RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
WHATSAPP - 8056206308

Application of Matrices and Determinants T1

12th Standard

Maths

Exam Time : 01:15:00 Hrs — Total Marks : 50

5 x 1 = 5

1) If $|adj(adj\ A)| = |A|^9$, then the order of the square matrix A is
(a) 3 (b) 4 (c) 2 (d) 5

2) If A is a 3 × 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T =
(a) A (b) B (c) I (d) B^T

3) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|adjB|}{|C|}$ =
(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

4) If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =
(a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

5) If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then 9I - A =
(a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

5 x 2 = 10

6) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

7) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

8) Find the rank of the following matrices by minor method:
$\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

9) Solve the following system of homogenous equations.
2x + 3y − z = 0, x − y − 2z = 0, 3x + y + 3z = 0

10) Solve : 2x - y = 3, 5x + y = 4 using matrices.

5 x 3 = 15

11) Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

12) Verify the property $(A^T)^{-1} = (A^{-1})$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

13) Find adj(adj A) if adj $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

14) Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form.

15) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x}$ + 2y = 12, $\frac{2}{x}$ + 3y = 13

4 x 5 = 20

16) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$, verify thatA(adj A)=(adj A)A = |A| I_3.

17) Find the inverse of A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

18) The prices of three commodities A, B and C are Rs.x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B . Person R purchases one unit of A and sells 3 unit of B and one unit of C . In the process, P, Q and R earn Rs.15,000, Rs.1,000 and Rs.4,000 respectively. Find the prices per unit of A, B and C . (Use matrix inversion method to solve the problem.)

19) Investigate for what values of λ and μ the system of linear equations
x + 2y + z = 7, x + y + λz = μ, x + 3y − 5z = 5 has
(i) no solution
(ii) a unique solution
(iii) an infinite number of solutions

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RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82. WHATSAPP - 8056206308

Application of Matrices and Determinants T2

12th Standard

Maths

Exam Time : 01:30:00 Hrs — Total Marks : 60

10 x 1 = 10

1) If A = $\begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then |adj (AB)| =

(a) -40 (b) -80 (c) -60 (d) -20

2) If P = $\begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3 × 3 matrix A and |A| = 4, then x is

(a) 15 (b) 12 (c) 14 (d) 11

3) If A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and A^{-1} = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

(a) 0 (b) -2 (c) -3 (d) -1

4) If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) adj A = $|A|A^{-1}$ (b) adj(AB) = (adj A)(adj B) (c) det A^{-1} = $(\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

5) If $(AB)^{-1}$ = $\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and A^{-1} = $\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

6) If A^TA^{-1} is symmetric, then A^2 =

(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

7) If A is a non-singular matrix such that A^{-1} = $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1}$ =

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

8) If A = $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

(a) $\frac{-4}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

9) If A = $\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = I , then B =

(a) $\left(\cos^2\frac{\theta}{2}\right)A$ (b) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (c) $\left(\cos^2\theta\right)I$ (d) $(\mathrm{Sin}^2\frac{\theta}{2})A$

10) If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and A(adj A) = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ then adj (AB) is

(a) 0 (b) $\sin\theta$ (c) $\cos\theta$ (d) 1

5 x 2 = 10

11) If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

12) If A is symmetric, prove that then adj Ais also symmetric.

13) Find the rank of the following matrices by minor method:
$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$

14) For any 2 x 2 matrix, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then find $|A|$.

15) Solve 6x - 7y = 16, 9x - 5y = 35 using (Cramer's rule).

5 x 3 = 15

16) If A = $\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

17) If A = $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A(adj A) = $|A|I_2$.

18) Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

19) Solve the following system of linear equations by matrix inversion method:
2x + 5y = −2, x + 2y = −3

20) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer’s rule to solve the problem).

5 x 5 = 25

21) If A = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

22) If A = $\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c , and hence A^{-1}.

23) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

24) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and

the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (30, 18) can you conclude that Chennai Super Kings won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

25) Find the value of k for which the equations $kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have

(i) no solution

(ii) unique solution

(iii) infinitely many solution

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RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
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Application of Matrices and Determinants T3

Date : 01-Jan-70

12th Standard

Maths

Exam Time : 01:30:00 Hrs

10 x 1 = 10

1) If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that λA^{-1} =A, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

2) If adj A = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and adj B = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

(a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

3) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

4) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$ (c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

5) Which of the following is/are correct?
(i) Adjoint of a symmetric matrix is also a symmetric matrix.
(ii) Adjoint of a diagonal matrix is also a diagonal matrix.
(iii) If A is a square matrix of order n and λ is a scalar, then adj(λA) = λ^n adj(A).
(iv) A(adjA) = (adjA)A = |A| I

(a) Only (i) (b) (ii) and (iii) (c) (iii) and (iv) (d) (i), (ii) and (iv)

6) If ρ(A) = ρ([A | B]), then the system AX = B of linear equations is

(a) consistent and has a unique solution (b) consistent (c) consistent and has infinitely many solution (d) inconsistent

7) If 0 ≤ θ ≤ π and the system of equations x + (sinθ)y - (cosθ)z = 0, (cosθ)x - y +z = 0, (sinθ)x + y - z = 0 has a non-trivial solution then θ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

8) The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{bmatrix}$. The system has infinitely many solutions if

(a) λ = 7, μ ≠ -5 (b) λ = 7, μ = 5 (c) λ ≠ 7, μ ≠ -5 (d) λ = 7, μ = -5

9) Let A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and 4B = $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(a) 2 (b) 4 (c) 3 (d) 1

10) If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj(adj A) is

(a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

4 x 2 = 8

11) If A is a non-singular matrix of odd order, prove that |adj A| is positive

12) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

13) Find the rank of the following matrices which are in row-echelon form :

$$\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

14) Find k if the equations x + 2y + 2z = 0, x - 3y - 3z = 0, 2x + y + kz = 0 have only the trivial solution.

4 x 3 = 12

15) Find the matrix A for which A$\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

16) If A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

17) A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was Rs.19,800 per month at the end of the first month after 3 years of service and Rs.23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)

18) A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself ? (Use Cramer’s rule to solve the problem).

6 x 5 = 30

19) If A = $\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c , and hence A^{-1}.

20) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is Rs.200. The cost of five dosai, four idlies and two vadais is Rs.250. The family has Rs.350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

21) Investigate the values of λ and μ the system of linear equations 2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + λz = μ, have

(i) no solution

(ii) a unique solution
(iii) an infinite number of solutions.

22) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.)

23) Solve the following system of homogenous equations.
$3x + 2y + 7z = 0,\ 4x - 3y - 2z = 0{,}5x + 9y + 23z = 0$

24) Solve the following systems of linear equations by Cramer's rule:
$3x + 3y - z = 11,\ 2x - y + 2z = 9,\ 4x + 3y + 2z = 25.$

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RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
WHATSAPP - 8056206308

Application of Matrices and Determinants 2 MARKS TEST

12th Standard

Maths

Exam Time : 01:30:00 Hrs | Total Marks : 60

30 x 2 = 60

1) If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

2) If A is a non-singular matrix of odd order, prove that |adj A| is positive

3) Find a matrix A if adj(A) = $\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.

4) If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

5) If A is symmetric, prove that then adj Ais also symmetric.

6) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal

7) Find the adjoint of the following:

$\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

8) Find the inverse (if it exists) of the following:

$\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

9) If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

10) Reduce the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form.

11) Find the rank of the following matrices which are in row-echelon form :

$\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

12) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

13) Find the rank of the following matrices by minor method:

$\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$

14) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$$

15) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$$

16) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$$

17) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$$

18) Solve the following system of homogenous equations.
$2x + 3y - z = 0$, $x - y - 2z = 0$, $3x + y + 3z = 0$

19) Find the rank of each of the following matrices:

$$\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$$

20) Find the rank of the following matrices which are in row-echelon form :

$$\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

21) Find the rank of the following matrices which are in row-echelon form :

$$\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

22) For any 2 x 2 matrix, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then find |A|.

