v=3-4i$ and $w=4+3i$, find u in rectangular form.
- 6) Find the least value of the positive integer n for which $(\sqrt{3} + i)^n$ real
- 7) If $z_1=3+4i, z_2=5-12i$, and $z_3=6+8i$, find $|z_1|, |z_2|, |z_3|, |z_1+z_2|, |z_2-z_3|$, and $|z_1+z_3|$
- 8) If $|z|=2$ show that $3 \leq |z+3+4i| \leq 7$
- 9) Show that the equation $z^2 = \bar{z}$ has four solutions.
- 10) Which one of the points $10-8i$, $11+6i$ is closest to $1+i$.
- 11) If $|z|=1$, show that $2 \leq |z^2-3| \leq 4$
- 12) Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions
- 13) Find the rectangular form of the complex numbers $\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right) \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$
- 14) If $\frac{1+z}{1-z} = \cos 2\theta + i \sin 2\theta$, show that $z = i \tan \theta$
- 15) Find the value of $\left(\frac{1 + \sin \frac{\pi}{10} + i \cos \frac{\pi}{10}}{1 + \sin \frac{\pi}{10} - i \cos \frac{\pi}{10}} \right)^{10}$
- 16) If $\omega \neq 1$ is a cube root of unity, show that $(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$
- 17) Obtain the Cartesian form of the locus of z in $|z|=|z-i|$
- 18) Find the quotient $\frac{2 \left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)}{4 \left(\cos \left(\frac{-3\pi}{2} \right) + i \sin \left(\frac{-3\pi}{2} \right) \right)}$ in rectangular form
- 19) Find the cube roots of unity.

20) If $z = (\cos\theta + i\sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$

Complex Numbers TEST 5

12th Standard

Maths

Exam Time : 01:00:00 Hrs

Total Marks : 60

20 x 3 = 60

- 1) Find the value of the real numbers x and y , if the complex number $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal
- 2) Find the values of the real numbers x and y , if the complex numbers $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal.
- 3) If $z_1=2+5i$, $z_2=-3-4i$, and $z_3=1+i$, find the additive and multiplicative inverse of z_1, z_2 and z_3
- 4) If $z_1=2-i$ and $z_2=-4+3i$, find the inverse of $z_1 z_2$ and $\frac{z_1}{z_2}$
- 5) Prove the following properties z is real if and only if $z = \bar{z}$
- 6) Show that $(2 + i\sqrt{3})^{10} - (2 - i\sqrt{3})^{10}$ is purely imaginary
- 7) If z_1, z_2 and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$ find the value of $\left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right|$
- 8) Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.
- 9) For any two complex number z_1 and z_2 such that $|z_1|=|z_2|=1$ and $z_1 z_2 \neq 1$, then show that $\frac{z_1 + z_2}{1 + z_1 z_2}$ is real number.
- 10) If $|z|=3$, show that $7 \leq |z + 6 - 8i| \leq 13$.
- 11) If $\left| z - \frac{2}{z} \right| = 2$ show that the greatest and least value of $|z|$ are $\sqrt{3} + 1$ and $\sqrt{3} - 1$ respectively.
- 12) If $z=x+iy$ is a complex number such that $\left| \frac{z-4i}{z+4i} \right| = 1$ show that the locus of z is real axis.
- 13) If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\dots(x_n+iy_n)=a+ib$, show that
 - i) $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\dots(x_n^2+y_n^2)=a^2+b^2$
 - ii) $\sum_{r=1}^n \tan^{-1} \left(\frac{y_r}{x_r} \right) = \tan^{-1} \left(\frac{b}{a} \right) + 2k\pi, k \in \mathbb{Z}$
- 14) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then show that
 - (i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$
 - (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$
- 15) If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1-2\omega, 1-2\omega^2$.

16) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta = \frac{\pi}{3}$.

17) Find the product $\frac{3}{2} \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \cdot 6 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$ in rectangular form

18) Find the fourth roots of unity.

