

## Applications of Integration

12th Standard

Maths

Exam Time : 00:40:00 Hrs

Total Marks : 40

40 x 1 = 40

- 1) The value of  $\int_{-4}^4 \left[ \tan^{-1} \left( \frac{x^2}{x^4+1} \right) + \tan^{-1} \left( \frac{x^4+1}{x^2} \right) \right] dx$  is  
 (a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$
- 2) The value of  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left( \frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x} \right) dx$  is  
 (a) 4 (b) 3 (c) 2 (d) 0
- 3) If  $f(x) = \int_0^x t \cos t \, dt$ , then  $\frac{dx}{dx}$   
 (a)  $\cos x - x \sin x$  (b)  $\sin x + x \cos x$  (c)  $x \cos x$  (d)  $x \sin x$
- 4) The area between  $y^2 x = 4$  and its latus rectum is  
 (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{5}{3}$
- 5) The value of  $\int_0^1 x(1-x)^{99} dx$  is  
 (a)  $\frac{1}{11000}$  (b)  $\frac{1}{10100}$  (c)  $\frac{1}{10010}$  (d)  $\frac{1}{10001}$
- 6) The value of  $\int_0^\pi \frac{dx}{1+5^{\cos x}}$  is  
 (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c)  $\frac{3\pi}{2}$  (d)  $2\pi$
- 7) The value of  $\frac{(n+2)}{(n)} = 90$  then n is  
 (a) 10 (b) 5 (c) 8 (d) 9
- 8) The value of  $\int_0^{\frac{\pi}{6}} \cos^3 3x \, dx$   
 (a)  $\frac{2}{3}$  (b)  $\frac{2}{9}$  (c)  $\frac{1}{9}$  (d)  $\frac{1}{3}$
- 9) The value of  $\int_0^\pi \sin^4 x \, dx$  is  
 (a)  $\frac{3\pi}{10}$  (b)  $\frac{3\pi}{8}$  (c)  $\frac{3\pi}{4}$  (d)  $\frac{3\pi}{2}$
- 10) The value of  $\int_0^\infty e^{-3x} x^2 \, dx$  is  
 (a)  $\frac{7}{27}$  (b)  $\frac{5}{27}$  (c)  $\frac{4}{27}$  (d)  $\frac{2}{27}$
- 11) If  $\int_a^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$  then a is  
 (a) 4 (b) 1 (c) 3 (d) 2
- 12) The volume of solid of revolution of the region bounded by  $y^2 = x(a-x)$  about x-axis is  
 (a)  $\pi a^2$  (b)  $\frac{\pi a^2}{4}$  (c)  $\frac{\pi a^2}{5}$  (d)  $\frac{\pi a^2}{6}$
- 13) If  $f(x)f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$  and  $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$ , then one of the possible value of a is  
 (a) 3 (b) 6 (c) 9 (d) 5
- 14) The value of  $\int_0^1 (\sin^{-1} x)^2 dx$   
 (a)  $\frac{\pi^2}{4} - 1$  (b)  $\frac{\pi^2}{4} + 2$  (c)  $\frac{\pi^2}{4} + 1$  (d)  $\frac{\pi^2}{4} - 2$

15) The value of  $\int_0^a (\sqrt{a^2 - x^2})^2 dx$

(a)  $\frac{\pi a^2}{16}$

(b)  $\frac{3\pi a^4}{16}$

(c)  $\frac{3\pi a^2}{8}$

(d)  $\frac{3\pi a^4}{8}$

16) If  $\int_0^x f(t)dt = x + \int_x^1 tf(t)dt$  then the value of  $f(1)$  is

(a)  $\frac{1}{2}$

(b) 2

(c) 1

(d)  $\frac{3}{4}$

17) The value of  $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$  is

(a)  $\frac{\pi}{6}$

(b)  $\frac{\pi}{2}$

(c)  $\frac{\pi}{4}$

(d)  $\pi$

18) The value of  $\int_{-1}^2 |x|dx$

(a)  $\frac{1}{2}$

(b)  $\frac{3}{2}$

(c)  $\frac{5}{2}$

(d)  $\frac{7}{2}$

19) For any value of  $n \in \mathbb{Z}$ ,  $\int_0^\pi e \cos^{2x} \cos^3[(2n+1)x] dx$  is

(a)  $\frac{\pi}{2}$

(b)  $\pi$

(c) 0

(d) 2

20) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x dx$  is

(a)  $\frac{3}{2}$

(b)  $\frac{1}{2}$

(c) 0

(d)  $\frac{2}{3}$

21) The value of  $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$

(a)  $\pi$

(b)  $\frac{\pi}{2}$

(c)  $\frac{\pi}{4}$

(d) 0

22) The value of  $\int_{\frac{\pi}{2}}^{\pi} \sqrt{\frac{1-\cos 2x}{2x}} dx$  is

(a)  $\frac{1}{2}$

(b) 2

(c) 0

(d) 1

23)  $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$  is

(a)  $\frac{\pi}{3}$

(b)  $\frac{\pi}{6}$

(c)  $\frac{\pi}{12}$

(d)  $-\frac{\pi}{6}$

24) If  $\int_0^{2a} f(x)dx = 2 \int_0^a f(x)dx$  then

(a)  $f(2a-x) = -f(x)$

(b)  $f(2a-x) = f(x)$

(c)  $f(x)$  is odd

(d)  $f(x)$  is even

25) The value of  $\int_{-\pi}^{\pi} \sin^3 x \cos^3 x dx$  is

(a) 0

(b)  $\pi$

(c)  $2\pi$

(d)  $4\pi$

26) The area enclosed by the curve  $y = \frac{x^2}{2}$ , the x-axis and the lines  $x=1$ ,  $x=3$  is

(a) 4

(b)  $8\frac{2}{3}$

(c) 13

(d)  $4\frac{1}{3}$

27) The area bounded by the parabola  $y = x^2$  and the line  $y = 2x$  is

(a)  $\frac{4}{3}$

(b)  $\frac{2}{3}$

(c)  $\frac{51}{3}$

(d)  $\frac{30}{3}$

28) The ratio of the volumes generated by revolving the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  about major and minor axes is