23) For the matrix A, if $A^3 = I$, then find A^{-1}.

24) If A is a square matrix such that $A^3 = I$, then prove that A is non-singular.

25) Show that the system of equations is inconsistent. $2x + 5y = 7$, $6x + 15y = 13$.

26) Flod the rank of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$.

27) Show that the equations $3x + y + 9z = 0$, $3x + 2y + 12z = 0$ and $2x + y + 7z = 0$ have nontrivial solutions also.

28) Find k if the equations $x + 2y + 2z = 0$, $x - 3y - 3z = 0$, $2x + y + kz = 0$ have only the trivial solution.

29) Solve : 2x - y = 3, 5x + y = 4 using matrices.
30) Solve 6x - 7y = 16, 9x - 5y = 35 using (Cramer's rule).

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Ravi home tutions

Application of Matrices and Determinants 3 MARKS

Date : 01-Jan-70

12th Standard

Maths

Exam Time : 01:30:00 Hrs

25 x 3 = 75

1) Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

2) Verify the property $(A^T)^{-1} = (A^{-1})$ with A = $\begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

3) If A = $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A(adj A) = $|A|I_2$.

4) If A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$

5) If adj(A) = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

6) Find adj(adj A) if adj A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

7) A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that A^TA^{-1} = $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

8) Find the matrix A for which A$\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

9) Given A = $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that AXB = C.

10) If A = $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

11) Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix} \begin{bmatrix} 20 & 4 \end{bmatrix}$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

12) Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form.

13) Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

14) Find the inverse of the non-singular matrix A = $\begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss-Jordan method.

15) Find the rank of the following matrices by row reduction method:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$$

16) Find the inverse of each of the following by Gauss-Jordan method:

$$\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$$

17) Solve the following system of linear equations, using matrix inversion method: 5x + 2y = 3, 3x + 2y = 5.

18) A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was Rs.19,800 per month at the end of the first month after 3 years of service and Rs.23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)

19) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

20) Solve the following systems of linear equations by Cramer’s rule:

$\frac{3}{x}$ + 2y = 12, $\frac{2}{x}$ + 3y = 13

21) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer’s rule to solve the problem).

22) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer’s rule to solve the problem).

23) A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself ? (Use Cramer’s rule to solve the problem).

24) Solve the system of linear equations, by Gaussian elimination method 4x + 3y + 6z = 25, x + 5y + 7z = 13, 2x + 9y + z = 1.

25) Test for consistency of the following system of linear equations and if possible solve: x - y + z = -9, 2x - 2y + 2z = -18, 3x - 3y + 3z + 27 = 0.

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Application of Matrices and Determinants 3 MARKS TEST

12th Standard

Maths

Exam Time : 01:30:00 Hrs | Total Marks : 50

17 x 3 = 51

1) Verify $(AB)^{-1} = B^{-1}A^{-1}$ with A = $\begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, B = $\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

2) If A = $\frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

3) Solve the following system of linear equations, using matrix inversion method:
5x + 2y = 3, 3x + 2y = 5.

4) Find the adjoint of the following:
$\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

5) Find the inverse (if it exists) of the following:
$\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

6) Find the rank of the following matrices by row reduction method:
$\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

7) Solve the following system of linear equations by matrix inversion method:
2x − y = 8, 3x + 2y = −2

8) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x}$ + 2y = 12, $\frac{2}{x}$ + 3y = 13

9) Solve: 2x + 3y = 10, x + 6y = 4 using Cramer's rule.

10) For what value of t will the system tx +3y - z = 1, x + 2y + z = 2, -tx + y + 2z = -1 fail to have unique solution?

11) Solve: 3x+ay =4, 2x+ay=2, a≠0 by Cramer's rule.