19) Simplify $\left(\frac{1 + \cos 2\theta + i \sin 2\theta}{1 + \cos 2\theta - i \sin 2\theta} \right)^{30}$

20) Simplify $\left(\sin \frac{\pi}{6} + i \cos \frac{\pi}{6} \right)^{18}$

Complex Numbers TEST 6

12th Standard

Maths

Exam Time : 01:15:00 Hrs

Total Marks : 75

25 x 3 = 75

- 1) Evaluate the following if $z=5-2i$ and $w= -1+3i$
 $z^2+2zw+w^2$
- 2) Evaluate the following if $z=5-2i$ and $w= -1+3i$
 $(z+w)^2$
- 3) Given the complex number $z=2+3i$, represent the complex numbers in Argand diagram
 z , $-iz$, and $z-iz$
- 4) If $z_1=3, z_2=-7i$, and $z_3=5+4i$, show that
 $(z_1+z_2)z_3=z_1z_3+z_2z_3$
- 5) Prove the following properties

$$\operatorname{Re}(z) = \frac{z + \bar{z}}{2} \text{ and } \operatorname{Im}(z) = \frac{z - \bar{z}}{2i}$$

- 6) Find the least value of the positive integer n for which $(\sqrt{3} + i)^n$ purely imaginary

- 7) Show that $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real

- 8) Obtain the Cartesian form of the locus of $z=x+iy$ in
 $\operatorname{Im}[(1-i)z+1]=0$

- 9) Obtain the Cartesian form of the locus of $z=x+iy$ in
 $|z+i|=|z-1|$

- 10) Obtain the Cartesian equation for the locus of $z=x+iy$ in
 $|z-4|^2-|z-1|^2=16$

- 11) Write in polar form of the following complex numbers

$$\frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$$

- 12) Find the rectangular form of the complex numbers

$$\frac{\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}}{2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}$$

- 13) If $\omega \neq 1$ is a cube root of unity, show that

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\dots(1+\omega^{2^{11}})=11.$$

- 14) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta = \frac{2\pi}{3}$.

- 15) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta = \frac{3\pi}{2}$.
- 16) Simplify the following:
 i^{1729}
- 17) Obtain the Cartesian form of the locus of z in
 $|2z-3-i|=3$
- 18) Given the complex number $z=3+2i$, represent the complex numbers z , iz , and $z+iz$ in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.
- 19) Show that $\left| \frac{z-3}{z+3} \right| = 2$ represent a circle.
- 20) Show that the complex numbers $3 + 2i$, $5i$, $-3 + 2i$ and $-i$ form a square.
- 21) If the complex number $2 + i$ and $1-2i$ are equidistant from $x + iy$ then show that $x+3y = 0$.
- 22) Find the locus of z if $\operatorname{Re}\left(\frac{z+1}{z-i}\right) = 0$ where $z=x+iy$.
- 23) Find the locus of Z if $|3z - 5| = 3|z + 1|$ where $z=x+iy$.
- 24) Find the locus of z if $\operatorname{Re}\left(\frac{\bar{z}+1}{\bar{z}-i}\right) = 0$.
- 25) If $\frac{(a+i)^2}{2a-i} = p+iq$, show that $p^2+q^2 = \frac{(a^2+i)^2}{4a^2+1}$.