(a) 4:9

(b) 9:4

(c) 2:3

(d) 3:2

29)  $\int_0^\infty e^{-mx} x^7 dx$  is

(a)  $\frac{m}{7^m}$

(b)  $\frac{7}{m^7}$

(c)  $\frac{m}{7^{m+1}}$

(d)  $\frac{7}{m^8}$

30) If  $\int_0^a f(x)dx + \int_0^a f(2a-x)dx =$

(a)  $\int_0^a f(x)dx$

(b)  $2 \int_0^a f(x)dx$

(c)  $\int_0^{2a} f(x)dx$

(d)  $\int_0^{2a} f(a-x)dx$

31)  $\int_{-1}^1 x dx = \dots\dots\dots$

(a) -1

(b) 1

(c) 0

(d) 2

- 32)  $\int_0^{2a} f(x)dx = 2 \int_0^{2a} f(x)dx$   
 (a)  $f(2a-x) = f(x)$  (b)  $f(a-x) = f(x)$  (c)  $f(x) = -f(-x)$  (d)  $f(-x) = f(x)$
- 33) The area enclosed by the curve  $y^2 = 4x$ , the x-axis and its latus rectum is ..... sq.units.  
 (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{16}{3}$
- 34) The area of the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$   
 (a)  $6\pi$  (b)  $36\pi$  (c)  $6\pi^2$  (d)  $36\pi^2$
- 35) The volume generated by the curve  $y^2 = 16x$  from  $x = 2$  to  $x = 3$  rotating about x - axis ..... cu. units  
 (a)  $72\pi$  (b)  $\frac{256 \times 19}{3} \pi$  (c)  $40\pi$  (d)  $80\pi$
- 36)  $\int_a^b f(x)dx = \dots\dots\dots$   
 (a)  $2 \int_0^a f(x)dx$  (b)  $\int_a^b f(a-x)dx$  (c)  $\int_b^a f(b-x)dx$  (d)  $\int_a^b f(a+b-x)dx$
- 37)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x}{2+\cos x} dx =$   
 (a) 0 (b) 2 (c)  $\log 2$  (d)  $\log 4$
- 38)  $\int_0^{\frac{\pi}{4}} \cos^3 2x dx =$   
 (a)  $\frac{2}{3}$  (b)  $\frac{1}{3}$  (c) 0 (d)  $\frac{2\pi}{3}$
- 39)  $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx = \dots\dots\dots$   
 (a)  $\frac{\pi}{2}$  (b) 0 (c)  $\frac{\pi}{4}$  (d)  $\pi$
- 40) The volume when  $y = \sqrt{3+x^2}$  from  $x = 0$  to  $x = 4$  is rotated about x-axis is .....  
 (a)  $100\pi$  (b)  $\frac{100\pi}{9}$  (c)  $\frac{100\pi}{3}$  (d)  $\frac{100}{3}$

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Exam Time : 00:40:00 Hrs

Total Marks : 60

20 x 2 = 40

- 1) Find an approximate value of  $\int_1^{1.5} x dx$  by applying the left-end rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .
- 2) Find an approximate value of  $\int_1^{1.5} (2-x) dx$  by applying the mid-point rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .

- 3) Evaluate the following definite integrals:

$$\int_3^4 \frac{dx}{x^2-4}$$

- 4) Evaluate the following definite integrals:

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$$

- 5) Evaluate the following integrals using properties of integration:

$$\int_{-5}^5 x \cos \left( \frac{e^x-1}{e^x+1} \right) dx$$

- 6) Evaluate the following integrals using properties of integration:

$$\int_0^{2\pi} x \log \left( \frac{3+\cos x}{3-\cos x} \right) dx$$

- 7) Evaluate the following integrals using properties of integration:

$$\int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$$

- 8) Evaluate the following integrals using properties of integration:

$$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}} dx$$

- 9) Evaluate the following:

$$\int_0^1 \frac{\sin(3\tan^{-1}x) \tan^{-1}x}{1+x^2} dx$$

- 10) Evaluate the following:

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \frac{dx}{1+5\cos^2 x}$$

- 11) Evaluate the following

$$\int_0^{\pi/2} \cos^7 x \, dx$$

- 12) Evaluate the following

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \sin^2 x \cos^4 x dx$$

- 13) Evaluate the following

$$\int_0^1 x^2 (1-x)^3 dx$$

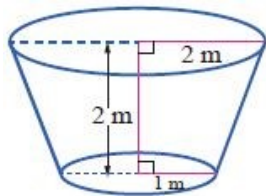
- 14) If  $\int_0^\infty e^{-ax^2} x^3 dx = 32$ ,  $a > 0$ , find  $a$

- 15) Find the area of the region bounded by the curve  $2+x-x^2+y=0$ , x-axis,  $x=-3$  and  $x=3$ .

- 16) Find the area of the region bounded by  $y = \tan x$ ,  $y = \cot x$  and the lines  $x=0$ ,  $x=\frac{\pi}{2}$ ,  $y=0$

- 17) The curve  $y = (x-2)^2+1$  has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ.

- 18) Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by  $y = e^{-2x}$ ,  $y = 0$ ,  $x = 0$  and  $x = 1$
- 19) The region enclosed between the graphs of  $y = x$  and  $y = x^2$  is denoted by R, Find the volume generated when R is rotated through  $360^\circ$  about x-axis.
- 20) Find, by integration, the volume of the container which is in the shape of a right circular conical frustum. 6. A watermelon has an ellipsoid shape which can be obtained by



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Exam Time : 01:15:00 Hrs

Total Marks : 50

25 x 2 = 50

- 1) Find an approximate value of  $\int_1^{1.5} x^2 dx$  by applying the right-end rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .

- 2) Evaluate the following integrals as the limits of sums.

$$\int_1^2 (4x^2 - 1) dx$$

- 3) Evaluate the following definite integrals:

$$\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$$

- 4) Evaluate the following definite integrals:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{\cos \theta} \sin^3 \theta d\theta$$

- 5) Evaluate the following integrals using properties of integration:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x \cos x + \tan^3 x + 1) dx$$

- 6) Evaluate the following integrals using properties of integration:

$$\int_0^1 |5x - 3| dx$$

- 7) Evaluate the following integrals using properties of integration:

$$\int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

- 8) Evaluate the following integrals using properties of integration:

$$\int_0^{\pi} \frac{x \sin x}{1 + \sin x} dx$$

- 9) Evaluate the following integrals using properties of integration:

$$\int_0^{\pi} x \left[ \sin^2(\sin x) + \cos^2(\cos x) \right] dx$$

- 10) Evaluate the following:

$$\int_0^1 x^3 e^{-2x} dx$$

- 11) Evaluate the following:

$$\int_{\frac{\pi}{6}}^{\frac{1}{e}} \frac{e^a \sin^{-1} x \sin^{-1} x}{\sqrt{1-x^2}} dx$$

- 12) Evaluate the following:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} x^2 \cos 2x dx$$

- 13) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 4 \sin^2 x}$

- 14) Evaluate the following

$$\int_0^{\pi/4} \sin^6 x dx$$

- 15) Evaluate the following

$$\int_0^{2\pi} \sin^7 \frac{x}{4} dx$$

- 16) Evaluate the following

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^3 \theta \cos^5 \theta d\theta$$

17) Evaluate the following:

$$\int_0^{\infty} x^5 e^{-3x} dx$$

18) Evaluate the following:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{e^{-\tan x}}{\cos^6 x} dx$$

19) Find the area of the region bounded by  $3x - 2y + 6 = 0$ ,  $x = -3$ ,  $x = 1$  and x-axis.