12) Verify $(AB)^{-1} = B^{-1} A^{-1}$ for A=$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and B=$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$.

13) Find,the rank of the matrix math $\begin{bmatrix} 4 & 4 & 0 & 3 \\ -2 & 3 & -1 & 5 \\ 1 & 4 & 8 & 7 \end{bmatrix}$.

14) Verify that $(A^{-1})^T = (A^T)^{-1}$ for A=$\begin{bmatrix} -2 & -3 \\ 5 & -6 \end{bmatrix}$.

15) Solve 2x - 3y = 7, 4x - 6y = 14 by Gaussian Jordan method.

16) Solve: x + y + 3z = 4, 2x + 2y + 6z = 7, 2x + y + z = 10.

17)

If the rank of the matrix $\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{bmatrix}$ is 2, then find λ.

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Application of Matrices and Determinants 5 MARKS TEST

Date : 01-Jan-70

12th Standard

Maths

Exam Time : 03:00:00 Hrs

25 x 5 = 125

1) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that A(adj A)=(adj A)A = |A| I_3.

2) If A = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O_2$. Hence, find A^{-1}.

3) If A = $\frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c , and hence A^{-1}.

4) If $F(\alpha)$ = $\begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$.

5) If A = $\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

6) Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

7) Find the inverse of A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

8) Solve the following system of equations, using matrix inversion method:
$2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$.

9) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

10) The prices of three commodities A, B and C are Rs.x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C . Person Q purchases 2 units of C and sells 3 units of A and one unit of B . Person R purchases one unit of A and sells 3 unit of B and one unit of C . In the process, P, Q and R earn Rs.15,000, Rs.1,000 and Rs.4,000 respectively. Find the prices per unit of A, B and C . (Use matrix inversion method to solve the problem.)

11) Solve, by Cramer’s rule, the system of equations
$x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$.

12) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is Rs.200. The cost of five dosai, four idlies and two vadais is Rs.250. The family has Rs.350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

13) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$, and $x - 1$, the remainders are 21,61 and 9 respectively. Find a,b and c. (Use Gaussian elimination method.)

14) An amount of Rs.65,000 is invested in three bonds at the rates of 6%,8% and 9% per annum respectively. The total annual income is Rs.4,800. The income from the third bond is Rs.600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

15) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6, 8)$,$(-2, -12)$, and $(3, 8)$. He wants to meet his friend at P(7,60) . Will he meet his friend? (Use Gaussian elimination method.)

16) Test for consistency of the following system of linear equations and if possible solve: $x + 2y - z = 3$, $3x - y + 2z = 1$, $x - 2y + 3z = 3$, $x - y + z + 1 = 0$

17) Test the consistency of the following system of linear equations $x - y + z = -9$, $2x - y + z = 4$, $3x - y + z = 6$, $4x - y + 2z = 7$.

18) Investigate for what values of λ and μ the system of linear equations $x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has
(i) no solution
(ii) a unique solution
(iii) an infinite number of solutions

19) Find the value of k for which the equations $kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have
(i) no solution
(ii) unique solution
(iii) infinitely many solution

20) Solve the system: $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$

21) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0$, $3x + (3\lambda - 8)y + 3z = 0$, $3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution.

22) If the system of equations $px + by + cz = 0$, $ax + qy + cz = 0$, $ax + by + rz = 0$ has a non-trivial solution and $p \neq a, q \neq b, r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

23) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x} - \frac{4}{y} - \frac{2}{z}$ - 0, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z}$ - 2 = 0, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z}$ + 1 = 0

24) Using determinants; find the quadratic defined by $f(x) = ax^2 + bx + c$, if $f(1) = 0$, $f(2) = -2$ and $f(3) = -6$.

25) For what value of λ, the system of equations $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ is consistent.