Complex Numbers TEST 7

12th Standard

Maths

Exam Time : 02:00:00 Hrs

Total Marks : 100

20 x 5 = 100

- 1) Show that $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real ii) $\left(\frac{19 + 9i}{5 - 3i}\right)^{15} - \left(\frac{8 + i}{1 + 2i}\right)^{15}$ is purely imaginary.
- 2) Let z_1, z_2 , and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$ prove that $\left| \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3} \right| = r$
- 3) If z_1, z_2 , and z_3 are three complex numbers such that $|z_1|=1, |z_2|=2, |z_3|=3$ and $|z_1 + z_2 + z_3|=1$, show that $|9z_1 z_2 + 4z_1 z_2 + z_2 z_3|=6$
- 4) If $z=x+iy$ is a complex number such that $\operatorname{Im} \left(\frac{2z+1}{iz+1} \right) = 0$ show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$
- 5) If $z=x+iy$ and $\arg \left(\frac{z-i}{z+2} \right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$
- 6) Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2} \right)^5 = -\sqrt{3}$
- 7) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that
 - i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$.
 - ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$
 - iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$
 - iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$
- 8) Solve the equation $z^3 + 27 = 0$.
- 9) Prove that the values of $\sqrt[4]{-1}$ are $\pm \frac{1}{\sqrt{2}}(1 \pm i)$. Let $z = (-1)$
- 10) If $z=x+iy$ and $\arg \left(\frac{z-1}{z+1} \right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$.
- 11) Suppose z_1, z_2 , and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1 = 1 + i\sqrt{3}$ then find z_2 and z_3

- 12) Find all cube roots of $\sqrt{3} + i$
- 13) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$
- 14) Simplify: $(1+i)^{18}$
- 15) Simplify: $(-\sqrt{3} + 3i)^{31}$
- 16) If $1, \omega, \omega^2$ are the cube roots of unity then show that $(1+5\omega^2+\omega^4)(1+5\omega+\omega^2)(5+\omega+\omega^5) = 64$
- 17) Show that $\left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{2\omega} + \left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{2\omega} = -1$
- 18) Verify that $\arg(1+i) + \arg(1-i) = \arg[(1+i)(1-i)]$
- 19) Find all the roots $(2 - 2i)^{\frac{1}{3}}$ and also find the product of its roots.
- 20) Find the radius and centre of the circle $z\bar{z} - (2+3i)z - (2-3i)\bar{z} + 9 = 0$ where z is a complex number.

Complex Numbers TEST 8

12th Standard

Maths

Exam Time : 02:00:00 Hrs

Total Marks : 75

15 x 1 = 15

1) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

- (a) 0 (b) 1 (c) -1 (d) i

2) The area of the triangle formed by the complex numbers z , iz , and $z+iz$ in the Argand's diagram is

- (a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

3) If $z = \frac{(\sqrt{3} + i)^3 (3i + 4)^2}{(8 + 6i)^2}$, then $|z|$ is equal to

- (a) 0 (b) 1 (c) 2 (d) 3

4) If $|z-2+i| \leq 2$, then the greatest value of $|z|$ is

- (a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$ (c) $\sqrt{5} - 2$ (d) $\sqrt{5} + 2$

5) If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

- (a) z (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

6) If $|z_1|=1, |z_2|=2, |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is

- (a) 1 (b) 2 (c) 3 (d) 4

7) z_1, z_2 and z_3 are complex number such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$ then $z_1^2+z_2^2+z_3^2$ is

- (a) 3 (b) 2 (c) 1 (d) 0

8) If $z=x+iy$ is a complex number such that $|z+2|=|z-2|$, then the locus of z is

- (a) real axis (b) imaginary axis (c) ellipse (d) circle

9) The principal argument of $(\sin 40^\circ + i \cos 40^\circ)^5$ is

- (a) -110° (b) -70° (c) 70° (d) 110°

10) If $\omega \neq 1$ is a cubic root of unity and $(1 + \omega)^7 = A + B\omega$, then (A,B) equals

- (a) (1,0) (b) (-1,1) (c) (0,1) (d) (1,1)

11) If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020} + \beta^{2020}$ is

- (a) -2 (b) -1 (c) 1 (d) 2

12) If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^2 \end{vmatrix} = 3k$, then k is equal to

- (a) 1 (b) -1 (c) $\sqrt{3}i$ (d) $-\sqrt{3}i$

13) If $\omega = cis \frac{2\pi}{3}$, then the number of distinct roots of

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$$

(a) 1 (b) 2 (c) 3 (d) 4

14) The value of $(1+i)(1+i^2)(1+i^3)(1+i^4)$ is

(a) 2 (b) 0 (c) 1 (d) i

15) If, $i^2 = -1$, then $i^1 + i^2 + i^3 + \dots$ up to 1000 terms is equal to

(a) 1 (b) -1 (c) i (d) 0

7 x 2 = 14

16) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

17) Find the following $\left|\frac{2+i}{-1+2i}\right|$

18) Find the square root of $6-8i$.