20) Find the area of the region bounded by the line  $y = 2x + 5$  and the parabola  $y = x^2 - 2x$ .

21) Find the area of the region bounded by the parabola  $y^2 = x$  and the line  $y = x - 2$

22) Father of a family wishes to divide his square field bounded by  $x = 0, x = 4, y = 4$  and  $y = 0$  along the curve  $y^2 = x$  and  $x^2 = y$  into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

23) Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by  $y = 2x^2$ ,  $y = 0$  and  $x = 1$ .

24) The region enclosed between the graphs of  $y = x$  and  $y = x^2$  is denoted by R, Find the volume generated when R is rotated through  $360^\circ$  about x-axis.

25) A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration.

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Exam Time : 01:30:00 Hrs

Total Marks : 75

25 x 3 = 75

- 1) Evaluate :  $\int_0^1 \frac{2x+7}{5x^2+9} dx$
- 2) Evaluate :  $\int_0^1 [2x] dx$  where  $[\cdot]$  is the greatest integer function
- 3) Evaluate :  $\int_0^{\frac{\pi}{2}} \frac{\sec x \tan x}{1+\sec^2 x} dx$
- 4) Evaluate :  $\int_0^9 \frac{1}{x+\sqrt{x}} dx$
- 5) Evaluate  $\int_1^2 \frac{x}{(x+1)(x+2)} dx$
- 6) Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{\cos \theta}{(1+\sin \theta)(2+\sin \theta)} d\theta$
- 7) Evaluate:  $\int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx$
- 8) Evaluate:  $\int_0^{1.5} [x^2] dx$  where  $[x]$  is the greatest integer function
- 9) Evaluate:  $\int_{-4}^4 [x+3] dx$ .
- 10) Show that  $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3} \log_e 2$
- 11) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sin 2x dx}{\sin^4 x + \cos^4 x} = \frac{\pi}{4}$
- 12) Prove that  $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{1}{ab} \tan^{-1} \left( \frac{a}{b} \right)$  where  $a, b > 0$
- 13) Evaluate :  $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$
- 14) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{x}{1+\sin x} dx$
- 15) Evaluate:  $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} x \cot x dx$ .
- 16) Prove that  $\int_0^{\frac{\pi}{2}} \log(1+\tan x) dx = \frac{\pi}{8} \log 2$ .
- 17) Show that  $\int_0^1 (\tan^{-1} x + \tan^{-1}(1-x)) dx = \frac{\pi}{2} - \log_e 2$
- 18) Evaluate  $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$ .
- 19) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$
- 20) Find the volume of a sphere of radius a.
- 21) Find the volume of a right-circular cone of base radius r and height h.
- 22) Find the volume of the spherical cap of height h cut of from a sphere of radius r.
- 23) Find the volume of the solid formed by revolving the region bounded by the parabola  $y = x^2$ , x-axis, ordinates  $x = 0$  and  $x = 1$  about the x-axis.
- 24) Find the volume of the solid formed by revolving the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$  about the major axis.
- 25) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded by the curves  $y = \log x$ ,  $y = 0$ ,  $x = 0$  and  $y = 2$ .



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- 1) Find an approximate value of  $\int_1^{1.5} x^2 dx$  by applying the right-end rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .
- 2) Evaluate  $\int_0^1 x dx$ , as the limit of a sum.
- 3) Evaluate :  $\int_0^1 \frac{2x+7}{5x^2+9} dx$
- 4) Evaluate :  $\int_0^9 \frac{1}{x+\sqrt{x}} dx$
- 5) Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{(1+\sin\theta)(2+\sin\theta)} d\theta$
- 6) Show that  $\int_0^\pi g(\sin x) dx = 2 \int_0^{\frac{\pi}{2}} g(\sin x) dx$ , where  $g(\sin x)$  is a function of  $\sin x$ .
- 7) If  $f(x) = f(a+x)$ , then  $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$
- 8) Evaluate:  $\int_0^a \frac{fx}{fx+f(a-x)} dx$ .
- 9) Evaluate the following definite integrals:  
 $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$
- 10) Evaluate the following definite integrals:  
 $\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta} \sin^3\theta d\theta$
- 11) Evaluate the following integrals using properties of integration:  
 $\int_0^{2\pi} x \log\left(\frac{3+\cos x}{3-\cos x}\right) dx$
- 12) Evaluate  $\int_0^1 e^{-2x}(1+x-2x^3) dx$
- 13) Evaluate:  $\int_{-1}^1 e^{-\lambda x}(1-x^2) dx$
- 14) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$
- 15) Evaluate  $\int_0^{\frac{\pi}{\sqrt{2}}} \frac{dx}{5+4\sin^2 x}$
- 16) Evaluate  $\int_0^1 x^5(1-x^2)^5 dx$
- 17) Evaluate the following  
 $\int_0^{\pi/4} \sin^6 x dx$
- 18) Evaluate the following  
 $\int_0^{2\pi} \sin^7 \frac{x}{4} dx$
- 19) Evaluate the following  
 $\int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^5 \theta d\theta$
- 20) Evaluate  $\int_0^\infty e^{-ax} x^n dx$ , where  $a > 0$ .
- 21) If  $\int_0^\infty e^{-ax^2} x^3 dx = 32$ ,  $a > 0$ , find  $a$