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RAVI MATHS TUITION CENTER, NEAR VILLIVAKKAM RLY STATION, CHENNAI – 82.
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Application of Matrices and Determinants 5 MARKS TEST

12th Standard

Maths

Exam Time : 02:30:00 Hrs | Total Marks : 100

20 x 5 = 100

1) If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the productsAB and BAand hence solve the system of equations x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.
2) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is y = ax^2 + bx + c with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (30, 18) can you conclude that Chennai Super Kings won the match?
   Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)
3) The upward speed v(t)of a rocket at time t is approximated by v(t) = at^2 + bt + c ≤ t ≤ 100 where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)
4) Solve the following systems of linear equations by Gaussian elimination method:
   (i) 2x − 2y + 3z = 2, x + 2y − z = 3,3x − y + 2z = 1
   (ii) 2x + 4y + 6z = 22, 3x + 8y + 5z = 27,−x + y + 2z = 2
5) Test for consistency of the following system of linear equations and if possible solve:
   4x − 2y + 6z = 8, x + y − 3z = −1, 15x − 3y + 9z = 21.
6) Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: x + y + z = a, x + 2y + 3z = b, 3x + 5y + 7z = c.
7) Test for consistency and if possible, solve the following systems of equations by rank method.
   i) x - y + 2z = 2, 2x + y + 4z = 7, 4x - y + z = 4
   ii) 3x + y + z = 2, x - 3y + 2z = 1, 7x - y + 4z = 5
   iii) 2x + 2y + z = 5, x - y + z = 1, 3x + y + 2z = 4
   iv) 2x - y + z = 2, 6x - 3y + 3z = 6, 4x - 2y + 2z = 4
8) Investigate the values of λ and μ the system of linear equations 2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + λz = μ, have
   (i) no solution
   (ii) a unique solution
   (iii) an infinite number of solutions.
9) Solve the system: x + y − 2z = 0, 2x − 3y + z = 0, 3x − 7y + 10z = 0, 6x − 9y + 10z = 0.
10) By using Gaussian elimination method, balance the chemical reaction equation:
   $C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.)

11) Solve the following system of homogenous equations.
$3x + 2y + 7z = 0$, $4x - 3y - 2z = 0$,$5x + 9y + 23z = 0$

12) Determine the values of λ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has
(i) a unique solution
(ii) a non-trivial solution

13) By using Gaussian elimination method, balance the chemical reaction equation:
$C_2 H_5 + O_2 \rightarrow H_2O + CO_2$

14) Find the inverse of each of the following by Gauss-Jordan method:

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

15) Solve the following system of linear equations by matrix inversion method:
$x + y + z - 2 = 0$, $6x - 4y + 5z - 31 = 0$, $5x + 2y + 2z = 13$.

16) Solve the following systems of linear equations by Cramer's rule:
$3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$.

17) Solve: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z}$ =2

18) The sum of three numbers is 20. If we multiply the third number by 2 and add the first number to the result we get 23. By adding second and third numbers to 3 times the first number we get 46. Find the numbers using Cramer's rule.

19) Show that the equations $-2x + y + z = a$, $x - 2y + z = b$, $x + y - 2z = c$ are consistent only if $a + b + c = 0$.

20) Using Gaussian Jordan method, find the values of λ and μ so that the system of equations $2x - 3y + 5z = 12$, $3x + y + \lambda z = \mu$, $x - 7y + 8z = 17$ has (i) unique solution (ii) infinite solutions and (iii) no solution.

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RAVI MATHS TUITION CENTER, GKM COLONY, CH-82. PH- 8056206308

MATRICES AND DETERMINANTS FULL TEST

12th Standard

Maths

Reg.No. : [ ][ ][ ][ ][ ][ ]

Exam Time : 03:00:00 Hrs — Total Marks : 90

ANSWER ALL — 20 X 1 = 20

1) If $|\text{adj}(\text{adj } A)| = |A|^9$, then the order of the square matrix A is