19) Show that the following equations represent a circle, and, find its centre and radius

$$|z - 2 - i| = 3$$

20) Show that $|3z-5+i|=4$ represents a circle, and, find its centre and radius.

21) Find the principal argument $\text{Arg } z$, when $z = \frac{-2}{1+i\sqrt{3}}$

22) Find the modulus of the following complex number $\frac{2-i}{1+i} + \frac{1-2i}{1-i}$

7 x 3 = 21

23) If $z_1=3, z_2=-7i$, and $z_3=5+4i$, show that $z_1(z_2+z_3)=z_1z_2+z_1z_3$

24) The complex numbers u, v , and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If $v=3-4i$ and

$w=4+3i$, find u in rectangular form.

25) Show that $(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary

26) For any two complex number z_1 and z_2 such that $|z_1|=|z_2|=1$ and $z_1z_2 \neq 1$, then show

that $\frac{z_1+z_2}{1+z_1z_2}$ is real number.

27) If $\left|z - \frac{2}{z}\right| = 2$ show that the greatest and least value of $|z|$ are $\sqrt{3}+1$ and $\sqrt{3}-1$ respectively.

28) If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\dots(x_n+iy_n) = a+ib$, show that

i) $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\dots(x_n^2+y_n^2) = a^2+b^2$

ii) $\sum_{r=1}^n \tan^{-1} \left(\frac{y_r}{x_r} \right) = \tan^{-1} \left(\frac{b}{a} \right) + 2k\pi, k \in \mathbb{Z}$

29) Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

$5 \times 5 = 25$

30) Show that $(2 + i\sqrt{3})^{10} + (2 - i\sqrt{3})^{10}$ is real ii)

$\left(\frac{19 + 9i}{5 - 3i}\right)^{15} - \left(\frac{8 + i}{1 + 2i}\right)^{15}$ is purely imaginary.

31) Let z_1, z_2 , and z_3 be complex numbers such that

$|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$ prove that

$\left| \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3} \right| = r$

32) If z_1, z_2 , and z_3 are three complex numbers such that $|z_1|=1, |z_2|=2, |z_3|=3$ and $|z_1 + z_2 + z_3|=1$, show that $|9z_1 z_2 + 4z_1 z_2 + z_2 z_3|=6$

33) If $z=x+iy$ is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus

of z is $2x^2 + 2y^2 + x - 2y = 0$

34) If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$

Complex Numbers TEST 9

12th Standard

Maths

Exam Time : 02:00:00 Hrs

Total Marks : 75

15 x 1 = 15

- 1) The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is
 (a) $1+i$ (b) i (c) 1 (d) 0
- 2) The conjugate of a complex number is $\frac{1}{i-2}$ /Then the complex number is
 (a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$
- 3) If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
- 4) If $\left|z - \frac{3}{z}\right| = 2$ then the least value $|z|$ is
 (a) 1 (b) 2 (c) 3 (d) 5
- 5) The solution of the equation $|z|-z=1+2i$ is
 (a) $\frac{3}{2} - 2i$ (b) $-\frac{3}{2} + 2i$ (c) $2 - \frac{3}{2}i$ (d) $2 + \frac{3}{2}i$
- 6) If z is a complex number such that $z \in C/R$ and $z + \frac{1}{z} \in R$ then $|z|$ is
 (a) 0 (b) 1 (c) 2 (d) 3
- 7) If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3
- 8) The principal argument of $\frac{3}{-1+i}$
 (a) $\frac{-5\pi}{6}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{-3\pi}{4}$ (d) $\frac{-\pi}{2}$
- 9) If $(1+i)(1+2i)(1+3i)\dots(1+ni)=x+iy$, then $2 \cdot 5 \cdot 10 \dots (1+n^2)$ is
 (a) 1 (b) i (c) x^2+y^2 (d) $1+n^2$
- 10) The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is
 (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