- 22) Find the area of the region bounded by the line  $7x - 5y = 35$ , x-axis and the lines  $x = -2$  and  $x = 3$ .
- 23) Find the area of the region bounded between the parabola  $y^2 = 4ax$  and its latus rectum
- 24) Find the area of the region bounded by x-axis, the curve  $y = \cos x$ , the lines  $x = 0$  and  $x = \pi$ .
- 25) Find the volume of a right-circular cone of base radius  $r$  and height  $h$ .
- 26) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded between the curve  $y = \frac{3}{4}\sqrt{x^2 - 16}$ ,  $x \geq 4$  the y-axis, and the lines  $y = 1$  and  $y = 6$ .
- 27) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded by the curves  $y = \log x$ ,  $y = 0$ ,  $x = 0$  and  $y = 2$ .
- 28) Evaluate  $\int_0^1 x(1 - x)^{10} dx$
- 29) Find the area bounded by  $x=0, x=6+5y-y^2$
- 30) Evaluate  $\int_0^{\frac{\pi}{2}} \cos^{10} x dx$

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Exam Time : 02:00:00 Hrs

Total Marks : 100

20 x 5 = 100

- 1) Evaluate  $\int_1^4 (2x^2 - 3) dx$ , as the limit of a sum
- 2) Evaluate  $\int_0^x x^2 \cos nx dx$ , where n is a positive integer.
- 3) Evaluate  $\int_0^1 e^{-2x}(1+x-2x^3)dx$
- 4) Evaluate:  $\int_{-1}^1 e^{-\lambda x}(1-x^2)dx$
- 5) Evaluate  $\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} (\sin^2 x + \cos^4 x) dx$
- 6) Find the values of the following:  
 $\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \sin^5 x \cos^4 x dx$
- 7) Evaluate  $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$
- 8) Evaluate  $\int_0^1 x^3(1-x)^4 dx$
- 9) Find the area of the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- 10) Find the area of the region bounded between the parabola  $y^2 = ax = 4$  and its latus rectum
- 11) Find the area of the region bounded by the y-axis and the parabola  $x = 5 - 4y - y^2$ .
- 12) Find the area of the region bounded by x-axis, the curve  $y = \cos x$ , the lines  $x = 0$  and  $x = \pi$ .
- 13) Find the area of the region bounded between the parabolas  $y^2 = x = 4$  and  $x^2 = y = 4$ .
- 14) Find the area of the region bounded between the parabola  $x^2 = y$  and the curve  $y = x$ .
- 15) Find the area of the region bounded by  $y = \cos x$ ,  $y = \sin x$ , the lines  $x = \frac{\pi}{4}$  and  $x = \frac{5\pi}{4}$ .
- 16) The region enclosed by the circle  $x^2 + y^2 = a^2$  is divided into two segments by the line  $x = h$ . Find the area of the smaller segment.
- 17) Find the area of the region in the first quadrant bounded by the parabola  $y^2 = x = 4$ , the line  $x + y = 3$  and y-axis
- 18) Find, by integration, the area of the region bounded by the lines  $5x - 2y = 15$ ,  $x + y + 4 = 0$  and the x-axis
- 19) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are  $(-1, 1)$ ,  $(3, 2)$ , and  $(0, 5)$  respectively
- 20) Using integration, find the area of the region which is bounded by x-axis, the tangent and normal to the circle  $x^2 + y^2 = 4$  drawn at  $(1, \sqrt{3})$

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Exam Time : 02:30:00 Hrs

Total Marks : 100

20 x 5 = 100

- 1) Evaluate  $\int_1^4 (2x^2 - 3) dx$ , as the limit of a sum
- 2) Evaluate:  $\int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x}) dx$
- 3) Show that  $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3} \log_e 2$
- 4) Prove that  $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{1}{ab} \tan^{-1} \left( \frac{a}{b} \right)$  where  $a, b > 0$
- 5) Evaluate  $\int_0^{\pi} \frac{x}{1+\sin x} dx$
- 6) Show that  $\int_0^1 (\tan^{-1} x + \tan^{-1}(1-x)) dx = \frac{\pi}{2} - \log_e 2$
- 7) Evaluate the following integrals using properties of integration:  
 $\int_0^{2\pi} \sin^4 x \cos^3 x dx$
- 8) Evaluate the following integrals using properties of integration:  
 $\int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt + \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$
- 9) Evaluate the following integrals using properties of integration:  
 $\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$
- 10) Evaluate the following integrals using properties of integration:  
 $\int_0^{\pi} x [\sin^2(\sin x) + \cos^2(\cos x)] dx$
- 11) Evaluate the following:  
 $\int_0^1 \frac{\sin(3\tan^{-1} x) \tan^{-1} x}{1+x^2} dx$
- 12) Find the area of the region bounded by  $y = \cos x$ ,  $y = \sin x$ , the lines  $x = \frac{\pi}{4}$  and  $x = \frac{5\pi}{4}$ .
- 13) Find, by integration, the area of the region bounded by the lines  $5x - 2y = 15$ ,  $x + y + 4 = 0$  and the x-axis
- 14) Using integration, find the area of the region which is bounded by x-axis, the tangent and normal to the circle  $x^2 + y^2 = 4$  drawn at  $(1, \sqrt{3})$
- 15) Find the area of the region bounded between the curves  $y = \sin x$  and  $y = \cos x$  and the lines  $x = 0$  and  $x = \pi$
- 16) Father of a family wishes to divide his square field bounded by  $x = 0$ ,  $x = 4$ ,  $y = 4$  and  $y = 0$  along the curve  $y^2 = x$  and  $x^2 = y$  into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.
- 17) Find the area of the region common to the circle  $x^2 + y^2 = 16$  and the parabola  $y^2 = 6x$
- 18) Find the area of the loop of the curve  $3ay^2 = x(x-a)^2$
- 19) Find the area of the region bounded by  $a^2 y^2 = a^2(a^2 - x^2)$
- 20) Find the area of the region enclosed by the two circles  $x^2 + y^2 = 1$  and  $(x-1)^2 + y^2 = 1$ .