(a) 3 (b) 4 (c) 2 (d) 5

2) If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|\text{adj }(AB)| =$

(a) -40 (b) -80 (c) -60 (d) -20

3) If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

4) If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$

(a) $(\cos^2\frac{\theta}{2})A$ (b) $(\cos^2\frac{\theta}{2})A^T$ (c) $(\cos^2\theta)I$ (d) $(\text{Sin}^2\frac{\theta}{2})A$

5) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$ (c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

6) If A^TA^{-1} is symmetric, then $A^2 =$

(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^{-1})^2$

7) If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

8) If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj } A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$ then adj (AB) is

(a) 0 (b) sin θ (c) cos θ (d) 1

9) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(a) 17 (b) 14 (c) 19 (d) 21

10) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3

11) If $\rho(A) = \rho([A \mid B])$, then the system AX = B of linear equations is

(a) consistent and has a unique solution (b) consistent (c) consistent and has infinitely many solution (d) inconsistent

12) If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

(a) $\frac{2\pi}{}$ (b) $\frac{3\pi}{}$ (c) $\frac{5\pi}{}$ (d) $\frac{\pi}{}$

13) If A is a 3 × 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T =

(a) A (b) B (c) I (d) B^T

14) If A = $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|adjB|}{|C|}$ = 6 4

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

15) If A$\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =

(a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

16) If A = $\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then 9I - A =

(a) A^{-1} (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

17) If A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is

(a) 0 (b) -2 (c) -3 (d) -1

18) If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) adj A = $|A|A^{-1}$ (b) adj(AB) = (adj A)(adj B) (c) det A^{-1} = $(\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

19) If A = [2 0 1] then the rank of AA^T is ______

(a) 1 (b) 2 (c) 3 (d) 0

20) The number of solutions of the system of equations 2x+y = 4, x - 2y = 2, 3x + 5y = 6 is

(a) 0 (b) 1 (c) 2 (d) infinitely many

ANSWER ANY 7 7 X 2= 14

21) If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

22) A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^TA^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

23) Given A = $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that AXB = C.

24) Find the inverse of each of the following by Gauss-Jordan method:

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

25) Solve the following system of homogenous equations.

2x + 3y − z = 0, x − y − 2z = 0, 3x + y + 3z = 0

26) If A is a non-singular matrix of odd order, prove that |adj A| is positive

27) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$$

28) For any 2 x 2 matrix, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ then find |A|.

29)

Show that the system of equations is inconsistent. $2x + 5y = 7$, $6x + 15y = 13$.

30) Find k if the equations $x + 2y + 2z = 0$, $x - 3y - 3z = 0$, $2x + y + kz = 0$ have only the trivial solution.

ANSWER ANY 7 $7 \times 3 = 21$

31) Solve the following system of equations, using matrix inversion method:

$2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$.

32) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the productsAB and BAand hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

33) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c \leq t \leq 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

34) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0$, $3x + (3\lambda - 8)y + 3z = 0$, $3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution.

35) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \longrightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.)

36) Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

37) Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

38) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

39) If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, show that $A^{-1} = \frac{1}{2}(A^2 - 3I)$.

40) Decrypt the received encoded message $[2 \quad -3][20 \quad 4]$ with the encryption matrix $\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.

ANSWER ANY 7 $7 \times 5 = 35$

41) Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form.

42) Solve, by Cramer's rule, the system of equations

$x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$.

43) Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions: $x + y + z = a$, $x + 2y + 3z = b$, $3x + 5y + 7z = c$.

44) Investigate for what values of λ and μ the system of linear equations

$x + 2y + z = 7$, $x + y + \lambda z = \mu$, $x + 3y - 5z = 5$ has

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions

45) If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_3$.

46) Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

47) Find the value of k for which the equations kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1 have

(i) no solution

(ii) unique solution

(iii) infinitely many solution

48) Investigate the values of λ and m the system of linear equations 2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + λz = μ, have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

49) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (30, 18) can you conclude that Chennai Super Kings won the match?

Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

50) If the system of equations px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0 has a non-trivial solution and $p \neq a, q \neq b, r \neq c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

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