- 11) The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is
- (a) -2 (b) -1 (c) 1 (d) 2
- 12) The value of $\left(\frac{1 + 3\sqrt{i}}{1 - \sqrt{3i}}\right)^{10}$
- (a) $\text{cis}\frac{2\pi}{3}$ (b) $\text{cis}\frac{4\pi}{3}$ (c) $-\text{cis}\frac{2\pi}{3}$ (d) $-\text{cis}\frac{4\pi}{3}$
- 13) If $\sqrt{a+ib} = x+iy$, then possible value of $\sqrt{a-ib}$ is
- (a) x^2+y^2 (b) $\sqrt{x^2+y^2}$ (c) $x+iy$ (d) $x-iy$
- 14) If $z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{6}$, then
- (a) $|z|=1, \arg(z) = \frac{\pi}{4}$ (b) $|z|=1, \arg(z) = \frac{\pi}{6}$ (c) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \frac{5\pi}{24}$ (d) $|z| = \frac{\sqrt{3}}{2}, \arg(z) = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$
- 15) The principal value of the amplitude of $(1+i)$ is
- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{12}$ (c) $\frac{3\pi}{4}$ (d) π
- 16) If $z_1=1-3i, z_2=4i$, and $z_3 = 5$, show that $(z_1+z_2)+z_3=z_1+(z_2+z_3)$
- 17) If $z_1=3-2i$ and $z_2=6+4i$, find $\frac{z_1}{z_2}$
- 18) Which one of the points $i, -2+i$, and 3 is farthest from the origin?
- 19) Find the square roots of $4+3i$
- 20) Obtain the Cartesian equation for the locus of $z=x+iy$ in $|z-4|=16$
- 21) If $\omega \neq 1$ is a cube root of unity, then show that
- $$\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = 1$$
- 22) Simplify the following $i \cdot 2i^3 \dots i^{2000}$
- 23) Find the values of the real numbers x and y , if the complex numbers $(3-i)x - (2-i)y + 2i + 5$ and $2x + (-1+2i)y + 3 + 2i$ are equal.
- 24) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z
- 25) If $z_1=3+4i, z_2=5-12i$, and $z_3 = 6+8$, find $|z_1|, |z_2|, |z_3|, |z_1+z_2|, |z_2-z_3|$, and $|z_1+z_3|$
- 26) If z_1, z_2 and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$ find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$
- 27) If $|z|=2$ show that $3 \leq |z+3+4i| \leq 7$
- 28) Which one of the points $10-8i, 11+6i$ is closest to $1+i$.
- 29) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then show that
- (i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

$$7 \times 2 = 14$$

$$7 \times 3 = 21$$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

5 x 5 = 25

30) Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$

31) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$.

ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

32) Solve the equation $z^3 + 27 = 0$.

33) Prove that the values of $\sqrt[4]{-1}$ are $\pm \frac{1}{\sqrt{2}}(1 \pm i)$. Let $z = (-1)$

34) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2 + y^2 = 1$.