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Exam Time : 02:30:00 Hrs

Total Marks : 90

20 x 1 = 20

1)

The value of  $\int_{-4}^4 \left[ \tan^{-1} \left( \frac{x^2}{x^4+1} \right) + \tan^{-1} \left( \frac{x^4+1}{x^2} \right) \right] dx$  is

- (a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$

2)

The value of  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left( \frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x} \right) dx$  is

- (a) 4 (b) 3 (c) 2 (d) 0

3)

If  $f(x) = \int_0^x t \cos t \, dt$ , then  $\frac{dx}{dx}$

- (a)  $\cos x - x \sin x$  (b)  $\sin x + x \cos x$  (c)  $x \cos x$  (d)  $x \sin x$

4)

The area between  $y^2 = x$  and its latus rectum is

- (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{5}{3}$

5)

The value of  $\int_0^1 x(1-x)^{99} dx$  is

- (a)  $\frac{1}{11000}$  (b)  $\frac{1}{10100}$  (c)  $\frac{1}{10010}$  (d)  $\frac{1}{10001}$

6)

The value of  $\int_0^{\pi} \frac{dx}{1+5\cos x}$  is

- (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c)  $\frac{3\pi}{2}$  (d)  $2\pi$

7)

The value of  $\frac{(n+2)}{(n)} = 90$  then n is

- (a) 10 (b) 5 (c) 8 (d) 9

8)

The value of  $\int_0^{\pi} \cos^3 x dx$

- (a)  $\frac{2}{3}$  (b)  $\frac{2}{9}$  (c)  $\frac{1}{9}$  (d)  $\frac{1}{3}$

9)

The value of  $\int_0^{\pi} \sin^4 x dx$  is

- (a)  $\frac{3\pi}{10}$  (b)  $\frac{3\pi}{8}$  (c)  $\frac{3\pi}{4}$  (d)  $\frac{3\pi}{2}$

10)

The value of  $\int_0^{\infty} e^{-3x} x^2 dx$  is

- (a)  $\frac{7}{27}$  (b)  $\frac{5}{27}$  (c)  $\frac{4}{27}$  (d)  $\frac{2}{27}$

11)

If  $\int_a^{\infty} \frac{1}{4+x^2} dx = \frac{\pi}{8}$  then a is

- (a) 4 (b) 1 (c) 3 (d) 2

12)

The volume of solid of revolution of the region bounded by  $y^2 = x(a-x)$  about x-axis is

- (a)  $\pi a^2$  (b)  $\frac{\pi a^2}{4}$  (c)  $\frac{\pi a^2}{5}$  (d)  $\frac{\pi a^2}{6}$

13)

If  $f(x)f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$  and  $\int_1^3 \frac{e^{\sin x}}{x} dx = \frac{1}{2}[f(a) - f(1)]$ , then one of the possible value of a is

- (a) 3 (b) 6 (c) 9 (d) 5

14)

The value of  $\int_0^1 (\sin^{-1} x)^2 dx$

- (a)  $\frac{\pi^2}{4} - 1$  (b)  $\frac{\pi^2}{4} + 2$  (c)  $\frac{\pi^2}{4} + 1$  (d)  $\frac{\pi^2}{4} - 2$

15)

The value of  $\int_0^a (\sqrt{a^2 - x^2})^2 dx$

- (a)  $\frac{\pi a^2}{16}$  (b)  $\frac{3\pi a^4}{16}$  (c)  $\frac{3\pi a^2}{8}$  (d)  $\frac{3\pi a^4}{8}$

16) If  $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$  then the value of  $f(1)$  is

- (a)  $\frac{1}{2}$  (b) 2 (c) 1 (d)  $\frac{3}{4}$

17) The value of  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sqrt{4-9x^2}}$  is

- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d)  $\pi$

18) The value of  $\int_{-1}^2 |x| dx$

- (a)  $\frac{1}{2}$  (b)  $\frac{3}{2}$  (c)  $\frac{5}{2}$  (d)  $\frac{7}{2}$

19) For any value of  $n \in \mathbb{Z}$ ,  $\int_0^\pi \cos^{2n} x \cos^3[(2n+1)x] dx$  is

- (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c) 0 (d) 2

20) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x dx$  is

- (a)  $\frac{3}{2}$  (b)  $\frac{1}{2}$  (c) 0 (d)  $\frac{2}{3}$

any 7

7 x 2 = 14

21) Find an approximate value of  $\int_1^{1.5} x^2 dx$  by applying the right-end rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .

22) Evaluate the following integrals as the limits of sums.

$$\int_1^2 (4x^2 - 1) dx$$

23) Evaluate the following definite integrals:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} e^x \left( \frac{1 + \sin x}{1 + \cos x} \right) dx$$

24) Evaluate the following integrals using properties of integration:

$$\int_0^{2\pi} x \log \left( \frac{3 + \cos x}{3 - \cos x} \right) dx$$

25) Evaluate the following integrals using properties of integration:

$$\int_0^1 |5x - 3| dx$$

26) Evaluate the following integrals using properties of integration:

$$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1 + \sqrt{\tan x}} dx$$

27) Evaluate the following:

$$\int_{\frac{1}{e}}^1 \frac{e^a \sin^{-1} x \sin^{-1} x}{\sqrt{1-x^2}} dx$$

28) Evaluate the following

$$\int_0^{\pi/4} \sin^6 x dx$$

29) Find the area of the region bounded by  $3x - 2y + 6 = 0$ ,  $x = -3$ ,  $x = 1$  and x-axis.

30) Find the area of the region bounded by the line  $y = 2x + 5$  and the parabola  $y = x^2 - 2x$ .

any 7

7 x 3 = 21

31) Evaluate:  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec x \tan x}{1 + \sec^2 x} dx$

32) Evaluate:  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos \theta}{(1 + \sin \theta)(2 + \sin \theta)} d\theta$



- 33) Show that  $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3} \log_e 2$
- 34) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sin 2x dx}{\sin^4 x + \cos^4 x} = \frac{\pi}{4}$
- 35) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{x}{1+\sin x} dx$
- 36) Prove that  $\int_0^{\frac{\pi}{2}} \log(1+\tan x) dx = \frac{\pi}{8} \log 2$ .
- 37) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$
- 38) Find the volume of a right-circular cone of base radius r and height h.
- 39) Find the volume of the solid formed by revolving the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$  about the major axis.
- 40) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded between the curve  $y = \frac{3}{4}\sqrt{x^2 - 16}$ ,  $x \geq 4$  the y-axis, and the lines  $y=1$  and  $y=6$ .
- any 7 7 x 5 = 35
- 41) Evaluate  $\int_1^4 (2x^2 - 3) dx$ , as the limit of a sum
- 42) Evaluate  $\int_0^1 e^{-2x}(1+x-2x^3) dx$
- 43) Evaluate  $\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^4 x) dx$
- 44) Evaluate  $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$
- 45) Find the area of the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- 46) Find the area of the region bounded by x-axis, the curve  $y = \cos x$ , the lines  $x = 0$  and  $x = \pi$ .
- 47) Find the area of the region bounded between the parabolas  $y^2 x = 4$  and  $x^2 y = 4$ .
- 48) Find the area of the region in the first quadrant bounded by the parabola  $y^2 x = 4$ , the line  $x+y=3$  and y-axis
- 49) Find, by integration, the area of the region bounded by the lines  $5x - 2y = 15$ ,  $x + y + 4 = 0$  and the x-axis
- 50) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are  $(-1,1)$ ,  $(3, 2)$ , and  $(0,5)$  respectively