Complex Numbers TEST 10

12th Standard

Maths

Exam Time : 02:30:00 Hrs

Total Marks : 100

20 x 1 = 20

- 1) If $a = 1+i$, then a^2 equals
 (a) $1-i$ (b) $2i$ (c) $(1+i)(1-i)$ (d) $i-1$
- 2) If $z = \frac{1}{(2+3i)^2}$ then $|z| =$
 (a) $\frac{1}{13}$ (b) $\frac{1}{5}$ (c) $\frac{1}{12}$ (d) none of these
- 3) If $z = 1 - \cos\theta + i \sin\theta$, then $|z| =$
 (a) $2 \sin \frac{\theta}{2}$ (b) $2 \cos \frac{\theta}{2}$ (c) $2|\sin \frac{\theta}{2}|$ (d) $2|\cos \frac{\theta}{2}|$
- 4) If $z = \frac{1}{1 - \cos\theta - i \sin\theta}$, the $\operatorname{Re}(z) =$
 (a) 0 (b) $\frac{1}{2}$ (c) $\cot \frac{\theta}{2}$ (d) $\frac{1}{2} \cot \frac{\theta}{2}$
- 5) If $x+iy = \frac{3+5i}{7-6i}$, they $y =$
 (a) $\frac{9}{85}$ (b) $-\frac{9}{85}$ (c) $\frac{53}{85}$ (d) none of these
- 6) The amplitude of $\frac{1}{i}$ is equal to
 (a) 0 (b) $\frac{\pi}{2}$ (c) $-\frac{\pi}{2}$ (d) π
- 7) The value of $(1+i)^4 + (1-i)^4$ is
 (a) 8 (b) 4 (c) -8 (d) -4
- 8) The complex number z which satisfies the condition $\left| \frac{1+z}{1-z} \right| = 1$ lies on
 (a) circle $x^2+y^2=1$ (b) x -axis (c) y -axis (d) the lines $x+y=1$
- 9) If $z = a + ib$ lies in quadrant then $\frac{\bar{z}}{z}$ also lies in the III quadrant if
 (a) $a > b > 0$ (b) $a < b < 0$ (c) $b < a < 0$ (d) $b > a > 0$
- 10) $\frac{1+e^{-i\theta}}{1+e^{i\theta}} =$
 (a) $\cos\theta + i \sin\theta$ (b) $\cos\theta - i \sin\theta$ (c) $\sin\theta - i \cos\theta$ (d) $\sin\theta + i \cos\theta$
- 11) If $z^n = \cos \frac{n\pi}{3} + i \sin \frac{n\pi}{3}$, then $z_1, z_2, \dots, z_6$ is
 (a) 1 (b) -1 (c) i (d) $-i$
- 12) If $x = \cos\theta + i \sin\theta$, then the value of $x^n + \frac{1}{x^n}$ is
 (a) $2 \cos\theta$ (b) $2i \sin n\theta$ (c) $2i \sin n\theta$ (d) $2i \cos n\theta$
- 13) If ω is the cube root of unity, then the value of $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$ is
 (a) 9 (b) -9 (c) 16 (d) 32
- 14) The points represented by $3 - 3i$, $4 - 2i$, $3 - i$ and $2 - 2i$ form _____ in the argand plane.
 (a) collinear points (b) Vertices of a parallelogram (c) Vertices of a rectangle (d) Vertices of a square
- 15) $(1+i)^3 =$ _____
 (a) $3 + 3i$ (b) $1 + 3i$ (c) $3 - 3i$ (d) $2i - 2$
- 16) $\frac{(\cos\theta + i \sin\theta)^6}{(\cos\theta - i \sin\theta)^5} =$ _____
 (a) $\cos 11\theta - i \sin 11\theta$ (b) $\cos 11\theta + i \sin 11\theta$ (c) $\cos\theta + i \sin\theta$ (d) $\cos \frac{6\theta}{5} + i \sin \frac{6\theta}{5}$

17) If $a = \cos\alpha + i \sin\alpha$, $b = -\cos\beta + i \sin\beta$ then $\left(ab - \frac{1}{ab}\right)$ is _____
 (a) $-2i \sin(\alpha - \beta)$ (b) $2i \sin(\alpha - \beta)$ (c) $2 \cos(\alpha - \beta)$ (d) $-2 \cos(\alpha - \beta)$

18) The conjugate of $\frac{1+2i}{1-(1-i)^2}$ is _____
 (a) $\frac{1+2i}{1-(1-i)^2}$ (b) $\frac{5}{1-(1-i)^2}$ (c) $\frac{1-2i}{1+(1+i)^2}$ (d) $\frac{1+2i}{1+(1-i)^2}$

19) The modular of $\frac{(-1+i)(1-i)}{1+i\sqrt{3}}$ is _____
 (a) $\sqrt{2}$ (b) 2 (c) 1 (d) $\frac{1}{2}$