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## Applications of Integration full test

12th Standard

Maths

Reg.No. : 

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Exam Time : 03:00:00 Hrs

Total Marks : 90

ANSWER ALL

20 x 1 = 20

- 1) The value of  $\int_{-4}^4 \left[ \tan^{-1} \left( \frac{x^2}{x^4+1} \right) + \tan^{-1} \left( \frac{x^4+1}{x^2} \right) \right] dx$  is  
 (a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$
- 2) The value of  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left( \frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x} \right) dx$  is  
 (a) 4 (b) 3 (c) 2 (d) 0
- 3) If  $f(x) = \int_0^x t \cos t \, dt$ , then  $\frac{dx}{dx}$   
 (a)  $\cos x - x \sin x$  (b)  $\sin x + x \cos x$  (c)  $x \cos x$  (d)  $x \sin x$
- 4) The area between  $y^2 x = 4$  and its latus rectum is  
 (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{5}{3}$
- 5) The value of  $\int_0^1 x(1-x)^{99} dx$  is  
 (a)  $\frac{1}{11000}$  (b)  $\frac{1}{10100}$  (c)  $\frac{1}{10010}$  (d)  $\frac{1}{10001}$
- 6) The value of  $\int_0^\pi \frac{dx}{1+5^{\cos x}}$  is  
 (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c)  $\frac{3\pi}{2}$  (d)  $2\pi$
- 7) The value of  $\frac{(n+2)}{(n)} = 90$  then n is  
 (a) 10 (b) 5 (c) 8 (d) 9
- 8) The value of  $\int_0^{\frac{\pi}{6}} \cos^3 3x dx$   
 (a)  $\frac{2}{3}$  (b)  $\frac{2}{9}$  (c)  $\frac{1}{9}$  (d)  $\frac{1}{3}$
- 9) The value of  $\int_0^\pi \sin^4 x dx$  is  
 (a)  $\frac{3\pi}{10}$  (b)  $\frac{3\pi}{8}$  (c)  $\frac{3\pi}{4}$  (d)  $\frac{3\pi}{2}$
- 10) The value of  $\int_0^\infty e^{-3x} x^2 dx$  is  
 (a)  $\frac{7}{27}$  (b)  $\frac{5}{27}$  (c)  $\frac{4}{27}$  (d)  $\frac{2}{27}$
- 11) If  $\int_a^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$  then a is  
 (a) 4 (b) 1 (c) 3 (d) 2
- 12) The volume of solid of revolution of the region bounded by  $y^2 = x(a-x)$  about x-axis is  
 (a)  $\pi a^2$  (b)  $\frac{\pi a^2}{4}$  (c)  $\frac{\pi a^2}{5}$  (d)  $\frac{\pi a^2}{6}$
- 13) If  $f(x)f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$  and  $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$ , then one of the possible value of a is  
 (a) 3 (b) 6 (c) 9 (d) 5
- 14) The value of  $\int_0^1 (\sin^{-1} x)^2 dx$   
 (a)  $\frac{\pi^2}{4} - 1$  (b)  $\frac{\pi^2}{4} + 2$  (c)  $\frac{\pi^2}{4} + 1$  (d)  $\frac{\pi^2}{4} - 2$
- 15) The value of  $\int_0^a (\sqrt{a^2 - x^2})^2 dx$   
 (a)  $\frac{\pi a^2}{16}$  (b)  $\frac{3\pi a^4}{16}$  (c)  $\frac{3\pi a^2}{8}$  (d)  $\frac{3\pi a^4}{8}$
- 16) If  $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$  then the value of f(1) is

- (a) (b) 2 (c) 1 (d)  $\frac{3}{4}$
- 17) The value of  $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$  is  $\frac{3}{4}$
- (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d)  $\pi$
- 18) The value of  $\int_{-1}^2 |x|dx$  is  $\frac{3}{2}$
- (a)  $\frac{1}{2}$  (b)  $\frac{3}{2}$  (c)  $\frac{5}{2}$  (d)  $\frac{7}{2}$
- 19) For any value of  $n \in \mathbb{Z}$ ,  $\int_0^\pi \cos^{2n} x \cos^3[(2n+1)x]$  is 0
- (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c) 0 (d) 2
- 20) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x dx$  is  $\frac{2}{3}$
- (a)  $\frac{3}{2}$  (b)  $\frac{1}{2}$  (c) 0 (d)  $\frac{2}{3}$

ANSWER ANY 7

7 x 2 = 14

21) Find an approximate value of  $\int_1^{1.5} x^2 dx$  by applying the right-end rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .

22) Evaluate the following integrals as the limits of sums.

$$\int_1^2 4x^2 - 1) dx$$

23) Evaluate the following definite integrals:

$$\int_0^{\frac{\pi}{2}} e^x \left( \frac{1+\sin x}{1+\cos x} \right) dx$$

24) Evaluate the following integrals using properties of integration:

$$\int_0^{2\pi} x \log \left( \frac{3+\cos x}{3-\cos x} \right) dx$$

25) Evaluate the following integrals using properties of integration:

$$\int_0^1 |5x - 3| dx$$

26) Evaluate the following integrals using properties of integration:

$$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}} dx$$

27) Evaluate the following:

$$\int_0^{\frac{1}{2}} \frac{e^a \sin^{-1} x \sin^{-1} x}{\sqrt{1-x^2}} dx$$

28) Evaluate the following

$$\int_0^{\pi/4} \sin^6 x dx$$

29) Find the area of the region bounded by  $3x - 2y + 6 = 0$ ,  $x = -3$ ,  $x = 1$  and x-axis.

30) Find the area of the region bounded by the line  $y = 2x + 5$  and the parabola  $y = x^2 - 2x$ .

ANSWER ANY 7

7 x 3 = 21

31) Evaluate:  $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x} dx$

32) Evaluate:  $\int_0^{\frac{\pi}{2}} \frac{\cos \theta}{(1+\sin \theta)(2+\sin \theta)} d\theta$

33) Show that  $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3} \log_e 2$

34) Prove that  $\int_0^{\frac{\pi}{4}} \frac{\sin 2x dx}{\sin^4 x + \cos^4 x} = \frac{\pi}{4}$

35) Evaluate  $\int_0^\pi \frac{x}{1+\sin x} dx$

36) Prove that  $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx = \frac{\pi}{8} \log 2$ .

- 37) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1+\sin^2 x}}$
- 38) Find the volume of a right-circular cone of base radius  $r$  and height  $h$ .
- 39) Find the volume of the solid formed by revolving the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$  about the major axis.

- 40) Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded between the curve  $y = \frac{3}{4}\sqrt{x^2 - 16}$ ,  $x \geq 4$  the y-axis, and the lines  $y = 1$  and  $y = 6$ .

ANSWER ANY 7

7 x 5 = 35

- 41) Evaluate  $\int_1^4 (2x^2 - 3) dx$ , as the limit of a sum
- 42) Evaluate  $\int_0^1 e^{-2x} (1 + x - 2x^3) dx$
- 43) Evaluate  $\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^4 x) dx$
- 44) Evaluate  $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx$
- 45) Find the area of the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- 46) Find the area of the region bounded by x-axis, the curve  $y = \cos x$ , the lines  $x = 0$  and  $x = \pi$ .
- 47) Find the area of the region bounded between the parabolas  $y^2 x = 4$  and  $x^2 y = 4$ .
- 48) Find the area of the region in the first quadrant bounded by the parabola  $y^2 x = 4$ , the line  $x + y = 3$  and y-axis
- 49) Find, by integration, the area of the region bounded by the lines  $5x - 2y = 15$ ,  $x + y + 4 = 0$  and the x-axis
- 50) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are  $(-1, 1)$ ,  $(3, 2)$ , and  $(0, 5)$  respectively

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## Applications of Integration

12th Standard

Maths

Exam Time : 02:30:00 Hrs

Total Marks : 100

20 x 1 = 20

- 1) The value of  $\int_{-4}^4 \left[ \tan^{-1} \left( \frac{x^2}{x^4+1} \right) + \tan^{-1} \left( \frac{x^4+1}{x^2} \right) \right] dx$  is  
 (a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$
- 2) If  $f(x) = \int_0^x t \cos t \, dt$ , then  $\frac{dx}{dx}$   
 (a)  $\cos x - x \sin x$  (b)  $\sin x + x \cos x$  (c)  $x \cos x$  (d)  $x \sin x$
- 3) The value of  $\int_0^1 x(1-x)^{99} dx$  is  
 (a)  $\frac{1}{11000}$  (b)  $\frac{1}{10100}$  (c)  $\frac{1}{10010}$  (d)  $\frac{1}{10001}$
- 4) The value of  $\frac{(n+2)}{(n)} = 90$  then n is  
 (a) 10 (b) 5 (c) 8 (d) 9
- 5) The value of  $\int_0^\pi \sin^4 x dx$  is  
 (a)  $\frac{3\pi}{10}$  (b)  $\frac{3\pi}{8}$  (c)  $\frac{3\pi}{4}$  (d)  $\frac{3\pi}{2}$
- 6) If  $\int_a^{\frac{1}{4+x^2}} dx = \frac{\pi}{8}$  then a is  
 (a) 4 (b) 1 (c) 3 (d) 2
- 7) If  $f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$  and  $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2} [f(a) - f(1)]$ , then one of the possible value of a is  
 (a) 3 (b) 6 (c) 9 (d) 5
- 8) The value of  $\int_0^a (\sqrt{a^2 - x^2})^2 dx$   
 (a)  $\frac{\pi a^2}{16}$  (b)  $\frac{3\pi a^4}{16}$  (c)  $\frac{3\pi a^2}{8}$  (d)  $\frac{3\pi a^4}{8}$
- 9) The value of  $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$  is  
 (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d)  $\pi$
- 10) For any value of  $n \in \mathbb{Z}$ ,  $\int_0^\pi e \cos^{2x} \cos^3[(2n+1)x] dx$  is  
 (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c) 0 (d) 2
- 11) The value of  $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$   
 (a)  $\pi$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d) 0
- 12)  $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$  is  
 (a)  $\frac{\pi}{3}$  (b)  $\frac{\pi}{6}$  (c)  $\frac{\pi}{12}$  (d)  $-\frac{\pi}{6}$
- 13) The value of  $\int_{-\pi}^\pi \sin^3 x \cos^3 x dx$  is  
 (a) 0 (b)  $\pi$  (c)  $2\pi$  (d)  $4\pi$
- 14) The area bounded by the parabola  $y = x^2$  and the line  $y = 2x$  is  
 (a)  $\frac{4}{3}$  (b)  $\frac{2}{3}$  (c)  $\frac{51}{3}$  (d)  $\frac{30}{3}$
- 15)  $\int_0^\infty e^{-mx} x^7 dx$  is  
 (a)  $\frac{7}{m^7}$  (b)  $\frac{7}{m^8}$  (c)  $\frac{7}{m^{m+1}}$  (d)  $\frac{7}{m^8}$

16) The area enclosed by the curve  $y^2 = 4x$ , the x-axis and its latus rectum is ..... sq.units.

- (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{16}{3}$

17) The volume generated by the curve  $y^2 = 16x$  from  $x = 2$  to  $x = 3$  rotating about x - axis ..... cu. units

- (a)  $72\pi$  (b)  $\frac{256 \times 19}{3} \pi$  (c)  $40\pi$  (d)  $80\pi$

18)  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x}{2 + \cos x} dx =$

- (a) 0 (b) 2 (c)  $\log 2$  (d)  $\log 4$

19)  $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx =$  .....

- (a)  $\frac{\pi}{2}$  (b) 0 (c)  $\frac{\pi}{4}$  (d)  $\pi$

20) The volume when  $y = \sqrt{3 + x^2}$  from  $x = 0$  to  $x = 4$  is rotated about x-axis is .....

- (a)  $100\pi$  (b)  $\frac{100\pi}{9}$  (c)  $\frac{100\pi}{3}$  (d)  $\frac{100}{3}$

$8 \times 2 = 16$

21) Evaluate the following integrals using properties of integration:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x dx$$

22) Find the values of the following:

$$\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x dx$$

23) Evaluate the following

$$\int_0^1 x^2(1-x)^3 dx$$

24) Find, by integration, the volume of the solid generated by revolving about the y-axis, the region enclosed by  $x^2 y = 1 +$  and  $y = 3$ .

25) Evaluate  $\int e^{3x} 3^{2x} 5^x dx$

26) Find the area enclosed between the parabola  $y^2 = 4ax$  and the line  $x = a$ ,  $x = 9a$ .

27) Find the area bounded by the curve  $y = \cos ax$  in one arc of the curve.