20) If $x = \cos\theta + i \sin\theta$, then $x^n + \frac{1}{x^n}$ is _____
 (a) $2 \cos n\theta$ (b) $2i \sin n\theta$ (c) $2^n \cos\theta$ (d) $2^n i \sin\theta$

10 x 2 = 20

21) Simplify the following

$$\sum_{n=1}^{10} i^{n+50}.$$

22) Write the following in the rectangular form:

$$\overline{3i} + \frac{1}{2-i}.$$

23) If $z = x + iy$, find the following in rectangular form.

$$\operatorname{Im}(3z + 4\bar{z} - 4i)$$

24) Find the square roots of

$$-5 - 12i.$$

25) Show that the following equations represent a circle, and, find its centre and radius

$$|3z - 6 + 12i| = 8$$

26) Simplify the following:

$$\sum_{n=1}^{102} i^n$$

27) Find the following $\left| \overline{(1+i)}(2+3i)(4i-3) \right|$

28) Find the following $\left| \frac{i(2+i)^3}{(1+i)^2} \right|$

29) Find the modulus and principal argument of the following complex numbers:

$$-\sqrt{3} - i$$

30) Represent the complex number $1 + i\sqrt{3}$ in polar form.

10 x 3 = 30

31) Find the cube roots of unity.

32) If $z = (\cos\theta + i \sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and

$$z^n - \frac{1}{z^n} = 2i \sin n\theta$$

33) Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$

$$z^2 + 2zw + w^2$$

34) If $z_1 = 3$, $z_2 = -7i$, and $z_3 = 5 + 4i$, show that

$$(z_1 + z_2)z_3 = z_1z_3 + z_2z_3$$

35) Show that $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real

36) Obtain the Cartesian form of the locus of $z=x+iy$ in $\text{Im}[(1-i)z+1]=0$

37) Obtain the Cartesian equation for the locus of $z=x+iy$ in $|z-4|^2 - |z-1|^2 = 16$

38) Find the rectangular form of the complex numbers

$$\frac{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$$

39) If $\omega \neq 1$ is a cube root of unity, show that

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\dots(1+\omega^{2^{11}}) = 11$$

40) Obtain the Cartesian form of the locus of z in $|2z-3-i|=3$

$$6 \times 5 = 30$$

41) Suppose z_1, z_2 , and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$ then find z_2 and z_3

42) Find all cube roots of $\sqrt{3} + i$

43) Solve the equation $z^3+8i=0$, where $z \in \mathbb{C}$

44) Simplify: $(1+i)^{18}$

45) Simplify: $(-\sqrt{3} + 3i)^{31}$

46) Show that $\left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{2\omega} + \left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{2\omega} = -1$

COMPLEX NUMBER FULL CHAPTER TEST

12th Standard

Maths

Reg.No. :

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Exam Time : 03:00:00 Hrs

Total Marks : 90

ANSWER ALL

20 x 1 = 20

1) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(a) 0

(b) 1

(c) -1

(d) i

2)

The conjugate of a complex number is $\frac{1}{i-2}$ /Then the complex number is(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$ 3) If $|z-2+i| \leq 2$, then the greatest value of $|z|$ is(a) $\sqrt{3}-2$ (b) $\sqrt{3}+2$ (c) $\sqrt{5}-2$ (d) $\sqrt{5}+2$

4)

If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) z

(b) $\bar{z}$ (c) $\frac{1}{z}$

(d) 1

5) If $|z_1|=1, |z_2|=2, |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is

(a) 1

(b) 2

(c) 3

(d) 4

6)

If z is a complex number such that $z \in C/R$ and $z + \frac{1}{z} \in R$ then $|z|$ is

(a) 0

(b) 1

(c) 2

(d) 3

7)

If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 3

8)

The principal argument of $\frac{3}{-1+i}$ (a) $-\frac{5\pi}{6}$ (b) $-\frac{2\pi}{3}$ (c) $-\frac{3\pi}{4}$ (d) $-\frac{\pi}{2}$ 9) The principal argument of $(\sin 40^\circ + i \cos 40^\circ)^5$ is(a) -110° (b) -70° (c) 70° (d) 110°

10)

The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

- (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

11) If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020} + \beta^{2020}$ is

- (a) -2 (b) -1 (c) 1 (d) 2

12) The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

- (a) -2 (b) -1 (c) 1 (d) 2

13)

If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^2 \end{vmatrix} = 3k$, then k is equal to

- (a) 1 (b) -1 (c) $\sqrt{3}i$ (d) $-\sqrt{3}i$

14)

The value of $\left(\frac{1+3\sqrt{i}}{1-\sqrt{3}i}\right)^{10}$

- (a) $\frac{2\pi}{3} cis$ (b) $\frac{4\pi}{3} cis$ (c) $\frac{2\pi}{3} - cis$ (d) $\frac{4\pi}{3} - cis$

15)

If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}$

- (a) 1 (b) 2 (c) 3 (d) 4

16) The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is

- (a) $1+i$ (b) i (c) 1 (d) 0

17) The area of the triangle formed by the complex numbers z, iz , and $z+iz$ in the Argand's diagram is

- (a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

18)

If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

- (a) 0 (b) 1 (c) 2 (d) 3

19) If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 3

20) The solution of the equation $|z|-z=1+2i$ is

(a) $\frac{3}{2} - 2i$

(b) $-\frac{3}{2} + 2i$

(c) $2 - \frac{3}{2}i$

(d) $2 + \frac{3}{2}i$

ANSWER ANY 7

$7 \times 2 = 14$

21) If $z_1=2+5i$, $z_2=-3-4i$, and $z_3=1+i$, find the additive and multiplicate inverse of z_1, z_2 and z_3

22) If $z=x+iy$, find the following in rectangular form.

$$\operatorname{Re}\left(\frac{1}{z}\right)$$

23)

The complex numbers u, v , and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v=3-4i$ and $w=4+3i$, find u in rectangular

form.

24) If $|z|=3$, show that $7 \leq |z+6-8i| \leq 13$.

25) If the area of the triangle formed by the vertices z, iz and $z+iz$ is 50 square units, find the value of $|z|$

26) Find the square roots of $4+3i$

27) Show that the following equations represent a circle, and, find its centre and radius|

$$|z-2-i|=3$$

28) Write in polar form of the following complex numbers

$$2 + i2\sqrt{3}$$

29) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

30)

If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$.

ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

ANSWER ANY 7

$7 \times 3 = 21$

31) Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

32) If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z

33) Solve the equation $z^3+8i=0$, where $z \in \mathbb{C}$

34) Find the cube roots of unity.

35) Simplify $\left(\frac{1+\cos 2\theta + i\sin 2\theta}{1+\cos 2\theta - i\sin 2\theta}\right)^{30}$

36) If $z = (\cos\theta + i\sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$

37) Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

38) Simplify the following:
 $i \ i^2 \ i^3 \dots i^{40}$

39) Given the complex number $z=3+2i$, represent the complex numbers z, iz , and $z+iz$ in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.

40) If $\omega \neq 1$ is a cube root of unity, show that $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\dots(1+\omega^{2^{11}}) = 11$.

ANSWER ANY 7

$7 \times 5 = 35$

41) If z_1, z_2 and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$ find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

42) Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

43) Let z_1, z_2 , and z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1+z_2+z_3 \neq 0$ prove

that $\left|\frac{z_1z_2 + z_2z_3 + z_3z_1}{z_1 + z_2 + z_3}\right| = r$

44) Find the quotient $\frac{2\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form

45)

If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, then show that $x^2+y^2=1$.

46)

Find the following $\left| \frac{-(1+i)(2+3i)(4i-3)}{(1+i)^2} \right|$

47)

Find the following $\left| \frac{i(2+i)^3}{(1+i)^2} \right|$

48)

If $z=x+iy$ is a complex number such that $\operatorname{Im}\left(\frac{2z+1}{iz+1}\right) = 0$ show that the locus of z is $2x^2+2y^2+x-2y=0$

49)

Show that $(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real ii) $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

50) Solve the equation $z^3+27=0$.