28) Find the area bounded by  $y = x^2 + 2$ , x-axis,  $x = 1$  and  $x = 2$ .

$8 \times 3 = 24$

29) Find an approximate value of  $\int_1^{1.5} x^2 dx$  by applying the right-end rule with the partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .

30) Evaluate :  $\int_0^1 \frac{2x+7}{5x^2+9} dx$

31) Evaluate  $\int_1^2 \frac{x}{(x+1)(x+2)} dx$

32) If  $f(x) = f(a+x)$ , then  $\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$

33) Evaluate  $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$ .

34) Evaluate the following definite integrals:

$$\int_0^1 \frac{1-x^2}{(1+x^2)^2} dx$$

35) Evaluate the following

$$\int_0^{2\pi} \sin^7 \frac{x}{4} dx$$

36) Find the area of the region bounded by  $3x - 2y + 6 = 0$ ,  $x = -3$ ,  $x = 1$  and x-axis.

$8 \times 5 = 40$

37) Evaluate  $\int_1^4 (2x^2 - 3) dx$ , as the limit of a sum

- 38) Evaluate:  $\int_{-4}^4 [x+3]dx$ .
- 39) Show that  $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3} \log_e 2$
- 40) Prove that  $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx = \frac{\pi}{8} \log 2$ .
- 41) Find the area of the region bounded by  $y = \cos x$ ,  $y = \sin x$ , the lines  $x = \frac{\pi}{4}$  and  $x = \frac{5\pi}{4}$ .
- 42) The region enclosed by the circle  $x^2 + y^2 = a^2$  is divided into two segments by the line  $x = h$ . Find the area of the smaller segment.
- 43) The curve  $y = (x - 2)^2 + 1$  has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ.
- 44) A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration.

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- 1) The value of  $\int_{-4}^4 \left[ \tan^{-1} \left( \frac{x^2}{x^4+1} \right) + \tan^{-1} \left( \frac{x^4+1}{x^2} \right) \right] dx$  is  
 (a)  $\pi$  (b)  $2\pi$  (c)  $3\pi$  (d)  $4\pi$
- 2) The value of  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left( \frac{2x^7-3x^5+7x^3-x+1}{\cos^2 x} \right) dx$  is  
 (a) 4 (b) 3 (c) 2 (d) 0
- 3) If  $f(x) = \int_0^x t \cos t \, dt$ , then  $\frac{dx}{dx}$   
 (a)  $\cos x - x \sin x$  (b)  $\sin x + x \cos x$  (c)  $x \cos x$  (d)  $x \sin x$
- 4) The area between  $y^2 x = 4$  and its latus rectum is  
 (a)  $\frac{2}{3}$  (b)  $\frac{4}{3}$  (c)  $\frac{8}{3}$  (d)  $\frac{5}{3}$
- 5) The value of  $\int_0^1 x(1-x)^{99} dx$  is  
 (a)  $\frac{1}{11000}$  (b)  $\frac{1}{10100}$  (c)  $\frac{1}{10010}$  (d)  $\frac{1}{10001}$
- 6) The value of  $\int_0^\pi \frac{dx}{1+5^{\cos x}}$  is  
 (a)  $\frac{\pi}{2}$  (b)  $\pi$  (c)  $\frac{3\pi}{2}$  (d)  $2\pi$
- 7) The value of  $\frac{(n+2)}{(n)} = 90$  then n is  
 (a) 10 (b) 5 (c) 8 (d) 9
- 8) The value of  $\int_0^{\frac{\pi}{6}} \cos^3 3x dx$   
 (a)  $\frac{2}{3}$  (b)  $\frac{2}{9}$  (c)  $\frac{1}{9}$  (d)  $\frac{1}{3}$
- 9) The value of  $\int_0^\pi \sin^4 x dx$  is  
 (a)  $\frac{3\pi}{10}$  (b)  $\frac{3\pi}{8}$  (c)  $\frac{3\pi}{4}$  (d)  $\frac{3\pi}{2}$
- 10) The value of  $\int_0^\infty e^{-3x} x^2 dx$  is  
 (a)  $\frac{7}{27}$  (b)  $\frac{5}{27}$  (c)  $\frac{4}{27}$  (d)  $\frac{2}{27}$
- 11) If  $\int_a^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$  then a is  
 (a) 4 (b) 1 (c) 3 (d) 2
- 12) The volume of solid of revolution of the region bounded by  $y^2 = x(a-x)$  about x-axis is  
 (a)  $\pi a^2$  (b)  $\frac{\pi a^2}{4}$  (c)  $\frac{\pi a^2}{5}$  (d)  $\frac{\pi a^2}{6}$
- 13) If  $f(x)f(x) = \int_1^x \frac{e^{\sin u}}{u} du, x > 1$  and  $\int_1^3 \frac{e^{\sin x^2}}{x} dx = \frac{1}{2}[f(a) - f(1)]$ , then one of the possible value of a is  
 (a) 3 (b) 6 (c) 9 (d) 5
- 14) The value of  $\int_0^1 (\sin^{-1} x)^2 dx$   
 (a)  $\frac{\pi^2}{4} - 1$  (b)  $\frac{\pi^2}{4} + 2$  (c)  $\frac{\pi^2}{4} + 1$  (d)  $\frac{\pi^2}{4} - 2$
- 15) The value of  $\int_0^a (\sqrt{a^2 - x^2})^2 dx$   
 (a)  $\frac{\pi a^2}{16}$  (b)  $\frac{3\pi a^4}{16}$  (c)  $\frac{3\pi a^2}{8}$  (d)  $\frac{3\pi a^4}{8}$
- 16) If  $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$  then the value of f(1) is  
 (a)  $\frac{1}{2}$  (b) 2 (c) 1 (d)  $\frac{3}{4}$
- 17) The value of  $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$  is  
 (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{2}$  (c)  $\frac{\pi}{4}$  (d)  $\pi$
- 18) The value of  $\int_{-1}^2 |x| dx$   
 (a)  $\frac{1}{2}$  (b)  $\frac{3}{2}$  (c)  $\frac{5}{2}$  (d)  $\frac{7}{2}$



19) For any value of  $n \in \mathbb{Z}$ ,  $\int_0^\pi \cos^{2n} x \cos^{2n+1} x \, dx$  is

(a)  $\frac{\pi}{2}$

(b)  $\pi$

(c) 0

(d) 2

20) The value of  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$  is

(a)  $\frac{3}{2}$

(b)  $\frac{1}{2}$

(c) 0

(d)  $\frac{2}{3}$

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